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NONLINEAR H_∞ CONTROL FOR CONTINUOUS-TIME RECURRENT NEURAL NETWORKS*

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Abstract

In this paper we investigate the nonlinear H_∞ control problem for recurrent neural network models connected to a recurrent neural network controller. Conditions for dissipativity with finite L_2-gain are derived and expressed as matrix inequalities, based on a two-hidden layer recurrent neural network in standard plant form. The matrix inequalities are obtained from a storage function of quadratic form or quadratic form plus integral terms. Narendra's dynamic backpropagation procedure for training on a set of specific reference inputs is modified with a dissipativity condition.

1 Introduction

In [12, 14] a modelbased neural control framework has been introduced with recurrent neural network models and controllers in state space form. The neural state space models are identified from I/O measurements on the plant and a linear or neural controller is designed, based upon the identified model. It has been shown how, in a discrete time framework, the closed-loop system forms can be represented as NL_q systems. In NL_q theory, sufficient conditions for global asymptotic stability, I/O stability with finite L_2-gain and robust performance have been derived. The I/O stability criteria, applied to the control scheme in standard plant form, correspond to nonlinear H_∞ control for discrete time multilayer recurrent neural networks. NL_q theory has been applied to the stabilization and control of nonlinear systems with a unique or multiple equilibria, periodic behaviour, quasi-periodic behaviour, unstable systems and chaos [12].

In this paper we derive conditions for the continuous time case. Recently there has been considerable interest in the H_∞ control problem for general nonlinear systems [1, 7, 8, 15, 16, 17, 21]. Here we want to study parametrizations of the nonlinear mappings by multilayer perceptrons in the model and the controller. It is well-known that multilayer perceptrons with one or more hidden layers are universal approximators in the sense that they can approximate any static continuous nonlinear function arbitrarily well on a compact interval [2, 6]. Hence the models and controllers stay general after parametrization by the multilayer perceptrons. On the other hand the layered structure of the resulting models and corresponding closed-loop schemes is exploited in order to derive matrix inequalities for dissipativity with finite L_2-gain. This is done for two specific choices of storage functions. Finally, this enables to modify Narendra's dynamic backpropagation [10] procedure with a dissipativity constraint.

This paper is organized as follows. In Section 2 we mention the standard H_∞ control problem for nonlinear systems. In Section 3 we discuss the H_∞ control problem for recurrent neural networks. In Section 4, matrix inequalities are derived for dissipativity with finite L_2-gain. Finally, in Section 5 Narendra's dynamic backpropagation is modified with a dissipativity constraint.

2 Standard H_∞ control problem for nonlinear systems

The standard H_∞ control problem is formulated with respect to general nonlinear systems in standard plant form

$$\begin{cases} \dot{p} &= f(p,u,w) \\ y &= g(p,u,w) \\ e &= h(p,u,w) \end{cases} \qquad (1)$$

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with state vector $p \in \mathbb{R}^{n_p}$, control input $u \in \mathbb{R}^m$, sensed output $y \in \mathbb{R}^l$, exogenous input $w \in \mathbb{R}^{n_w}$, regulated output $e \in \mathbb{R}^{n_e}$ and continuously differentiable nonlinear mappings $f(.)$, $g(.)$, $h(.)$. We assume $f(0,0,0) = 0$, $g(0,0,0) = 0$, $h(0,0,0) = 0$.

The nonlinear H_∞ suboptimal control problem with disturbance attenuation level γ is to find a nonlinear dynamic output feedback controller

$$\left\{ \begin{array}{rcl} \dot{z} & = & m(z,y) \\ u & = & n(z,y) \end{array} \right. \tag{2}$$

with state vector $z \in \mathbb{R}^{n_z}$ and $m(0,0) = 0$, $n(0,0) = 0$, such that the closed-loop system for (1)(2) has L_2-gain less or equal than γ, in the sense that

$$\int_0^T \|e(t)\|_2^2 \, dt \leq \gamma^2 \int_0^T \|w(t)\|_2^2 \, dt \tag{3}$$

for all functions $w(.) \in L_2[0,T]$ and all $T \geq 0$, where $e(.)$ corresponds to the response of the closed-loop system with initial condition $p(0) = 0$, $z(0) = 0$. The optimal H_∞ control problem consists of finding the smallest value γ that satisfies this property. The solution is characterized e.g. by means of the Hamilton-Jacobi inequality in a continuously differentiable storage function, following the theory of dissipative systems. Some references on the work on nonlinear H_∞ control are e.g. [1, 7, 8, 15, 16, 17, 21].

3 Nonlinear H_∞ control for recurrent neural networks

Let us consider the following class of nonlinear models:

$$\left\{ \begin{array}{rcl} \dot{x} & = & A_m x + B_m u + W_{AB} \sigma(V_A x + V_B u) + K\epsilon \\ y & = & C_m x + \epsilon \end{array} \right. \tag{4}$$

with input $u \in \mathbb{R}^m$, output $y \in \mathbb{R}^l$, state vector $x \in \mathbb{R}^n$ and interconnection matrices $W_{AB} \in \mathbb{R}^{n \times n_{h_x}}$, $V_A \in \mathbb{R}^{n_{h_x} \times n}$, $V_B \in \mathbb{R}^{n_{h_x} \times m}$ and $A_m \in \mathbb{R}^{n \times n}$, $B_m \in \mathbb{R}^{n \times m}$, $C_m \in \mathbb{R}^{l \times n}$. The model can be considered as the continuous time version of neural state space models [12, 13]. K is a steady state Kalman gain. The signal $\epsilon = y - \hat{y}$ is the prediction error. The parameters of the model (4) can be learned from I/O measurements on the system by considering (4) as a recurrent neural network and applying dynamic backpropagation [10], where the gradient of the cost function follows from a sensitivity model. Also, for certain systems the structure of the nonlinear differential equation is known a priori (e.g. from physical laws) and one might find an approximation (4) by representing the unknown nonlinearity of the system through the multilayer perceptron and train it in a static way.

Let us consider then the following nonlinear dynamic output feedback controller, parametrized by multilayer perceptrons:

$$\left\{ \begin{array}{rcl} \dot{z} & = & E_c z + F_c y + F_{c_2} d + W_{EF} \sigma(V_E z + V_F y + V_{F_2} d) \\ u & = & W_{GH} \sigma(V_G z + V_H y + V_{H_2} d) \end{array} \right. \tag{5}$$

with controller state $z \in \mathbb{R}^{n_z}$, reference input $d \in \mathbb{R}^l$ and $E_c \in \mathbb{R}^{n_z \times n_z}$, $F_c \in \mathbb{R}^{n_z \times l}$, $F_{c_2} \in \mathbb{R}^{n_z \times l}$, $W_{EF} \in \mathbb{R}^{n_z \times n_{h_z}}$, $V_E \in \mathbb{R}^{n_{h_z} \times n_z}$, $V_F \in \mathbb{R}^{n_{h_z} \times l}$, $V_{F_2} \in \mathbb{R}^{n_{h_z} \times l}$, $W_{GH} \in \mathbb{R}^{m \times n_{h_u}}$, $V_G \in \mathbb{R}^{n_{h_u} \times n_z}$, $V_H \in \mathbb{R}^{n_{h_u} \times l}$, $V_{H_2} \in \mathbb{R}^{n_{h_u} \times l}$ with n_{h_u}, n_{h_z} the number of hidden neurons in the hidden layers. Considering the closed-loop system of (4), connected to (5), one obtains the state equation:

$$\dot{p} = A_* p + B_* \sigma(N_* p + H_* \sigma(C_* p + D_{*_2} w) + D_{*_1} w) + D_{*_0} w, \tag{6}$$

with state vector $p = [x; z]$, exogenous input $w = [\epsilon; d]$ and

$$\begin{array}{c} A_* = \left[\begin{array}{cc} A_m & 0 \\ F_c C_m & E_c \end{array} \right], B_* = \left[\begin{array}{cc} W_{AB} & B_m W_{GH} \\ 0 & 0 \end{array} \right] \\ N_* = \left[\begin{array}{cc} V_A & 0 \\ V_H C_m & V_G \end{array} \right], H_* = \left[\begin{array}{cc} V_B W_{GH} & 0 \\ 0 & 0 \end{array} \right] \\ C_* = \left[\begin{array}{cc} V_H C_m & V_G \\ 0 & 0 \end{array} \right], D_{*_2} = \left[\begin{array}{cc} V_H & V_{H_2} \\ 0 & 0 \end{array} \right] \\ D_{*_1} = \left[\begin{array}{cc} 0 & 0 \\ V_H & V_{H_2} \end{array} \right], D_{*_0} = \left[\begin{array}{cc} K & 0 \\ V_F & V_{F_2} \end{array} \right]. \end{array} \tag{7}$$

This motivates the study of the following two-hidden layer recurrent neural network:

$$\left\{ \begin{array}{rcl} \dot{p} & = & Ap + B\sigma_1(Np + H\sigma_2(Cp + D_2 w) + D_1 w) + D_0 w \\ e & = & Ep + F\sigma_3(Mp + J\sigma_4(Gp + L_2 w) + L_1 w) + L_0 w \end{array} \right. \tag{8}$$

with external input $w \in \mathbb{R}^{n_w}$, output $e \in \mathbb{R}^{n_e}$ and state vector $p \in \mathbb{R}^{n_p}$. For the static nonlinearities one has the sector conditions $\sigma_1(.) \in [0, k_1]$, $\sigma_2(.) \in [0, k_2]$, $\sigma_3(.) \in [0, k_3]$, $\sigma_4(.) \in [0, k_4]$ [9, 19]. The corresponding number of hidden neurons are n_{h_i} $(i = 1, 2, 3, 4)$ with interconnection matrices of appropriate dimension.

Let us consider for the system (8) the supply rate

$$s(w,e) = \gamma^2 w^T w - e^T e \tag{9}$$

with finite L_2-gain γ. The system (8) with supply rate (9) is called then dissipative if for all locally square integrable w and all $t_f \geq 0$, one has

$$\int_0^{t_f} s(w(t), e(t)) \, dt \geq 0 \tag{10}$$

with $p(0) = 0$ and $s(w,e)$ evaluated along the trajectory of (8) [4, 5, 20]. The system (8) is dissipative with respect to the supply rate (9) if there exists a storage function $\phi(p) : \mathbb{R}^n \mapsto \mathbb{R}$ satisfying $\phi(p) > 0$ for $p \neq 0$ and $\phi(0) = 0$ and

$$\dot{\phi}(p) \leq s(w,e), \qquad \forall w \in \mathbb{R}^{n_w}, \forall e \in \mathbb{R}^{n_e}. \tag{11}$$

The nonlinear H_∞ control problem for recurrent neural networks, consists then of finding a neural controller (5), given the recurrent neural network model (4), such that condition (11) is satisfied, where the control problem is formulated in the standard plant framework (8).

4 Matrix inequalities for dissipativity with finite L_2-gain

In order to derive matrix inequalities as sufficient conditions for dissipativity with finite L_2-gain of the two-hidden layer recurrent neural network (8), we investigate two types of storage functions: quadratic form and quadratic form plus integral terms. The next Theorem is related to the storage function:

$$\phi(p) = p^T P p, \tag{12}$$

with $P = P^T > 0$.

Theorem 1. *Let $T = \text{diag}\{\tau_j\}$, $\Phi = \text{diag}\{\phi_i\}$, $\Lambda = \text{diag}\{\lambda_l\}$, $\Theta = \text{diag}\{\theta_k\}$ be diagonal matrices with $\tau_j > 0$, $\phi_i > 0$, $\lambda_l > 0$, $\theta_k > 0$ with $i = 1, ..., n_{h_1}$; $j = 1, ..., n_{h_2}$; $k = 1, ..., n_{h_3}$, $l = 1, ..., n_{h_4}$. Then if there exist $P = P^T > 0$ and diagonal matrices T, Φ, Λ, Θ such that the matrix inequality*

$$Y = Y^T = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} & Y_{15} & Y_{16} \\ . & Y_{22} & Y_{23} & Y_{24} & Y_{25} & Y_{26} \\ . & . & Y_{33} & Y_{34} & Y_{35} & Y_{36} \\ . & . & . & Y_{44} & Y_{45} & Y_{46} \\ . & . & . & . & Y_{55} & Y_{56} \\ . & . & . & . & . & Y_{66} \end{bmatrix} < 0 \tag{13}$$

with

$$\begin{aligned} Y_{11} &= A^T P + PA + E^T E \\ Y_{12} &= PB + k_1 N^T \Phi \\ Y_{13} &= k_2 C^T T \\ Y_{14} &= k_3 M^T \Theta + E^T F^T \\ Y_{15} &= k_4 G^T \Lambda \\ Y_{16} &= PD_0 + E^T L_0 \\ \\ Y_{22} &= -2\Phi \\ Y_{23} &= k_1 \Phi H \\ Y_{24} &= 0 \\ Y_{25} &= 0 \\ Y_{26} &= k_1 \Phi D_1 \\ \\ Y_{33} &= -2T \\ Y_{34} &= 0 \\ Y_{35} &= 0 \\ Y_{36} &= k_2 T D_2 \\ \\ Y_{44} &= -2\Theta + F^T F \\ Y_{45} &= k_3 \Theta J \\ Y_{46} &= k_3 \Theta L_1 + F^T L_0 \\ \\ Y_{55} &= -2\Lambda \\ Y_{56} &= k_4 \Lambda L_2 \\ \\ Y_{66} &= -\gamma^2 I + L_0^T L_0 \end{aligned}$$

is satisfied, the two-hidden layer recurrent neural network (8) is dissipative with respect to supply rate (9) and storage function (12).

Proof: Let us first define $z_2 = Cp + D_2 w$, $z_1 = Np + H\sigma_2(z_2) + D_1 w$, $z_4 = Gp + L_2 w$, $z_3 = Mp + J\sigma_4(z_4) + L_1 w$. Then we derive under what condition $\dot{\phi} - s(w, e) \leq 0$ holds. Using the $\mathcal{S}$-procedure [3] and the inequalities from the sector conditions of the nonlinearities one obtains:

$$\begin{aligned} &[Ap + B\sigma_1(z_1) + D_0 w]^T Pp + p^T P[Ap + B\sigma_1(z_1) + D_0 w] \\ &-2\sigma_2(z_2)^T T[\sigma_2(z_2) - k_2 Cp - k_2 D_2 w] \\ &-2\sigma_1(z_1)^T \Phi[\sigma_1(z_1) - k_1 Np - k_1 H\sigma_2(z_2) - k_1 D_1 w] \\ &-2\sigma_4(z_4)^T \Lambda[\sigma_4(z_4) - k_4 Gp - k_4 L_2 w] \\ &-2\sigma_3(z_3)^T \Theta[\sigma_3(z_3) - k_3 Mp - k_3 J\sigma(z_4) - k_3 L_1 w] \\ &+[Ep + F\sigma_3(z_3) + L_0 w]^T [Ep + F\sigma_3(z_3) + L_0 w] \\ &+\gamma^2 w^T w < 0. \end{aligned}$$

Defining $\xi = [p; \sigma_1(z_1); \sigma_2(z_2); \sigma_3(z_3); \sigma_4(z_4); w]$, one obtains the quadratic form

$$\xi^T Y \xi < 0,$$

which should be satisfied $\forall \xi$, yielding the matrix inequality $Y < 0$.

□

In order to be able to derive the next Theorem, we assume $D = D_0$, $D_1 = 0$, $D_2 = 0$, $L_i = 0$ $(i = 0, 1, 2)$ and $F = 0$ in (8):

$$\begin{cases} \dot{p} &= Ap + B\sigma_1(Np + H\sigma_2(Cp)) + Dw \\ e &= Ep. \end{cases} \tag{14}$$

We investigate the storage function

$$\begin{aligned} \phi(p) &= p^T Pp + \textstyle\sum_{i=1}^{n_{h_1}} 2\omega_i . \int_0^{z_i} \sigma_{1_i}(\rho) k_1 d\rho + \\ &\quad \textstyle\sum_{j=1}^{n_{h_2}} 2\gamma_j . \int_0^{e_j} \sigma_{2_j}(\rho) k_2 d\rho \end{aligned} \tag{15}$$

with $z = Np + H\sigma_2(Cp)$, $P = P^T > 0$ and $\omega_i > 0$, $\gamma_j > 0$ (note the resemblance with a Lur'e-Postnikov Lyapunov function).

Theorem 2. *Let $\Gamma = \text{diag}\{\gamma_j\}$, $\Omega = \text{diag}\{\omega_i\}$, $T = \text{diag}\{\tau_j\}$, $\Phi = \text{diag}\{\phi_i\}$ be diagonal matrices with $\gamma_j > 0$, $\omega_i > 0$, $\tau_j > 0$, $\phi_i > 0$ for $i = 1, ..., n_{h_1}$ and $j = 1, ..., n_{h_2}$ and consider a constant $\alpha > 0$. Assume the following condition on the slope of $\sigma_2(.)$:*

$$0 \leq \frac{d\sigma_{2_j}}{dy_j} \leq k_3, \; k_3 \geq 1, \; \forall j \tag{16}$$

in addition to the sector conditions $\sigma_1(.) \in [0, k_1]$, $\sigma_2(.) \in [0, k_2]$. *Furthermore, let*

$$Y = Y^T = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ . & Y_{22} & Y_{23} & Y_{34} \\ . & . & Y_{33} & Y_{34} \\ . & . & . & Y_{44} \end{bmatrix} \quad (17)$$

with

$$\begin{aligned} Y_{11} &= A^T P + PA + E^T E \\ Y_{12} &= PB + k_1 A^T N^T \Omega + k_1 N^T \Phi \\ Y_{13} &= k_2 A^T C^T \Gamma + k_2 C^T T \\ Y_{14} &= PD \\ \\ Y_{22} &= k_1 \Omega N B + k_1 B^T N_T \Omega \\ Y_{23} &= k_2 B^T C^T \Gamma + k_1 H^T \Phi \\ Y_{24} &= k_1 \Omega N D \\ \\ Y_{33} &= -2T - \alpha I \\ Y_{34} &= k_2 \Gamma C D \\ \\ Y_{44} &= -\gamma^2 I \end{aligned}$$

and

$$Z = \begin{bmatrix} A^T C^T & 0 & 0 & 0 \\ B^T C^T & \Omega H & 0 & 0 \\ 0 & 0 & \frac{1}{2}\sqrt{\alpha} I & \frac{1}{2}\sqrt{\alpha} I \\ D^T C^T & 0 & 0 & 0 \end{bmatrix},$$

where Z *is assumed to be a full rank matrix. Then, if there exist a* $P = P^T > 0$ *and diagonal matrices* Γ, Ω, T, Φ *such that the matrix inequality*

$$Y + k_1 k_3 \, Z Z^T < 0 \quad (18)$$

is satisfied, the two-hidden layer recurrent neural network (14) is dissipative with respect to supply rate (9) and storage function (15).

Proof: Let us first define $z_2 = Cp$, $z_1 = Np + H\sigma_2(z_2)$. We investigate then under what condition $\dot{\phi} - s(w, e) \leq 0$ holds. Using the $\mathcal{S}$-procedure [3] and the inequalities from the sector conditions of the nonlinearities one obtains:

$$\begin{aligned} &[Ap + B\sigma_1(z_1) + Dw]^T Pw + p^T P[Ap + B\sigma_1(z_1) + Dw] \\ &+2k_2\sigma_2(z_2)^T \Gamma C[Ap + B\sigma_1(z_1) + Dw] \\ &+2k_1\sigma_1(z_1)^T \Omega[NAp + NB\sigma_1(z_1) + NDw \\ &+H\tfrac{d\sigma_2}{dz_2} C(Ap + B\sigma_1(z_1) + Dw)] \\ &-2\sigma_2(z_2)^T T[\sigma_2(z_2) - k_2 Cp] \\ &-2\sigma_1(z_1)^T \Phi[\sigma_1(z_1) - k_1 Np - k_1 H\sigma_2(z_2)] \\ &-\gamma^2 w^T w + p^T E^T E p < 0. \end{aligned}$$

Defining $\xi = [p; \sigma_1(z_1); \sigma_2(z_2); w]$, one obtains the quadratic form:

$$\xi^T (Y + k_1 Z \Lambda Z^T) \xi < 0,$$

with

$$\Lambda = \begin{bmatrix} 0 & \frac{d\sigma_2}{dz_2} & 0 & \\ \frac{d\sigma_2}{dz_2} & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \leq k_3 I.$$

Multiplication from the left and right with the full rank matrix Z gives $Z\Lambda Z^T \leq k_3 Z Z^T$, finally yielding the matrix inequality (18).

□

5 Neural control design

After the derivation of the matrix inequalities (13) and (18), one can ask the question how to use these criteria for design of a recurrent neural network controller. One can use the criteria on itself to find a controller or one may modify Narendra's dynamic backpropagation with a dissipativity constraint (see also NL_q theory [12]).

The H_∞ optimal control problem with respect to the quadratic storage function (Theorem 1) is formulated as

$$\min_{\theta_c, P, T, \Phi, \Lambda, \Theta} \gamma \quad \text{s.t.} \quad Y(\theta_c, P, T, \Phi, \Lambda, \Theta, \gamma) < 0 \quad (19)$$

where θ_c denotes the controller parameter vector. One is seeking the minimal disturbance attenuation level γ then such that the matrix inequality is satisfied. In a similar way one can use the matrix inequality from Theorem 2.

A popular method in neural control is Narendra's dynamic backpropagation [10], which consists of training a neural controller on a set of specific reference inputs. In NL_q theory [12, 14] this has been modified with an internal or I/O stability constraint, in order to obtain closed-loop stability. A similar method can be applied here for continuous-time recurrent neural networks:

$$\min_{\theta_c, P, T, \Phi, \Lambda, \Theta} J(\theta_c) \quad \text{s.t.} \quad Y(\theta_c, P, T, \Phi, \Lambda, \Theta, \gamma^*) < 0. \quad (20)$$

Here γ^* is a fixed disturbance attenuation level and $J(\theta_c)$ is a tracking error, defined on one or a set of specific reference inputs. The gradient of the cost function in (20) follows from classical dynamic backpropagation with sensitivity model. On the other hand, the constraint might become non-differentiable when the two largest eigenvalues of Y coincide. Algorithms for non-convex non-differentiable optimization for this purpose have been described e.g. in [11]. A real-life working example of such an approach in NL_q theory, applied to a ball and beam system, has been reported in [18].

6 Conclusion

We considered the H_∞ control problem for control schemes consisting of continuous-time recurrent neural network models and controllers. Conditions for dissipativity with

finite L_2-gain of a two-hidden layer recurrent neural network in standard plant form have been derived. The conditions are expressed as matrix inequalities, based on a quadratic storage function or a quadratic storage function plus integral terms. Neural control design that is based on these matrix inequalities has been discussed, including a modification of Narendra's dynamic backpropagation procedure with a dissipativity constraint.

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