Subspace Identification of Bilinear Systems Subject to White Inputs

Wouter Favoreel, Bart De Moor, *Senior Member, IEEE*, and Peter Van Overschee

***Abstract*— In the present paper the authors generalize linear subspace identification theory to an analog theory for the subspace identification of bilinear systems. A major assumption they make is that the inputs of the system should be white and mutually independent. It is shown that in that case most of the properties of linear subspace identification theory can be extended to similar properties for bilinear systems. The link between the presented bilinear subspace method and Kalman filter theory is made. Finally, the practical relevance of the method is illustrated by making a direct comparison between linear and bilinear subspace identification methods when applied on data from a model of a distillation column.**

***Index Terms*—Bilinear systems, subspace identification.**

I. Introduction—Problem Description

THE IDENTIFICATION of nonlinear systems has been a long-standing topic of research. One class of nonlinear systems, bilinear systems, has been studied extensively. In the literature, numerous papers describe the use of bilinear systems in various fields of application such as engineering, economics, biology, ecology, etc. [1]–[5]. Besides the fact that bilinear systems are universal approximators among nonlinear systems with linear inputs [6], bilinear systems have shown to be natural models to approximate many dynamical processes. For example, in most chemical processes the controls are flow rates, and from first principles (mass and heat balances) we know that these will appear in the system equations as products with the state variables (which are typically temperatures or concentrations). The following continuous-time bilinear differential equation describes the mass balance of an arbitrary system. It expresses that the concentration x of a component in the system is a function of the incoming and outgoing flows q_i and q_o (the inputs) with their respective concentrations x, x_i, and x_o (the states)

$$M\frac{dx}{dt} = \sum_i q_i x_i - \sum_o q_o x_o.$$

This is a bilinear equation since there are products between the controlled variables, which are the flows q_i and q_o, and the states, i.e., the concentrations.

Important examples of industrial processes that can be well described by bilinear systems are CST-reactors [2] and distillation columns [7]. A bilinear model will therefore be a better approximation of reality than a linear model. This will be illustrated analytically and by means of simulation in Section IV through an example of a distillation column. The example is interesting since distillation is the most important chemical separation process in industry. In practice it is very important to have accurate models since in such processes enormous amounts of energy are consumed.

In this paper, we consider time-invariant multi-input/multi-input (MIMO) bilinear systems with the following state-space representation

$$\begin{aligned} x_{k+1} &= Ax_k + Nu_k \otimes x_k + Bu_k + w_k, \\ y_k &= Cx_k + Du_k + v_k \end{aligned} \tag{1}$$

with the state $x_k \in \mathbb{R}^n$, the input $u_k \in \mathbb{R}^m$, and the output $y_k \in \mathbb{R}^l$. The matrix N characterizing the bilinearity of the system is defined as $N = [N_1 \quad N_2 \quad \cdots \quad N_m] \in \mathbb{R}^{n \times nm}$, $N_i \in \mathbb{R}^{n \times n}$, $i = 1, \cdots, m$ and the Kronecker product of two arbitrary vectors $a \in \mathbb{R}^p$ and $b \in \mathbb{R}^q$ as $a \otimes b = [a_1 b^T \cdots a_p b^T]^T \in \mathbb{R}^{pq}$.

Assumption 1: The inputs u_k are assumed to be observed and white. Moreover, they should be mutually independent and independent of the measurement noise v_k and the process noise w_k.

The covariance matrix of these sequences is

$$E\left[\begin{pmatrix} u_p \\ w_p \\ v_p \end{pmatrix} \begin{pmatrix} u_q^T & w_q^T & v_q^T \end{pmatrix}\right] = \begin{bmatrix} I_m & 0 & 0 \\ 0 & Q & S \\ 0 & S^T & R \end{bmatrix} \delta_{pq} \geq 0. \tag{2}$$

The assumption that the input u_k should be white might at first seem somewhat restrictive. Nevertheless, the practical use of it is straightforward since in many cases, when one tries to identify a system, white noise is used because it has a rich spectrum and will therefore excite a large number of system modes.

Manuscript received November 15, 1996; revised October 9, 1997 and July 2, 1998. Recommended by Associate Editor, J. C. Spall. The work of W. Favoreel was supported by the IWT. This work was supported by the IT-program of the Flemish Institute for Scientific and Technological Research in Industry (IWT), the Flemish Government (Concerted Research Action: Model-Based Information Processing Systems), the Fund for Scientific Research Flanders (G.0292.95 Matrix Algorithms and Differential Geometry for Adaptive Signal Processing, System Identification, and Control), the FWO-Vlaanderen Onderzoeksgemeenschappen (Identification and Control of Complex Systems and Advanced Numerical Methods for Mathematical Modeling), the Belgian Government (Interuniversity Attraction Poles IUAP-17: Modeling and Control of Dynamical Systems, IUAP-50: Automation in Design and Production), the European Commission (Human Capital and Mobility Network: System Identification and Modeling Network), and the European Research Network for System Identification (SCIENCE-ERNSI).

The authors are with the Department of Electrical Engineering-ESAT/SISTA, National Fund for Scientific Research, Katholieke Universiteit Leuven, B-3001 Leuven, Belgium (e-mail: wouter.favoreel@esat.kuleuven.ac.be).

Publisher Item Identifier S 0018-9286(99)04527-4.

0018–9286/99$10.00 © 1999 IEEE

Authorized licensed use limited to: Katholieke Universiteit Leuven. Downloaded on October 14, 2008 at 06:54 from IEEE Xplore. Restrictions apply.

We are concerned with the following problem:

- **Given** measurements of the inputs u_k and the outputs y_k of the unknown bilinear system (1), (2), **determine** the system matrices A, B, C, D, and N as well as an estimate of the noise covariance matrices Q, R, and S.

As for linear systems there exist different system identification methods for bilinear systems in the literature. Some methods give an approximate solution by using orthogonal basis functions [8] while other methods, such as prediction error methods [9], [10], try to find the exact solution. Until now, no papers were presented in the literature that use subspace identification methods to identify bilinear systems. As will be explained below, subspace identification techniques present some important advantages over prediction error methods.

The results presented here can be labeled under subspace identification methods because a state-space representation of the unknown system is found by projecting the row spaces of matrices constructed from input and output data only. Subspace identification methods have their origins in various themes of system theory and linear algebra such as realization theory, Kalman filtering, signal processing, etc. We give the following references to the subspace identification literature [11]–[14]. One of the main reasons that made subspace identification so popular is that they have some very interesting advantages when compared to other system identification methods. For instance, they are very well suited for the identification of systems with multiple inputs and multiple outputs. They are noniterative (so no convergence problems arise), fast, and numerically robust (since only based on basic techniques of linear algebra). Especially in the field of process industry, the method is very popular. Many papers appeared where subspace identification was successfully applied [15]–[18]. However, most of the subspace identification methods are limited to the case where the system is linear. An extension has been proposed for Wiener systems [19], i.e., a linear system followed by a static nonlinearity. In this paper we propose a subspace method for the identification of bilinear systems with white mutually independent inputs.

The work in this paper is based on linear subspace identification theory [12], [14]. In [12] the results presented in this paper have been derived for linear systems. Here, these results are extended for bilinear systems with white independent inputs. We also used results from bilinear stochastic realization theory [20]. There the problem of finding a state-space representation of the bilinear system (1) and (2) is considered. It is assumed that the Volterra kernels of the system are known, which is not a very realistic assumption in practice. In the present paper this result is extended to the case where only input and output measurements of the system are known. Moreover, the link between bilinear subspace identification and Kalman filter theory is made. Some previous results can be found in [21] and [22].

The outline of the paper is as follows. In the next section, the notations used in the paper are presented. Section III contains the main results. First the Kalman filter equations for a bilinear system subject to white inputs are derived. This is needed later on where it is shown that the states estimated with the bilinear subspace methods are Kalman filter states of the unknown bilinear system. Further, some important results of bilinear stochastic realization theory [20] are extended to a more general class of bilinear systems, and finally, the new subspace algorithm for the identification of bilinear systems with white inputs is presented. Section IV illustrates the practical use of the algorithm by the hand of some examples.

II. Matrices used in Bilinear Subspace Identification

The following matrices play a crucial role in bilinear subspace identification: $\forall\, p > q \in \mathbb{N}$ and $j \gg p,\, q$ we define

$$\begin{aligned} X_p &= (x_p \quad x_{p+1} \quad \cdots \quad x_{p+j-1}) \\ U_p &= (u_p \quad u_{p+1} \quad \cdots \quad u_{p+j-1}) \\ Y_p &= (y_p \quad y_{p+1} \quad \cdots \quad y_{p+j-1}) \\ X_{p|q} &= \begin{pmatrix} X_{p-1|q} \\ U_p \odot X_{p-1|q} \end{pmatrix} \\ Y_{p|q} &= \begin{pmatrix} Y_p \\ Y_{p-1|q} \\ U_p \odot Y_{p-1|q} \end{pmatrix} \qquad (3) \\ Y_{q|p} &= \begin{pmatrix} Y_q \\ Y_{q+1|p} \\ U_q \odot Y_{q+1|p} \end{pmatrix} \qquad (4) \end{aligned}$$

with initial conditions $X_{p|p} = \binom{X_p}{U_p \odot X_p}$, $Y_{p|p} = Y_p$. The Khatri–Rao product of two matrices $F \in \mathbb{R}^{r\times j}$ and $G \in \mathbb{R}^{s\times j}$ is defined as the column-wise Kronecker product [23] $F \odot G = [f_1 \otimes g_1 \quad f_2 \otimes g_2 \quad \cdots \quad f_j \otimes g_j]$. The dimensions of the above-defined data matrices are determined by $d_i = \sum_{r=1}^{i} (m+1)^{r-1} l$. We have that $\{Y_{p|q},\, Y_{q|p}\} \in \mathbb{R}^{d_{p-q+1}\times j}$. We can also define $U_{p|q}$ and $U_{q|p}$ in a similar way, simply by replacing Y by U in the definitions of $Y_{p|q}$ and $Y_{q|p}$.

We are also interested in the first column of $Y_{p|q}$, $U_{p|q}$, $Y_{q|p}$, and $U_{q|p}$. For example, for $Y_{p|q}$ and $U_{p|q}$ we define

$$y_{p|q} = \begin{pmatrix} y_p \\ y_{p-1|q} \\ u_p \otimes y_{p-1|q} \end{pmatrix}, \qquad u_{p|q} = \begin{pmatrix} u_p \\ u_{p-1|q} \\ u_p \otimes u_{p-1|q} \end{pmatrix}$$

with initial conditions $y_{p|p} = y_p$ and $u_{p|p} = u_p$.

For statistical reasons, we assume throughout the whole paper that the data is ergodic and stationary. Moreover, the number of available data is supposed to be infinite ($j \to \infty$).

Every bilinear system of the form (1) and (2) can be considered as the sum of two subsystems, one only containing deterministic variables (index d) and the other containing all stochastic variables (index s). The state and output of (1) are then simply the sum of the states and outputs of both subsystems, respectively:

$$\begin{aligned} x_k &= x_k^d + x_k^s \\ y_k &= y_k^d + y_k^s. \end{aligned}$$

- The *deterministic* subsystem is described by

$$\begin{aligned} x_{k+1}^d &= Ax_k^d + Nu_k \otimes x_k^d + Bu_k \\ y_k^d &= Cx_k^d + Du_k. \end{aligned}$$

Authorized licensed use limited to: Katholieke Universiteit Leuven. Downloaded on October 14, 2008 at 06:54 from IEEE Xplore. Restrictions apply.

It should be noted that since u_k is observed, it is considered to be a deterministic variable. A remarkable property of bilinear systems is that the notion of reachability can be defined in an analog way as for linear systems [24]. The reachability matrix Δ_n^d is defined as

$$\begin{aligned}\Delta_n^d &= (B \quad |A\Delta_{n-1}^d| \quad N_1\Delta_{n-1}^d \quad \cdots \quad N_m\Delta_{n-1}^d)\\ \Delta_1^d &= B.\end{aligned}$$

- The following equations describe the *stochastic* subsystem:

$$\begin{aligned}x_{k+1}^s &= Ax_k^s + Nu_k \otimes x_k^s + w_k\\ y_k^s &= Cx_k^s + v_k\end{aligned} \tag{5}$$

and

$$\begin{aligned}\Sigma^s &= E\left[x_k^s (x_k^s)^T\right] = A\Sigma^s A^T + \sum_{p=1}^{m} N_p \Sigma^s N_p^T + Q\\ \Lambda_0^s &= E\left[y_k^s (y_k^s)^T\right] = C\Sigma^s C^T + R\\ G^s &= E\left[x_{k+1}^s (y_k^s)^T\right] = A\Sigma^s C^T + S.\end{aligned}$$

- Let us now consider the sum of stochastic and deterministic system, i.e., the original system (1) and (2). We define

$$\Sigma = E[x_k x_k^T] = A\Sigma A^T + \sum_{p=1}^{m} N_p \Sigma N_p^T + BB^T + Q \tag{6}$$

$$\Lambda_0 = E\left[y_k (y_k)^T\right] = C\Sigma C^T + DD^T + R \tag{7}$$

$$G = E\left[x_{k+1} y_k^T\right] = A\Sigma C^T + BD^T + S. \tag{8}$$

The reachability and the observability matrix of (1) are defined as

$$\begin{aligned}\Delta_n &= (G \quad |A\Delta_{n-1}| \quad N_1\Delta_{n-1} \quad \cdots \quad N_m\Delta_{n-1})\\ \Delta_1 &= G\\ \Gamma_n &= (C^T \quad |A^T\Gamma_{n-1}^T| \quad N_1^T\Gamma_{n-1}^T \quad \cdots \quad N_m^T\Gamma_{n-1}^T)^T\\ \Gamma_1 &= C.\end{aligned} \tag{9}$$

We also define the following important matrices:

$$L_i = E\left[y_{i-1|0}(y_{i-1|0})^T\right] \tag{10}$$

$$H_i = E\left[y_{i-1|0}(u_{i-1|0})^T\right]. \tag{11}$$

III. Main Results

In this section we first show how the Kalman filter estimates of the bilinear system (1) and (2) can be expressed as a linear combination of well-chosen products of inputs and outputs. These products are the columns in the data matrices (3) and (4). This result will play a very important role in Section III-C where it is shown that the state estimates derived in the bilinear subspace identification algorithm are Kalman filter estimates. In the next subsection stochastic realization theory of bilinear MIMO systems subject to white inputs is explained. This is necessary since subspace identification is heavily based on realization theory. In the last subsection the final algorithm for bilinear subspace identification of bilinear MIMO systems subject to white inputs is presented.

A. Kalman Filter

In this subsection, we derive the Kalman filter equations of the system (1) and (2). Further, these equations are expressed in a noniterative way. The use of doing so will become clear in the following subsections, where it is shown that the state estimates obtained by the subspace identification algorithm correspond to Kalman filter state estimates of the unknown system. The results presented here were inspired by linear subspace identification [12], [25] and optimal filtering theory [26].

Theorem 1—Kalman Filter:

1) The Kalman filter state estimates $\hat{x}_k$ of the system (1) and (2) are the solution of the following recursive formulas:

$$\hat{x}_{k+1} = A\hat{x}_k + Nu_k \otimes \hat{x}_k + Bu_k + K_k(y_k - C\hat{x}_k - Du_k) \tag{12}$$

where

$$K_k = \left(G^s - AP_kC^T\right)\left(\Lambda_0^s - CP_kC^T\right)^{-1} \tag{13}$$

$$\begin{aligned}P_k = AP_{k-1}A^T + \sum_{p=1}^{m} N_p P_{k-1} N_p^T + \left(G^s - AP_{k-1}C^T\right)\\ \cdot \left(\Lambda_0^s - CP_{k-1}C^T\right)^{-1}\left(G^s - AP_{k-1}C^T\right)^T.\end{aligned} \tag{14}$$

2) If we assume the initial conditions $\hat{x}_0 = 0$ and $P_0 = \Sigma^s - \Sigma$, then P_k and $\hat{x}_k$ are given by the following nonrecursive equations:

$$\begin{aligned}\hat{x}_k = \Delta_k^d u_{k-1|0} + \left(\Delta_k - \Delta_k^d H_k^T\right)\left(L_k - H_k H_k^T\right)^{-1}\\ \cdot (y_{k-1|0} - H_k u_{k-1|0})\end{aligned} \tag{15}$$

$$\begin{aligned}P_k = \overline{A}_k \overline{P}_k \overline{A}_k^T + \left(\Delta_k - \Delta_k^d H_k^T\right)\left(L_k - H_k H_k^T\right)^{-1}\\ \cdot \left(\Delta_k - \Delta_k^d H_k^T\right)^T\end{aligned} \tag{16}$$

where

$$\begin{aligned}\overline{P}_k &= \begin{pmatrix}\overline{P}_{k-1} & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \overline{P}_{k-1}\end{pmatrix} \in \mathbb{R}^{(m+1)^k n \times (m+1)^k n}\\ \overline{P}_0 &= P_0\\ \overline{A}_k &= (A\overline{A}_{k-1} \quad N_1\overline{A}_{k-1} \quad \cdots \quad N_m\overline{A}_{k-1})\\ \overline{A}_0 &= I.\end{aligned}$$

Proof: Since a bilinear system can be considered as a time-varying linear system, the derivation of (12)–(14) is done in the same way as for linear systems [26]. It should be noted that due to the assumption of the whiteness of u_k, a lot of simplifications can be made in the derivation. The only difference with the Kalman filter equations of time-invariant linear systems is an additional term in (12) and (14) depending on the system matrix N.

The proof of (15) and (16) is based on the results in [25] and quite straightforward but tedious. The interested reader can find the full mathematical details in [27]. ■

Authorized licensed use limited to: Katholieke Universiteit Leuven. Downloaded on October 14, 2008 at 06:54 from IEEE Xplore. Restrictions apply.

B. Realization Theory of Bilinear Systems Subject to White Inputs

In this subsection we extend the existing bilinear stochastic realization theory of [20] to a more general class of bilinear systems. In [20], only stochastic single-input/single-output (SISO) systems of type (5) are considered, i.e., the stochastic subsystem previously defined. Here, these results are extended for general MIMO bilinear systems (1) and (2).

Bilinear stochastic realization theory assumes that the covariance matrix

$$E\left[\begin{pmatrix} y_{i|2i-1} \\ y_{i-1|0} \end{pmatrix}\begin{pmatrix} y_{i-1|0} \\ u_{i-1|0} \end{pmatrix}^T\right] = \begin{pmatrix} \Gamma_i\Delta_i & \Gamma_i\Delta_i^d \\ L_i & H_i \end{pmatrix} \tag{17}$$

with i such that $\sum_{p=1}^{i-1}(m+1)^{p-1}l \geq n$ is known [20]. By calculating the singular value decomposition (SVD) of $E\left[y_{i|2i-1}\begin{pmatrix} y_{i-1|0} \\ u_{i-1|0}\end{pmatrix}^T\right]$, the extended observability matrix Γ_i and the extended reachability matrices Δ_i and Δ_i^d can be found. Exploiting the special structure, it is perfectly possible to find a permutation of rows such that Γ_{i-1} can be extracted from Γ_i and $E[y_{i+1|2i-1}(y_{i-1|0})^T]$ from $E[y_{i|2i-1}(y_{i-1|0})^T]$ (see Appendix I). The bilinear realization then becomes[1]

$$\begin{aligned} A &= \Gamma_{i-1}^{\dagger} E\left[y_{i+1|2i-1}y_{i-1|0}^T\right]\Delta_i^{\dagger} \\ N_1 &= \Gamma_{i-1}^{\dagger} E\left[u_{i,1}y_{i+1|2i-1}y_{i-1|0}^T\right]\Delta_i^{\dagger} \\ &\vdots \\ N_m &= \Gamma_{i-1}^{\dagger} E\left[u_{i,m}y_{i+1|2i-1}y_{i-1|0}^T\right]\Delta_i^{\dagger} \\ B &= \text{first } m \text{ columns of } \Delta_i^d \\ C &= \text{first } l \text{ rows of } \Gamma_i \\ D &= \text{first } l \text{ rows and } m \text{ columns of } H_i. \end{aligned}$$

It is easily verified that with (14) and (15), P_i can be written as

$$\begin{aligned} P_i = \Delta_i^d\left(\Delta_i^d\right)^T + \left(\Delta_i - \Delta_i^d H_i^T\right)\left(L_i - H_i H_i^T\right)^{-1} \\ \cdot \left(\Delta_i - \Delta_i^d H_i^T\right)^T \end{aligned}$$

which is an estimate of the state covariance matrix Σ. An estimate of the noise covariance matrices Q, R, and S can now be found with (6)–(8)

$$\begin{aligned} Q_i &= P_i - AP_iA^T - \sum_{p=1}^{m} N_pP_iN_p^T - BB^T \\ R_i &= \Lambda_0 - CP_iC^T - DD^T \\ S_i &= G - AP_iC^T - BD^T. \end{aligned}$$

A major drawback of realization theory is the fact that the prior knowledge of the covariance matrix (17) is required. The bilinear subspace identification algorithm presented in the following sections permits the calculation of the system matrices A, B, C, D, and N as well as an estimate of the noise covariance matrices Q, R, and S directly from the measured data, i.e., without any explicit determination or prior knowledge of covariances.

[1] † denotes the Moore–Penrose pseudoinverse.

C. Main Theorem of Bilinear Subspace Identification

In this subsection we derive the main subspace identification theorem of bilinear systems subject to white mutually independent inputs. This requires the following definition of projections of matrices.

Definition 1—Projection of Semi-Infinite Matrices: Given two matrices $P_1 \in \mathbb{R}^{p_1\times j}$ and $P_2 \in \mathbb{R}^{p_2\times j}$, with $j \to \infty$, the orthogonal projection of the row space of P_1 into the row space of P_2 is defined as

$$P_1/P_2 = P_1P_2^{\dagger}P_2 \in \mathbb{R}^{p_1\times j}.$$

The main theorem of the paper is now derived. It expresses that from the projection of the previously defined data matrices (3) and (4), not only can the extended observability matrix Γ_i be retrieved but also an estimate $\hat{X}_i$ of the state sequence X_i. Moreover, the link between the estimated state sequence $\hat{X}_i$ and the previously derived Kalman filter is made. This is a very important result since it permits the full understanding of the consistency of the final identification result.

Theorem 2—Bilinear Subspace Identification: Under the assumption that there is an infinite amount of measurements available ($j \to \infty$), with the projection $\mathcal{O}_i$ defined as

$$\mathcal{O}_i = Y_{i|2i-1}\Big/\begin{pmatrix} Y_{i-1|0} \\ U_{i-1|0}\end{pmatrix}$$

and the SVD

$$\mathcal{O}_i = \begin{pmatrix} U_1 & U_2 \end{pmatrix}\begin{pmatrix} S_1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} V_1^T \\ V_2^T \end{pmatrix}$$

we have the following.

1) The projection $\mathcal{O}_i$ equals
$$\mathcal{O}_i = \Gamma_i\hat{X}_i \tag{18}$$
where Γ_i is the extended observability matrix and $\hat{X}_i$ the sequence of Kalman filter state estimates (15) of the unknown bilinear system.
2) The order n of the system (1) and (2) is the number of nonzero singular values.
3) The extended observability matrix Γ_i and the estimated state sequence $\hat{X}_i$ can be directly calculated from the SVD (18)
$$\Gamma_i = U_1S_1^{1/2}$$
and
$$\hat{X}_i = S_1^{1/2}V_1^T.$$

Proof: See Appendix II. ■

The above theorem certainly requires some comment. The fact that the estimated state sequence $\hat{X}_i$ is a Kalman filter state sequence is of major importance and can be interpreted in the following way:

$$\hat{X}_i = \begin{pmatrix} \hat{x}_i & \hat{x}_{i+1} & \cdots & \hat{x}_{i+j-1} \end{pmatrix}. \tag{19}$$

This sequence can be interpreted as the solution to a bank of nonsteady state Kalman filters. This means that every element of $\hat{X}_i$ is the solution to the Kalman filter equations (12)–(14) where every filter has the same initial conditions $P_0 = \Sigma^s - \Sigma$ and $\hat{x}_0 = 0$ but uses different values of the inputs u_k and the outputs y_k. These values are simply the i past values of the

Authorized licensed use limited to: Katholieke Universiteit Leuven. Downloaded on October 14, 2008 at 06:54 from IEEE Xplore. Restrictions apply.

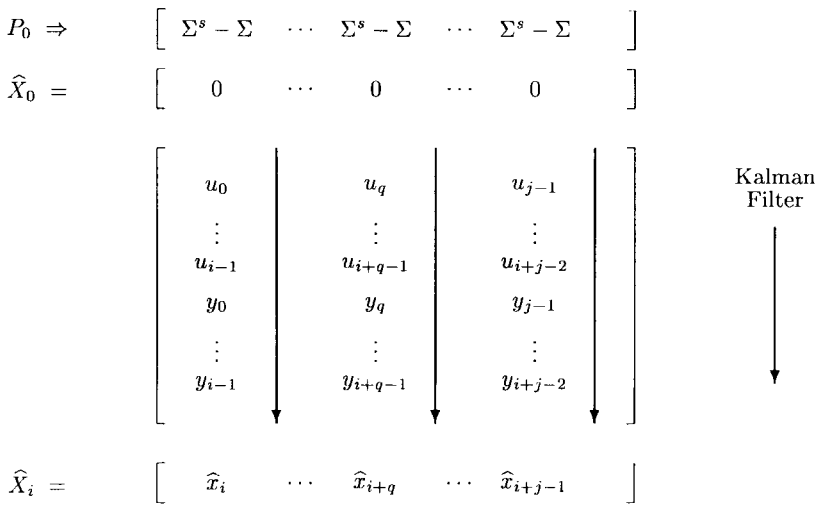


Fig. 1. Interpretation of the estimated state sequence $\hat{X}_i$, obtained from the bilinear subspace identification algorithm, as the solution to a bank of Kalman filters. When the system matrices A, B, C, D, Q, R, S would be known, the state $\hat{x}_{i+q}$ could be determined from the Kalman filter (12)–(14) as follows. Start the filter at time q, with an initial state estimate $\hat{x}_q = 0$ and initial state covariance matrix $P_q = \Sigma - \Sigma^s$. Now iterate the Kalman filter over i time steps (the vertical arrow down). The Kalman filter will then return a state estimate $\hat{x}_{i+q}$. When repeating this procedure for each of the j columns we obtain the estimated state sequence $\hat{X}_i$. We thus speak about a *bank* of Kalman filters. The major observation in subspace algorithms is that the system matrices A, B, C, D, Q, R, S do not have to be known to determine the state sequence $\hat{X}_i$. It can be determined directly from input–output data as shown in Theorem 2.

inputs u_k and the outputs y_k. This idea has been represented in a more graphical way in Fig. 1.

D. Subspace Identification Algorithm

Now that the main theorem has been presented, we generalize the linear subspace identification theory in [12] and [25] to a *bilinear* subspace identification algorithms. As mentioned before, we only consider bilinear systems subject to white mutually independent inputs. Nevertheless, this is a very important class of input signals for system identification purposes.

How can we now find the system parameters A, B, C, D, N and the noise covariance matrices Q, R, S? Let us therefore take a look at the innovation representation of the system (1) and (2) corresponding to the Kalman filter equations (12)–(14)

$$\begin{aligned} \hat{x}_{k+1} &= A\hat{x}_k + Nu_k \otimes \hat{x}_k + Bu_k + K_k e_k \\ y_k &= C\hat{x}_k + Du_k + e_k \end{aligned} \tag{20}$$

with the innovations e_k. If the state estimates $\hat{x}_k$ would be known, we would be able to solve the above set of equations for the unknown system parameters. However, this cannot be done by taking arbitrary estimates of the states. Only those states that are consecutive solutions $\hat{x}_k$ and $\hat{x}_{k+1}$ of the Kalman filter equations (12)–(14) will be consistent with the innovation representation (20) and lead to an unbiased estimate of A, B, C, D, N, and K_k.

In the previous theorem we have seen that a sequence of Kalman filter estimates $\hat{X}_i$ of the state sequence X_i can be found by projecting the row space of the matrix $Y_{i|2i-1}$ into the row space of the matrix $\binom{Y_{i-1|0}}{U_{i-1|0}}$. In order to determine the state sequence $\hat{X}_{i+1}$ that is the solution of the same Kalman filter bank, i.e., with the same initial conditions P_0, $\hat{x}_0$, and the same inputs u_k and outputs y_k, we have to calculate the projection (see Theorem 2 and Fig. 1)

$$\begin{aligned} \mathcal{O}_{i+1} &= Y_{i+1|2i-1} \Big/ \begin{pmatrix} Y_{i|0} \\ U_{i|0} \end{pmatrix} \\ &= \Gamma_{i-1} \hat{X}_{i+1}. \end{aligned} \tag{21}$$

Since both sequences $\hat{X}_i$ and $\hat{X}_{i+1}$ are solutions, respectively, at time step i and $i+1$ of the same Kalman filter bank (i.e., with the same initial conditions and the same inputs u_k and outputs y_k), they are related to each other through the innovation representation (20) of the system. We then have that

$$\begin{aligned} \hat{X}_{i+1} &= A\hat{X}_i + NU_i \odot \hat{X}_i + BU_i + K_i\rho_i \\ Y_i &= C\hat{X}_i + DU_i + \rho_i \end{aligned}$$

with the innovations sequence $\rho_i = (e_i \quad e_{i+1} \quad \cdots \quad e_{i+j-1})$. These relations can also be written as

$$\begin{pmatrix} \hat{X}_{i+1} \\ Y_i \end{pmatrix} = \begin{pmatrix} A & N & B \\ C & 0 & D \end{pmatrix} \begin{pmatrix} \hat{X}_i \\ U_i \odot \hat{X}_i \\ U_i \end{pmatrix} + \begin{pmatrix} K_i \\ I_l \end{pmatrix} \rho_i \tag{22}$$

which has to be solved as a *linear* least squares problem for the unknown system matrices A, B, C, D, and N. This problem could also be solved as a *constrained* linear least squares problem since we know that the solution contains a zero matrix.

Since the state sequences $\hat{X}_i$ and $\hat{X}_{i+1}$ are solutions of the same bank of Kalman filters, the estimates of A, B, C, D, and N are unbiased. Moreover, the residuals $\binom{K_i\rho_i}{\rho_i}$ of the least squares problem (22) permit us to calculate an estimate of the noise covariance matrices Q, R, and S

$$\begin{pmatrix} Q_i & S_i \\ S_i^T & R_i \end{pmatrix} = \frac{1}{j} \left[\begin{pmatrix} K_i\rho_i \\ \rho_i \end{pmatrix} \begin{pmatrix} K_i\rho_i \\ \rho_i \end{pmatrix}^T \right].$$

Interesting also is that by calculating the noise covariances Q_i, R_i, and S_i in this way, the positive semidefiniteness of

Authorized licensed use limited to: Katholieke Universiteit Leuven. Downloaded on October 14, 2008 at 06:54 from IEEE Xplore. Restrictions apply.

the noise covariance matrix is guaranteed

$$\begin{pmatrix} Q_i & S_i \\ S_i^T & R_i \end{pmatrix} \geq 0.$$

This is important in the sense that when a covariance is not positive semidefinite, it cannot correspond to a physical noise signal.

Further, it should be noted that both state sequences $\hat{X}_i$ and $\hat{X}_{i+1}$ must be calculated in the same state-space basis. This is not guaranteed by simply calculating the SVD of $\mathcal{O}_i$ and $\mathcal{O}_{i+1}$ separately. However, $\hat{X}_i$ and $\hat{X}_{i+1}$ can be forced into the same basis as follows. First, we find $\hat{X}_i$ and Γ_i from the SVD of $\mathcal{O}_i$ as indicated in the main theorem. Then Γ_{i-1} is determined from Γ_i (see Appendix I). Finally, the state sequence at time step $i+1$ is determined from (21) $\hat{X}_{i+1} = \Gamma_{i-1}^{\dagger}\mathcal{O}_{i+1}$.

Another interesting characteristic of the algorithm is that, though linear subspace identification assumes that $il > n$ where l is the number of outputs, here it is sufficient that i is such that $d_{i-1} = \sum_{p=1}^{i-1}(m+1)^{p-1}l > n$. For instance, for a SISO bilinear system of order seven, we only need $i = 4$, whereas for linear subspace identification, an i-coefficient of at least seven would be necessary. A drawback, however, is that due to the exponential growth rate of the matrices, i will always have to be small. Therefore, the estimate of the noise covariances will not be very accurate. Only for larger values of i the Kalman filter will reach the steady state and the estimate of the noise covariances will converge to the real value (see also Fig. 1 for an interpretation).

Finally, the algorithm can be summarized as in Fig. 2. It is important to note that it is a direct extension of linear subspace identification [12]. Indeed, if the product terms between inputs and outputs are omitted in the data matrices (3) and (4), one finds indeed the Hankel matrices used in linear subspace identification. As a result, the present algorithm also has the same nice features: the algorithm is noniterative, unbiased, no parameterization problems for MIMO systems, and numerically robust.

Bilinear subspace identification algorithm

1. *Calculate the projections:*

$$\mathcal{O}_i = Y_{i|2i-1} / \begin{pmatrix} Y_{i-1|0} \\ U_{i-1|0} \end{pmatrix},$$

$$\mathcal{O}_{i+1} = Y_{i+1|2i-1} / \begin{pmatrix} Y_{i|0} \\ U_{i|0} \end{pmatrix}.$$

2. *Calculate the SVD of $\mathcal{O}_i$:*

$$\mathcal{O}_i = \Gamma_i \hat{X}_i = \begin{pmatrix} U_1 & U_2 \end{pmatrix} \begin{pmatrix} S_1 & 0 \\ 0 & S_2 \end{pmatrix} \begin{pmatrix} V_1^T \\ V_2^T \end{pmatrix},$$

determine the order n of the system by examining the number of dominant singular values and find

$$\Gamma_i = U_1 S_1^{1/2}, \quad \hat{X}_i = S_1^{1/2} V_1.$$

3. *Determine Γ_{i-1} from Γ_i (see Appendix A).*
4. *Calculate the estimated state vector sequences at time step $i+1$:*

$$\hat{X}_{i+1} = \Gamma_{i-1}^{\dagger}\mathcal{O}_{i+1}.$$

5. *Solve the linear least squares problem:*

$$\begin{pmatrix} \hat{X}_{i+1} \\ Y_{i|i} \end{pmatrix} = \begin{pmatrix} A & N & B \\ C & 0 & D \end{pmatrix} \begin{pmatrix} \hat{X}_i \\ U_i \odot \hat{X}_i \\ U_i \end{pmatrix} + \begin{pmatrix} \rho_w \\ \rho_v \end{pmatrix}$$

to obtain the system matrices A, B, C, D, N and the residuals $\begin{pmatrix} \rho_w \\ \rho_v \end{pmatrix}$.
6. *Calculate the estimate of the noise covariance matrices Q, R and S:*

$$\begin{pmatrix} Q_i & S_i \\ S_i^T & R_i \end{pmatrix} = \begin{pmatrix} \rho_w \\ \rho_v \end{pmatrix} \begin{pmatrix} \rho_w^T & \rho_v^T \end{pmatrix} \geq 0.$$

Fig. 2. N4SID algorithm for the subspace identification of MIMO bilinear systems with white inputs.

IV. Examples

A. A Simple Example

Here we show how a simple second-order bilinear system with two inputs and two outputs can be identified using the results presented in the paper. The system matrices are

$$A = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.3 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$N_1 = \begin{pmatrix} 0.6 & 0 \\ 0 & 0.4 \end{pmatrix}, \qquad N_2 = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.5 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, \qquad D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

and the noise covariance matrices are

$$Q = \begin{pmatrix} 0.01 & 0 \\ 0 & 0.01 \end{pmatrix}, \qquad R = \begin{pmatrix} 0.01 & 0 \\ 0 & 0.01 \end{pmatrix}$$

$$S = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

The input signal we used is a pseudo-random binary sequence (PRBS) of length 4095 and amplitude one. In order to have an idea of the order of the system, one should examine the number of dominant singular values of $\mathcal{O}_i = Y_{i|2i-1} / \begin{pmatrix} Y_{i-1|0} \\ U_{i-1|0} \end{pmatrix}$ (see Fig. 3). Examining this figure, we choose a second-order bilinear model: $n = 2$, i.e., the order of the original system. The identified model is validated by comparing the eigenvalues of A, N_1, and N_2 of the model with those of the original system. From Table I one can see that these eigenvalues are very close. If the j-parameter were infinite, they would be identical. As mentioned in Section III-D the estimate of the noise covariance matrices is not very accurate since the i-parameter is small.

B. Binary Distillation Column

In this example we illustrate the use of bilinear systems in the field of chemical industry by comparing a linear model with a bilinear model.

If we look at the equations describing the physical behavior of a distillation column the bilinear nature of the process is quite easy to show. A cross section of a tray in a distillation column is represented in Fig. 4.

Authorized licensed use limited to: Katholieke Universiteit Leuven. Downloaded on October 14, 2008 at 06:54 from IEEE Xplore. Restrictions apply.

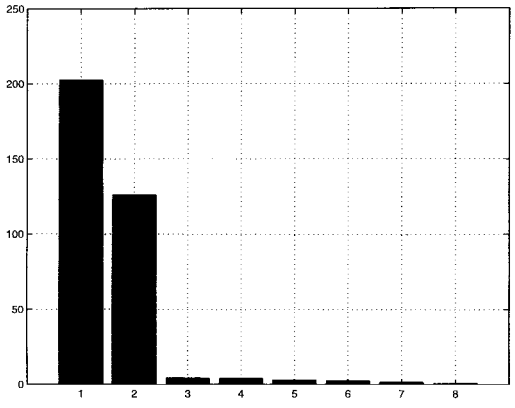


Fig. 3. Singular values of $\mathcal{O}_i$, $i = 2$, and $j = 4095$.

TABLE I
EIGENVALUES OF THE A AND THE N MATRICES: LEFT—ORIGINAL SYSTEM; RIGHT—MODEL

	Original system	Model
A	0.5, 0.3	0.5001, 0.2979
N_1	0.6, 0.4	0.4020, 0.5994
N_2	0.2, 0.5	0.1914, 0.5016

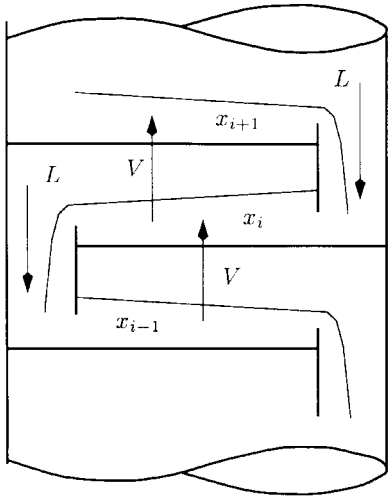
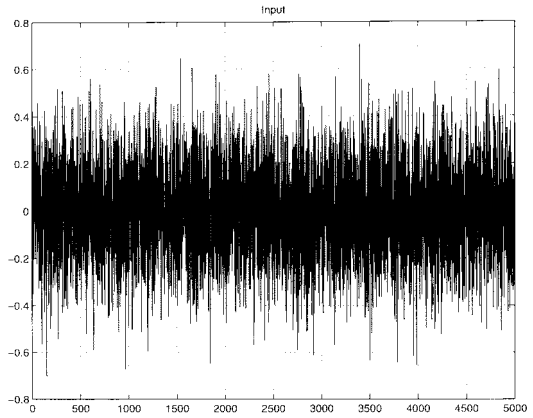


Fig. 4. Cross section of a tray in a distillation column. The arrows indicate the flows entering and leaving the tray. V is the vapor raising in the column and L the liquid flow going down the column. The concentrations in the liquid phase on tray $i-1$, i, and $i+1$ are x_{i-1}, x_i, and x_{i+1}, respectively.

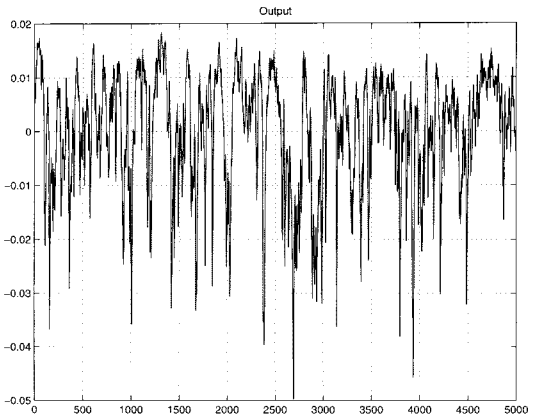
In case we have a binary distillation column (i.e., where two components have to separated), the partial mass balance of an arbitrary tray i of the column can be described by the following equation:

$$M\frac{dx_i}{dt} = Vy_{i-1} + Lx_{i+1} - Vy_i - Lx_i$$

where V is the gas flow going up in the column and L the liquid flow going down. The concentration of the gas y_i in thermodynamic equilibrium with the liquid phase can be well approximated around an operating point by as a linear function



(a)



(b)

Fig. 5. (a) The input is a white signal on the reflux rate. (b) The output X_d is the change in distillate concentration.

of the concentration in the liquid phase x_i:

$$y_i = K_i x_i.$$

In that case the partial mass balance becomes

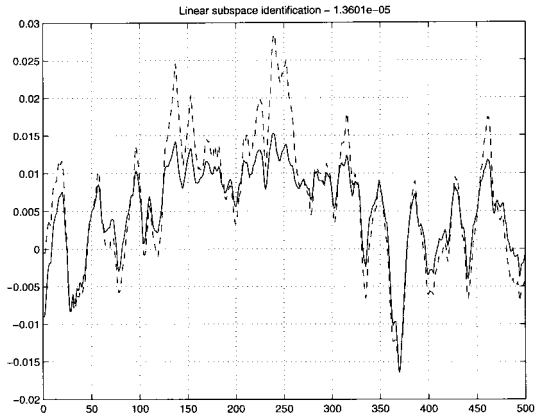
$$\frac{dx_i}{dt} = \frac{K_i}{M}Vx_{i-1} + \frac{1}{M}Lx_{i+1} - \frac{K_i}{M}Vx_i - \frac{1}{M}Lx_i$$

which is a bilinear equation since the flows L and V are controlled variables and the concentrations on the different trays x_{i-1}, x_i, and x_{i+1} are the states of the system.

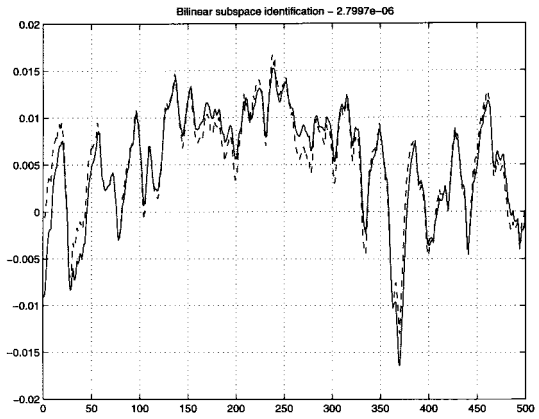
Let us now make a direct comparison between linear and bilinear subspace identification by identifying a full order nonlinear model of a distillation column. Using the same input–output data set we show that a bilinear model obtained with the bilinear subspace identification algorithm is more suited to approximate the dynamics of the distillation column than a model obtained with linear subspace methods.

The input we consider is the reflux rate of the column and is excited with a white signal, while the output is the purity x_d of the distillate collected at the top of the column (Fig. 5). A bilinear model of a certain order has more parameters than a linear model of the same order (due to the N-matrix). Therefore, to assure a better comparison between the two methods, we estimate a linear model of order six and a bilinear model of order three.

Authorized licensed use limited to: Katholieke Universiteit Leuven. Downloaded on October 14, 2008 at 06:54 from IEEE Xplore. Restrictions apply.



(a)



(b)

Fig. 6. The real output (full line) is compared with the output of the model (dashed line). Before identification, the mean value of the data was set to zero: (a) sixth-order linear model and (b) third-order bilinear model. The fit is clearly better with a bilinear model.

V. Conclusions and Open Problems

In this paper, we studied subspace identification of MIMO bilinear systems with additional measurement and process noise. The main assumption we made is that the inputs of the system should be white and mutually independent. We generalized bilinear stochastic realization theory to a more general class of MIMO bilinear systems. Further, we showed that the orthogonal projection of the row space of well-determined data matrices provides Kalman filter state estimates of the unknown bilinear system. This important result permitted us to extend the existing linear subspace algorithms to completely analog methods for the identification of bilinear systems. Therefore, the algorithm also presents the same interesting characteristics as linear subspace identification. Finally, we illustrated the power and practical interest of the theory by means of some examples. It was shown analytically and by means of a simulation example that bilinear models are more suited to representing the dynamics of a distillation column than linear models.

Further research is being done on this subject. The assumption of whiteness of the input can be quite restrictive in certain cases. We try to extend the present theories for bilinear systems without any input constraints [28]. More general classes of nonlinearities could also be considered such as quadratic systems, where products between states are allowed. Further, the problem of finding error bounds on the identified parameters in the framework of subspace identification is still unresolved.

Appendix I
Bilinear Observability Matrix

Here we give an algorithm that allows for the calculation of the extended observability matrix Γ_{i-1} from Γ_i. Matlab notations are used

$$\begin{aligned}
&\texttt{function}\ \Gamma_{i-1} = f(\Gamma_i)\\
&\quad s = \texttt{size}\,(\Gamma_i,\,1)\\
&\quad \texttt{if}\ s > 1\\
&\qquad \Gamma_{i-1} = \Big[\Gamma_i(1,:);\\
&\qquad\qquad f\left(\Gamma_i\left(2\colon \frac{s+1}{2},:\right)\right);\\
&\qquad\qquad f\left(\Gamma_i\left(\frac{s+1}{2}\colon s,:\right)\right)\Big]\\
&\quad \texttt{else}\\
&\qquad \Gamma_{i-1} = [\,];\\
&\quad \texttt{end}.
\end{aligned}$$

Appendix II
Proof of Theorem 2

Following Definition 1:

$$\begin{aligned}
\mathcal{O}_i &= Y_{i|2i-1} \Big/ \begin{pmatrix} Y_{i-1|0} \\ U_{i-1|0} \end{pmatrix}\\
&= Y_{i|2i-1} \begin{pmatrix} Y_{i-1|0} \\ U_{i-1|0} \end{pmatrix}^T \left[\begin{pmatrix} Y_{i-1|0} \\ U_{i-1|0} \end{pmatrix} \begin{pmatrix} Y_{i-1|0} \\ U_{i-1|0} \end{pmatrix}^T \right]^{-1}\\
&\quad \cdot \begin{pmatrix} Y_{i-1|0} \\ U_{i-1|0} \end{pmatrix}.
\end{aligned}$$

With (10), (11), (17), and $\Delta = L_i - H_i H_i^T$ this projection becomes

$$\begin{aligned}
\mathcal{O}_i &= (\Gamma_i \Delta_i \quad \Gamma_i \Delta_i^d) \begin{pmatrix} L_i & H_i \\ H_i^T & I \end{pmatrix}^{-1} \begin{pmatrix} Y_{i-1|0} \\ U_{i-1|0} \end{pmatrix}\\
&= \Gamma_i (\Delta_i \quad \Delta_i^d) \begin{pmatrix} \Delta^{-1} & -\Delta^{-1} H_i \\ -H_i^T \Delta^{-1} & I + H_i^T \Delta^{-1} H_i \end{pmatrix}\\
&\quad \cdot \begin{pmatrix} Y_{i-1|0} \\ U_{i-1|0} \end{pmatrix}\\
&= \Gamma_i \Big\{ \Delta_i^d U_{i-1|0} + \left(\Delta_i - \Delta_i^d H_i^T\right) \left(L_i - H_i H_i^T\right)^{-1}\\
&\qquad \cdot \left(Y_{i-1|0} - H_i U_{i-1|0}\right) \Big\}.
\end{aligned}$$

With (15) and (19) we then have that

$$\mathcal{O}_i = \Gamma_i \hat{X}_i.$$

This proves the first part of the theorem. The second and third part directly follow from the properties of the SVD.

Authorized licensed use limited to: Katholieke Universiteit Leuven. Downloaded on October 14, 2008 at 06:54 from IEEE Xplore. Restrictions apply.

REFERENCES

[1] M. España and I. D. Landau, "Reduced order models of distillation columns," *Automatica*, vol. 14, pp. 345–357, 1978.

[2] R. R. Mohler, *Nonlinear Systems—V.2: Applications to Bilinear Systems.* Englewood Cliffs, NJ: Prentice-Hall, 1991.

[3] D. Yu and D. N. Shields, "A bilinear fault detection observer," *Automatica*, vol. 32, no. 11, pp. 1597–1602, 1996.

[4] D. Williamson, "Observation of bilinear systems with application to biological control," *Automatica*, vol. 13, pp. 243–254, 1977.

[5] Y. Ying, M. Rao, and S. X. Shen, "Bilinear decoupling control and its industrial application," in *Proc. American Control Conf.*, Chicago, IL, June 24–26, 1992, pp. 1163–1167.

[6] R. W. Brockett, "Volterra series and geometric control theory," *Automatica*, vol. 12, pp. 167–176, 1976.

[7] K. Lo, E. S. Yoon, Y. K. Yeo, and H. K. Song, "Application of a bilinear long-range predictive control method in a distillation process," in *Proc. IFAC Dynamics and Control of Chemical Reactors*, 1992, pp. 249–254.

[8] V. R. Karanam, P. A. Frick, and R. R. Mohler, "Bilinear system identification by Walsh functions," *IEEE Trans. Automat. Contr.*, vol. 23, pp. 709–713, 1978.

[9] M. M. Gabr, "A recursive (on-line) identification of bilinear systems," *Int. J. Contr.*, vol. 44, no. 4, pp. 911–917, 1985.

[10] F. Fnaiech and L. Ljung, "Recursive identification of bilinear systems," *Int. J. Contr.*, vol. 45, no. 2, pp. 453–470, 1987.

[11] W. E. Larimore, "Canonical variate analysis in identification, filtering and adaptive control," in *Proc. 29th Conf. Decision and Control*, 1990, pp. 596–604.

[12] P. Van Overschee and B. De Moor, *Subspace Identification for Linear Systems: Theory, Implementation, Applications.* Dordrecht, The Netherlands: Kluwer, 1996.

[13] M. Verhaegen and P. Dewilde, "Subspace identification—Part I: The output-error state space model identification class of algorithms," *Int. J. Contr.*, vol. 56, pp. 1187–1210, 1992.

[14] B. De Moor, P. Van Overschee, and W. Favoreel, *Numerical Algorithms for Subspace State Space System Identification—An Overview.* Boston, MA: Birkhauser, 1998.

[15] W. Favoreel, B. De Moor, and P. Van Overschee, "Subspace state space system identification for industrial processes," in *Proc. 5th IFAC Symp. Dynamics and Control of Process Systems*, Corfu, Greece, June 8–10, 1998, pp. 322–330.

[16] T. Bastogne, H. Noura, A. Richard, and J. M. Hittinger, "Application of subspace methods to the identification of a winding process," in *Proc. 4th European Control Conf.*, Brussels, Belgium, July 1–4, 1997.

[17] K. Liu and R. E. Skelton, "Identification and control of NASA's ACES structure," in *Proc. American Control Conf.*, Boston, MA, 1991, pp. 3000–3006.

[18] S. Bittanti, E. Gatti, G. Ripamonti, and S. M. Savaresi, "An application of subspace-based techniques to nuclear spectroscopy," in *Proc. 11th IFAC Symp. System Identification*, Kitakyushu, Japan, July 8–11, 1997, vol. 4, pp. 1695–1700.

[19] D. Westwick and M. Verhaegen, "Identifying MIMO Wiener systems using subspace model identification methods," *Signal Processing*, vol. 52, pp. 235–258, 1996.

[20] U. B. Desai, "Realization of bilinear stochastic systems," *IEEE Trans. Automat. Contr.*, vol. 31, pp. 189–192, 1986.

[21] W. Favoreel, B. De Moor, and P. Van Overschee, "A bilinear extensions of subspace identification for systems subject to white inputs," in *Proc. American Control Conf.*, Albuquerque, NM, June 4–6, 1997, vol. WM07-3.

[22] _____, "Subspace identification of balanced deterministic bilinear systems subject to white inputs," in *Proc. 4th European Control Conf.*, Brussels, Belgium, July 1–4, 1997, vol. ECC585.

[23] C. G. Khatri and C. R. Rao, "Solutions to some function equations and their applications to characterization of probability distributions," *Indian J. Stat.*, vol. 30, pp. 167–180, 1968.

[24] A. Isidori and A. Ruberti, *Realization Theory of Bilinear Systems*, D. Q. Mayne and R. W. Brockett, Eds. Dordrecht, The Netherlands: Reidel, 1973, pp. 83–130.

[25] P. Van Overschee and B. De Moor, "N4SID: Subspace algorithms for the identification of combined deterministic and stochastic systems," *Automatica*, vol. 30, no. 1, pp. 75–93, 1994.

[26] B. D. O. Anderson and J. B. Moore, *Optimal Filtering.* Englewood Cliffs, NJ: Prentice-Hall, 1979.

[27] W. Favoreel, B. De Moor, and P. Van Overschee, "Subspace identification of bilinear systems subject to white inputs," Katholieke Universiteit Leuven, Tech. Rep. 96-53, ftp://ftp.esat.kuleuven.ac.be/pub/SISTA/favoreel/reports/report9653I.ps.-gz, 1996.

[28] _____, "Subspace identification of linear systems," in *Proc. MTNS'98*, Padova, Italy, June 6–10, 1998.



Wouter Favoreel was born in Kortrijk, West-Vlaanderen, Belgium, on July 12, 1971. He received the degree of mechanical engineering in control theory in 1994 from the Faculté Polytechnique de Mons, Belgium. In 1995, he received the Master of Science in control engineering from the Universität Stuttgart, Germany.

Currently, he is a Research Assistant of IWT and working toward his doctoral thesis at the Katholieke Universiteit Leuven, Belgium. His research interests include the joint multivariable system identification and control of linear and bilinear systems.



Bart De Moor (M'86–SM'93) was born in Halle, Brabant, Belgium, on July 12, 1960. He received the Ph.D. degree in 1988 at the Katholieke Universiteit Leuven, Belgium.

He was a Visiting Research Associate at the Departments of Computer Science and Electrical Engineering of Stanford University, CA, in 1989. He is currently a Senior Research Associate of the Fund for Scientific Research Flanders (FWO-Vlaanderen) and Associate Professor at the Department of Electrical Engineering of the Katholieke Universiteit Leuven. He is also the main Advisor of Science and Technology of the Flemish Minister for Science Policy. His research interests include numerical linear algebra (generalized SVD, max-algebra, and complementarity problems, tensor algebra), system identification (subspace methods, structured total least squares), control theory (robust control, neural nets), and signal processing. He has more than 200 papers in international journals and conference proceedings and received several national and international awards for his work. He is a member of several boards of administrators of (inter)national scientific, cultural, and commercial organizations.



Peter Van Overschee was born in Leuven, Brabant, Belgium, on October 28, 1966. He received the degree of electro-mechanical engineering in control theory in 1989 at the Katholieke Universiteit Leuven, Belgium. In 1990, he received the Master of Science in electrical engineering at Stanford University, CA. At the Katholieke Universiteit Leuven, he obtained the doctoral degree with the work "Subspace Identification: Theory—Implementation—Applications" in 1995.

His current research interests include the theory and application of multivariable system identification. He was also involved in the design, development, and implementation of the identification toolbox for Xmath (Integrated Systems Inc., Santa Clara, CA).

In 1994, he won the biannual Belgian Siemens Award and in 1996 the Tri-Annual Best Paper Award of Automatica, a journal of the International Federation of Automatic Control (at the IFAC World Congress, San Francisco, 1996).

Authorized licensed use limited to: Katholieke Universiteit Leuven. Downloaded on October 14, 2008 at 06:54 from IEEE Xplore. Restrictions apply.