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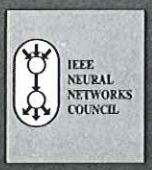
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Robust NL_q Neural Control Theory

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Abstract

In this paper we present sufficient conditions for global asymptotic stability and I/O stability (finite L_2-gain) of multilayer recurrent neural networks, with parametric uncertainties upon the interconnection matrices. This is done by considering perturbed NL_q systems. NL_qs are discrete time nonlinear dynamical systems in state space form, containing q layers with alternating linear and static nonlinear operators that satisfy a sector condition. Within the present framework, linear fractional transformations with real diagonal uncertainty block are interpreted as perturbed NL_qs with $q = 1$. While in μ robust control theory uncertainties upon nominal linear models are investigated, uncertainties upon nominal nonlinear models can be studied in NL_q neural control theory.

1. Introduction

Sufficient conditions for global asymptotic stability and I/O stability with finite L_2-gain for neural control systems have been derived in [9, 11, 13, 15]. Models and controllers were considered in neural state space form. In this way the closed-loop model is represented in so-called NL_q form. NL_q systems are discrete time nonlinear state space models, containing alternating linear and static nonlinear operators that satisfy a sector condition. NL_q systems are closely related to multilayer recurrent neural networks. Many problems arising in systems, control and neural networks can be interpreted as special cases [12]. It has been shown how several types of nonlinear behaviour (unique or multiple equilibria, periodic, quasi-periodic or chaotic) can be stabilized and controlled using NL_q theory [9, 16, 17]. A real-life example of top-down neural control design using NL_q theory has been given in [19], where the method is demonstrated for a ball/beam system.

In this paper we consider the problem of robust performance, i.e. conditions for I/O stability with finite L_2-gain in case there are parametric uncertainties upon one of the interconnection matrices of the neural network model or neural controller. This leads to perturbed NL_q systems, which correspond to nominal *nonlinear* models, with uncertainty modelled as a feedback perturbation upon this nominal model. Similar representations are used in modern robust control theory such as μ theory [5, 7], where uncertainties upon nominal *linear* models are modelled by means of a feedback perturbation through LFTs (Linear Fractional Transformations). It turns out that in the case of real parametric uncertainties the state space upper bound test in μ theory can be considered as a special case of the conditions of robust performance for perturbed NL_q systems.

This paper is organized as follows. In Section 2 we review some aspects on NL_q systems. In Section 3 the link between NL_qs and neural control schemes is presented. In Section 4 we discuss perturbed NL_q systems and present I/O stability criteria with finite L_2-gain. In Section 5 the link with μ theory is explained.

2. NL_q systems

The following nonlinear discrete time system is called an NL_q system and has been introduced in [9, 11, 12, 13, 14, 15]

$$\begin{cases} p_{k+1} &= \Gamma_1(V_1\,\Gamma_2(V_2\,...\Gamma_q(V_q\,p_k + B_q w_k) + ... \\ & \quad B_2 w_k) + B_1 w_k) \\ e_k &= \Lambda_1(W_1\Lambda_2(W_2...\Lambda_q(W_q p_k + D_q w_k) + ... \\ & \quad D_2 w_k) + D_1 w_k). \end{cases} \tag{1}$$

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Here $\Gamma_i(p_k, w_k)$, $\Lambda_i(p_k, w_k)$ $(i = 1, ..., q)$ are diagonal matrices with diagonal elements $\gamma_j(p_k, w_k)$, $\lambda_j(p_k, w_k) \in [0, 1]$, depending continuously on the variables $p_k \in \mathbb{R}^{n_p}$, $w_k \in \mathbb{R}^{n_w}$. Hence $\|\Gamma_i\| \leq 1$, $\|\Lambda_i\| \leq 1$ (for 1,2 or ∞ norm and $i = 1, 2, ..., q$). Furthermore $e_k \in \mathbb{R}^{n_e}$. The matrices V_i, W_i, B_i, D_i are constant. The term 'NL_q' system refers to the alternating sequence of nonlinear and linear operators acting on the input arguments p_k, w_k: the diagonal matrices Γ_i, Λ_i depend on p_k, w_k and refer to 'N' in 'NL_q', the constant matrices V_i, W_i, B_i, D_i refer to 'L' in 'NL_q'. The index q refers to the number of nonlinear operations in this alternating sequence (or 'layers').

An equivalent representation to (1) is

$$\begin{bmatrix} p_{k+1} \\ e_k^{ext} \end{bmatrix} = \prod_{i=1}^{q} \Omega_i(p_k, w_k) R_i \begin{bmatrix} p_k \\ w_k \end{bmatrix} \tag{2}$$

with $\Omega_i = \mathrm{diag}\{\Gamma_{i,e}, \Lambda_{i,e}, 0\}$ $(i = 1, ..., q)$ and $R_i = \mathrm{blockdiag}\{M_i, N_i, 0\}$, $R_q = [M_q; N_q; 0]$ $(i = 1, ..., q-1)$ where $\Gamma_{1,e} = \Gamma_1$, $\Gamma_{i,e} = \mathrm{diag}\{\Gamma_i, I\}$, $M_1 = [V_1\ B_1]$, $M_i = [V_i\ B_i; 0\ I]$, $\Lambda_{1,e} = \Lambda_1$, $\Lambda_{i,e} = \mathrm{diag}\{\Lambda_i, I\}$, $N_1 = [W_1\ D_1]$, $N_i = [W_i\ D_i; 0\ I]$ $(i = 2, ..., q)$. Furthermore e_k^{ext} corresponds to the output e_k augmented with a number of zero elements in order to make $\prod_{i=1}^{q} R_i$ square. Note that $\|\Omega_i\| \leq 1$ because $\|\Gamma_i\| \leq 1$ and $\|\Lambda_i\| \leq 1$.

Such NL_q systems are closely related to multilayer recurrent networks of the form

$$\begin{cases} p_{k+1} &= \sigma(V_1\,\sigma(V_2\,...\sigma(V_q\,p_k + B_q w_k) + ... \\ & \quad B_2 w_k) + B_1 w_k) \\ e_k &= \sigma(W_1\,\sigma(W_2...\sigma(W_q p_k + D_q w_k) + ... \\ & \quad D_2 w_k) + D_1 w_k) \end{cases} \tag{3}$$

where $\sigma(.)$ is a static nonlinear operator that satisfies the sector condition $[0, 1]$. Special cases for $\sigma(.)$ are e.g. $\tanh(.)$ and $\mathrm{sat}(.)$ (defined as $\mathrm{sat}(x) = x$ if $|x| \leq 1$ and $\mathrm{sat}(x) = \mathrm{sign}(x)$ if $|x| \geq 1$). In order to illustrate this, consider the following Hopfield network with synchronous updating $x_{k+1} = \sigma(W x_k)$. This can be written as the NL_1 system $x_{k+1} = \Gamma(x_k) W x_k$ with $\Gamma = \mathrm{diag}\{\gamma_i\}$ and $\gamma_i = \sigma(w_i^T x_k)/(w_i^T x_k)$.

3 Neural control systems as NL_q systems

In [10] neural state space models were proposed for nonlinear system identification

$$\begin{cases} \hat{x}_{k+1} &= W_{AB}\sigma(V_A \hat{x}_k + V_B u_k + \beta_{AB}) + K\epsilon_k \\ y_k &= W_{CD}\sigma(V_C \hat{x}_k + V_D u_k + \beta_{CD}) + \epsilon_k \end{cases} \tag{4}$$

which is a nonlinear state space model, parametrized by multilayer perceptrons. Here $u_k \in \mathbb{R}^m$, $y_k \in \mathbb{R}^l$ and $\hat{x}_k \in \mathbb{R}^n$ are the input, output and state vector respectively. W_* and V_* denote the interconnection matrices, with compatible dimensions and β_* are bias vectors. As activation function $\sigma(.)$ we take the hyperbolic tangent function $\tanh(.)$, which is applied elementwise (diagonal nonlinearity). The matrix K is a steady state Kalman gain, in order to take into account process noise. Prediction error algorithms for identifying (4) from I/O data are discussed in [10]. Narendra's dynamic backpropagation procedure is applied in order to compute the gradient of the cost function.

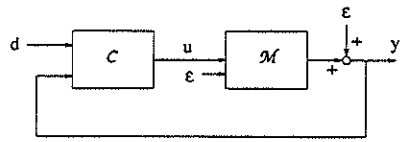


Figure 1. Control scheme with a recurrent neural state space model $\mathcal{M}$ and a recurrent neural state space controller C. The model and controller are both general nonlinear dynamical systems, parametrized by multilayer perceptrons. The closed loop system can be represented as an NL_q system.

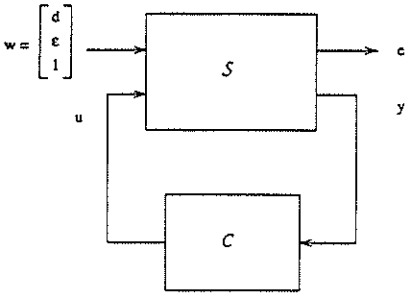


Figure 2. According to modern control theory, the neural control scheme is represented in standard plant form with augmented plant $\mathcal{S}$. c_k is the regulated output, w_k the exogenous input and p_k the state of the NL_q system. The constant in the exogenous input is due to the bias vectors in the multilayer perceptrons.

In order to design a neural controller based upon the identified model (like in Nguyen & Widrow's emulator approach) the following nonlinear dynamic output feedback controller is considered, parametrized by multilayer perceptrons:

$$\begin{cases} z_{k+1} &= W_{EF}\sigma(V_E z_k + V_F y_k + V_{F_2} d_k + \beta_{EF}) \\ u_k &= W_{GH}\sigma(V_G z_k + V_H y_k + V_{H_2} d_k + \beta_{GH}). \end{cases} \tag{5}$$

Here $d_k \in \mathbb{R}^l$ is the desired output and $z_k \in \mathbb{R}^{n_z}$ is the state of the neural controller. Again W_*, V_* denote interconnection matrices and β_* bias vectors. Supposing that $V_D = 0$, $V_H = 0$, $V_{H_2} = 0$, $\beta_{GD} = 0$, $\beta_{GH} = 0$ and defining additional state variables $\xi_k = \sigma(V_G \hat{x}_k)$ and $\eta_k = \sigma(V_G z_k)$ one obtains the following state equation for the closed loop system

$$\begin{cases} \hat{x}_{k+1} &= W_{AB}\sigma(V_A \hat{x}_k + V_B W_G \eta_k + \beta_{AB}) + K\epsilon_k \\ z_{k+1} &= W_{EF}\sigma(V_E z_k + V_F W_G \xi_k + V_F \epsilon_k + V_{F_2} d_k \\ &\quad + \beta_{EF}) \\ \xi_{k+1} &= \sigma(V_G W_{AB}\sigma(V_A \hat{x}_k + V_B W_G \eta_k + \beta_{AB}) + \\ &\quad V_G K \epsilon_k) \\ \eta_{k+1} &= \sigma(V_G W_{EF}\sigma(V_E z_k + V_F W_{GD} \xi_k + V_F \epsilon_k + \\ &\quad V_{F_2} d_k + \beta_{EF})) \end{cases} \tag{6}$$

which is in standard plant form [1, 5] an NL_2 system with state vector $p_k = [\hat{x}_k; z_k; \xi_k; \eta_k]$ and exogenous input $w_k = [d_k; \epsilon_k; 1]$ (Fig.1-2).

Also linear controllers or parametrizations by multilayer perceptrons with two hidden layers could be considered, leading to NL_q systems for the closed-loop system as well [9, 11].

4. Perturbed NL_qs and robust performance

Sufficient conditions for global asymptotic stability and I/O stability with finite L_2-gain of NL_q systems can be found in [9, 11, 12, 13, 14, 15]. Let us now consider parametric uncertainties upon one of the matrices R_l in (2) (which could e.g. correspond to parametric uncertainties upon the neural network model or controller). The NL_q system is perturbed as follows:

$$\begin{bmatrix} p_{k+1} \\ e_k^{ext} \end{bmatrix} = \prod_{i=1}^{l-1} \Omega_i R_i \; \Omega_l (R_l^{nom} + \Delta R_l) \prod_{j=l+1}^{q} \Omega_j R_j \begin{bmatrix} p_k \\ w_k \end{bmatrix}. \tag{7}$$

Suppose the perturbation on R_l is through one of the interconnection matrices $X \in \mathbb{R}^{n \times m}$ and that $x_{ij} \in [\underline{x}_{ij}, \overline{x}_{ij}]$. According to [8], by defining $x_{ij}^{nom} = (\overline{x}_{ij} + \underline{x}_{ij})/2, r_{ij} = (\overline{x}_{ij} - \underline{x}_{ij})/2$ one obtains $x_{ij} = x_{ij}^{nom} + \delta_{ij} r_{ij}$, $\delta_{ij} \in [-1, 1]$. The matrix $X = X^{nom} + \Delta X$ can then be written with $\Delta X = S_x \Delta_x R_x$ where $\Delta_x \in \mathbb{R}^{nm \times nm}$ is a real diagonal matrix that satisfies $\|\Delta_x\| \leq 1$ and

$$\begin{aligned} \Delta_x &= \text{diag}\{\delta_{11}, ..., \delta_{1m}, \delta_{21}, ..., \delta_{2m}, \delta_{n1}, ..., \delta_{nm}\} \\ S_x &= \text{blockdiag}\{1_{1\times m}, ..., 1_{1\times m}\} \\ R_x &= [\text{diag}\{r_{11}\}; \text{diag}\{r_{21}\}; ...; \text{diag}\{r_{n1}\}] \end{aligned}$$

for $i = 1, ..., m$. Finally, it is assumed that there exist matrices U_{R_l}, V_{R_l} such that $R_l(X)$ with a perturbed X can be written as $R_l(X) = R_l^{nom} + U_{R_l} \Delta_x V_{R_l}$. The perturbed system (7) can then be written as (Fig.3)

$$\begin{cases} \begin{bmatrix} p_{k+1} \\ e_k^{ext} \\ s_k \end{bmatrix} &= \prod_{i=1}^{q} \Upsilon_i T_i \begin{bmatrix} p_k \\ w_k \\ r_k \end{bmatrix} \\ r_k &= \Delta_x s_k, \qquad \|\Delta_x\| \leq 1 \end{cases} \tag{8}$$

with

$$\Upsilon_i = \begin{bmatrix} \Omega_i & \\ & I \end{bmatrix}, T_i = \begin{bmatrix} R_i & \\ & I \end{bmatrix} (i \neq l),$$

$$T_l = \begin{bmatrix} R_l^{nom} & U_{R_l} \\ V_{R_l} & 0 \end{bmatrix}$$

where $\|\Upsilon_i\| \leq 1 \quad (i = 1, ..., q)$.

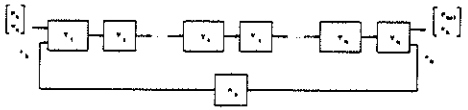


Figure 3. Perturbed NL_q system: parametric uncertainties upon the matrices of the nominal NL_q system are represented by means of a feedback perturbation Δ_x.

Lemma 1 [Perturbed NL_qs are NL_{q+1}s]. *The perturbed NL_q (8) can be written as an NL_{q+1} system with $T_{q+1} = I$ and $\Upsilon_{q+1} = \text{diag}\{I, I, \Delta_x\}$ as*

$$\begin{bmatrix} p_{k+1} \\ e_k^{ext} \\ s_k \end{bmatrix} = \prod_{i=1}^{q} \Upsilon_i T_i \begin{bmatrix} I & & \\ & I & \\ & & \Delta_x \end{bmatrix} I \begin{bmatrix} p_k \\ w_k \\ s_k \end{bmatrix} \tag{9}$$

with Δ_x real diagonal and $\|\Delta_x\|_2 \leq 1$.

□

This follows from (8) by elimination of r_k.

Theorem 1 [Diagonal scaling - perturbed NL_qs]. *Let*

$$T_{tot} = \begin{bmatrix} 0 & T_2 & & & 0 \\ & 0 & T_3 & & \\ & & \ddots & & \\ & & & 0 & T_q \\ T_1 & & & & 0 \end{bmatrix}$$

with $T_i \in \mathbb{R}^{n_{t_i} \times n_{t_{i+1}}}$, $n_{t_1} = n_{t_{q+1}}$ and $D_{tot} = \text{blockdiag}\{D_2, D_3, ..., D_q, D_{S_1}\}$, where $D_{S_1} = \text{blockdiag}\{D_{1_*}, I, D_{1_*}\}$, $D_{1_{*,b}}$ and D_i $(i = 2, .., q)$ are diagonal matrices with nonzero diagonal elements. Given the perturbed NL_q system (8), if there exist diagonal matrices D_i such that

$$\|D_{tot} T_{tot} D_{tot}^{-1}\|_2^q < 1 \tag{10}$$

then for all real diagonal matrices Δ_x that satisfy $\|\Delta_x\| \leq 1$, there exist positive constants c_1, c_2 such that

$$c_2(1 - \beta_D^2)\|p\|_2^2 + \|e\|_2^2 \leq \beta_D^2 \|w\|_2^2 + c_1 \|p_0\|_2^2 \tag{11}$$

provided that $\{w_k\}_{k=0}^{\infty} \in l_2$.

□

A proof of this Theorem can be found in [9, 11], together with 'sharper' versions related to diagonal dominance instead of diagonal scaling. It can be shown that the state space upper bound test of μ theory, related to the case of a real diagonal uncertainty block, follows as a special case from the derived Theorems for $q = 1$ (one layer) [9, 11]. The proof of Theorem 1 is based on a proof given by Packard & Doyle in [7].

5. Links with μ theory

In μ theory the influence of parametric uncertainties and unmodeled dynamics on the system's performance is studied with respect to a nominal linear model. Uncertainty is formulated then by means of LFTs. For a standard plant with exogenous input w_k, regulated output e_k and state p_k a state space formulation for an LFT looks like (Fig.4)

$$\begin{cases} \begin{bmatrix} x_{k+1} \\ e_k^{ext} \\ s_k \end{bmatrix} &= M \begin{bmatrix} x_k \\ w_k \\ r_k \end{bmatrix} \\ r_k &= \Delta \, s_k \end{cases} \tag{12}$$

with
$M = [M_{11} \; M_{12} \; M_{13}; M_{21} \; M_{22} \; M_{23}; M_{31} \; M_{32} \; M_{33}]$.

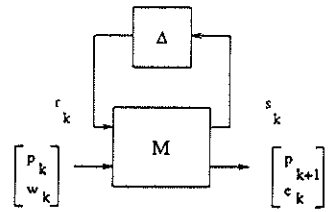


Figure 4. Linear fractional transformation (LFT) related to a standard plant. Under certain conditions this can interpreted as a special case of a perturbed NL_q system for the one-layer case $q = 1$.

In general a block uncertainty structure is considered for $\Delta \in \mathbf{\Delta} \subset \mathbb{C}^{n_{tot} \times n_{tot}}$ (see e.g. [5, 7])

$$\begin{aligned} \mathbf{\Delta} = \; &\{\text{diag}[\delta_1 I_{r_1}, ..., \delta_s I_{r_s}, \Delta_{s+1}, ..., \Delta_{s+f}] : \delta_i \in \mathbb{C}, \\ &\Delta_{s+j} \in \mathbb{C}^{m_j \times m_j}; i = 1, ..., s; j = 1, ..., f\} \end{aligned} \tag{13}$$

where $\sum_{i=1}^{s} r_i + \sum_{j=1}^{f} m_j = n_{tot}$ for consistency. The uncertainty blocks are real valued or complex valued or mixed: parametric uncertainties are real in nature, while unmodeled dynamics lead to complex values.

Theorem 2 [Packard & Doyle]. [7] *Given the LFT (12), if*

$$\min_{D_S} \|D_S M D_S^{-1}\|_2 = \beta < 1 \tag{14}$$

with $D_S = \text{diag}\{D_1, I, D\}$ and D_1, D diagonal matrices with positive elements, then the following holds for zero initial state

$$\|e\|_2^2 \leq \beta^2 \|w\|_2^2 \tag{15}$$

if $\{w_k\}_{k=0}^{\infty} \in l_2$.

□

Theorem 3 [Connection between Theorem 1-2]. *Theorem 2 is a special case of Theorem 1 under the following conditions*

1. *$q = 1$ and $\Upsilon_1 = I$*
2. *$M_{33} = 0$*
3. *Uncertainty block Δ is real*
4. *Diagonal Δ block: $r_i = 0(\forall i)$, $m_j = 1(\forall j)$, $s = 0$, $f = n_{tot}$ in (13).*

□

This is readily checked by comparing the formulas. Hence the Theorem by Packard & Doyle is a special case within NL_q theory, provided the uncertainty block is real diagonal. Like there is a close relationship between robust stability and robust performance in μ theory, this is also the case in NL_q theory. The latter becomes clear through the concept of dissipativity [9, 13, 14]. This link was also mentioned before e.g. [3, 18] but for continuous time systems. As a result conditions for robust stability and robust performance of neural control systems (robust with respect to real parametric uncertainties) can be studied in NL_q theory. Like for many problems arising in control theory (see [2] for an overview), stability conditions for NL_qs can also be formulated as LMIs, which are attractive from a computational point of view (see e.g. [6] for a general theory of polynomial-time interior point algorithms in convex programming). On the other hand, the neural controller synthesis problem leads to non-convex optimization problems (compare with $D-K$ iteration schemes for control design in μ theory).

6. Conclusion

In this paper we derived sufficient conditions for robust I/O stability with finite L_2-gain for NL_q systems. This was done by considering perturbed NL_qs, which are NL_qs with parametric uncertainties upon one of the interconnection matrices. Perturbed NL_qs can be considered as an extension of LFTs towards nominal nonlinear instead of linear models. It is shown in what sense for the case of real parametric uncertainty, the state space upper bound test in μ theory can be considered as a special case of the theorems for perturbed NL_qs.

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