KU LEUVEN

STADIUS
Center for Dynamical Systems, Signal Processing and Data Analytics

Citation/Reference	**13-14** Forouzan A., Moonen M., Maes J., Guenach M., ``Dynamic bandplanning for vectored DSL", *IEEE Transactions on Communications* , vol. 62, no. 1, Jan. 2014, pp. 302-315
Archived version	Author manuscript: Final publisher's version / pdf
Published version	insert link to the published version of your paper http://dx.doi.org/10.1109/TCOMM.2013.120413.130063
Journal homepage	insert link to the journal homepage of your paper http://www.http://ieeexplore.ieee.org
IR	url in Lirias https://lirias.kuleuven.be/handle/123456789/425894

(article begins on next page)



Dynamic Bandplanning for Vectored DSL

Amir R. Forouzan, *Senior Member, IEEE*, Marc Moonen, *Fellow, IEEE*, Jochen Maes, *Senior Member, IEEE*, and Mamoun Guenach, *Senior Member, IEEE*

***Abstract*—There are two types of crosstalk in digital subscriber line (DSL) systems, namely near-end crosstalk (NEXT) and far-end crosstalk (FEXT). NEXT is usually much stronger than FEXT. Therefore, high-speed DSL systems transmit in preplanned disjoint downstream (DS) and upstream (US) frequency bands to avoid NEXT. Although easy for implementation, such a fixed bandplan can lead to inefficient bandwidth usage, depending on the DS and US bit rate requirements and the loop topology, particularly in so-called vectored DSL systems, which include signal coordination for FEXT cancellation. In dynamic bandplanning (DBP), each frequency band is allocated to either DS, US, or to both directions depending on the aforementioned parameters. In this paper, we consider optimal DBP for vectored DSL with linear as well as with nonlinear transmitter/receiver structures. We propose an optimal DBP algorithm for systems with disjoint DS and US bands. For systems with overlapping bands, the problem of finding the optimal transmitter/receiver filters is nonconvex and we propose two iterative algorithms based on recent optimum spectrum balancing schemes from the literature. We also study the effect of echo and US NEXT cancellation on the system performance. Finally, simulation results are provided to demonstrate that our algorithms can indeed significantly increase the achievable bit rates.**

***Index Terms*—Bandplanning, crosstalk cancellation, digital subscriber line (DSL), echo cancellation, FEXT, NEXT, resource allocation, vectoring.**

I. Introduction

ONE of the main impairments of digital subscriber line (DSL) systems is crosstalk. Due to crosstalk, i.e., the electromagnetic induction between the twisted wire pairs inside a telephone cable, the signal transmitted by one DSL modem is also received by another (victim) modems. There are two types of crosstalk in DSL systems, namely, near-end crosstalk (NEXT) and far-end crosstalk (FEXT). NEXT is the crosstalk received by a (victim) modem from a transmitting modem located at the same cable end, while FEXT is the crosstalk received from a transmitting modem located at the other end of the cable (Fig. 1). NEXT is usually much stronger than FEXT, because FEXT travels a much longer distance in the cable and hence undergoes a much larger attenuation.

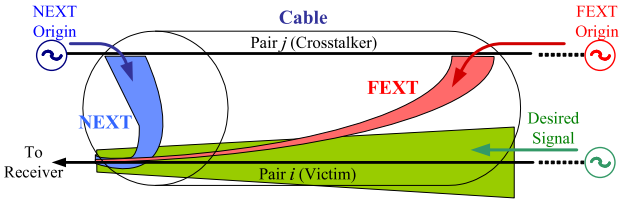


Fig. 1. Near-end and far-end crosstalk in DSL.

In recent years, the bit rate requirement of broadband services has experienced a significant boost mainly because of the increasing popularity of video-on-demand, live streaming, and on-line gaming. It has been estimated that the sum DS and US bit rate requirement of a residential customer could easily reach a few hundred Mbps up to one Gbps in a few years. The capacity of the twisted wire pair is not sufficient to deliver these bit rates over very long distances. Therefore, future DSL systems will connect consumers to the nearby cabinets in the streets located no farther than a few hundred meters. The DSL network is then connected to the backbone network using fiber-to-the curb technology. Such a topology is still much more cost efficient than a complete fiber-to-the home topology.

Earlier DSL systems used single-carrier modulation and overlapping frequency bands for the DS and US transmission. For example, SHDSL [1] used a frequency band from near DC to about 280 kHz (the 3-dB bandwidth) in both directions. However, as NEXT coupling is small at these low frequencies, NEXT did not have a catastrophic impact on the performance. Advanced high-speed DSL systems, however, use much higher frequencies. For example, VDSL2 bandplans 997E30 and 998E30 [2], [3] use up to 30 MHz of bandwidth. At these frequencies, NEXT can easily cease the communication. Another impairment in DSL is echo. Echo is the reflection of the transmitted signal in a twisted wire pair. In full duplex transmission, echo can even be much stronger than NEXT and hence should also be avoided or canceled. Therefore, current DSL systems use disjoint DS and US bands to avoid NEXT and echo. This well-known technique is called frequency-division duplexing (FDD)[1].

Manuscript received January 23, 2013; revised August 10 and November 9, 2013. The editor coordinating the review of this paper and approving it for publication was S. Galli.

This research work was carried out in part at the ESAT Laboratory of KU Leuven, in the frame of Concerted Research Action GOA-MaNet, The Belgian Programme on Interuniversity Attraction Poles initiated by the Belgian Federal Science Policy Office IUAP P6/04 (DYSCO, 2007-2011) and IUAP P7/19 (BESTCOM, 2012-2017), KU Leuven Research Council CoE PFV/10/002 'Optimization in Engineering' (OPTEC), IWT Project 'PHANTER: PHysical layer and Access Node TEchnology Revolutions: enabling the next generation broadband network'. The scientific responsibility is assumed by its authors.

A. R. Forouzan is with the Dept. of Electrical Engineering, University of Isfahan, Iran. He was with the Dept. of Electrical Engineering (ESAT-SISTA), KU Leuven, Leuven, Belgium working on this research project until Feb. 2012 (e-mail: a.forouzan@eng.ui.ac.ir).

M. Moonen is with the Dept. of Electrical Engineering (ESAT-SISTA) and IBBT Future Health Dept., KU Leuven, Leuven, Belgium (e-mail: Marc.Moonen@esat.kuleuven.be).

J. Maes and M. Guenach are with Bell Labs, Alcatel-Lucent, Antwerp, Belgium (e-mail: jochen.maes@alcatel-lucent.be, guenach@ieee.org).

Digital Object Identifier 10.1109/TCOMM.2013.120413.130063

[1]A similar technique is time-division duplexing (TDD) in which DS and US transmission take place at different time slots. Echo can be avoided using TDD but in order to also avoid NEXT, the users have to be synchronized such that all use the same time slots for DS and US transmission which is impossible in case of cable unbundling. For this main reason and a number of other reasons such as a smaller latency, current DSL systems use FDD.

0090-6778/14$31.00 © 2014 IEEE

When NEXT and echo are avoided, FEXT becomes the dominant impairment. Recently, MIMO crosstalk cancellation techniques, a.k.a. vectoring techniques, have attracted a lot of attention for FEXT cancellation [4], [5]. Using state-of-the art vectoring techniques [6]–[14], FEXT can be canceled almost perfectly and it is possible to closely approximate FEXT-free performance.

When FEXT is effectively canceled, the next promising step towards increasing the achievable bit rates in DSL systems can be dynamic bandplanning (DBP). In DBP, each frequency band is allocated to either US, DS, or both directions depending on the channel conditions and instant bit rate requirements.

In this paper, we consider optimal DBP for vectored DSL with linear as well as with nonlinear transmitter/receiver structures. We consider DBP for systems with disjoint DS and US bands as well as for systems with overlapping bands. Moreover, since the transmitted symbols in the DS direction are available at the line-termination (LT) side, it is possible to remove US NEXT using vectoring, and so we will also consider echo and US NEXT cancellation for systems with overlapping bands. We assume analog echo reduction (e.g., using hybrid circuits [15]) as well as echo cancellation in the digital domain. Perfect echo cancellation is possible in the digital domain, however, analog-to-digital converters with a higher resolution are needed.

We propose an optimal DBP algorithm for systems with disjoint bands. For systems with overlapping bands, finding the optimal vectoring scheme is a non-convex problem. We propose two iterative algorithms to solve this problem. In the first algorithm, the DS and US optimization are separated. The US optimization is based on the MAC-OSB[2] algorithm [9] and the DS optimization is based on the BC-OSB[3] algorithm [10]. In the second algorithm, the DS and US optimization are not completely separated. When the US parameters (US transmit powers and receiver filters) are optimized, DS transmit powers are also optimized using the IF/MAC-OSB[4] algorithm [16]. Similarly, when the DS parameters (DS transmit powers and transmitter filters) are optimized, US transmit powers are also optimized using the IF/BC-OSB[5] algorithm [17]. We compare the performance and computational complexity of these algorithms using computer simulations and point out a few techniques for their computational complexity reduction. Our simulation results show that these algorithms can increase the achievable bit rates significantly, particularly when echo and US NEXT cancellation are used.

Although in this paper we focus on DBP for vectored DSL, it should be noted that benefits of DBP are not merely limited to vectored DSL. Basically, DBP can be used for systems in which the power spectral density (PSD) of the users is static or systems in which spectrum balancing is feasible among users [18].

The paper is organized as follows. In Sec. II, we describe the DSL transmission model and review some results on vectoring. Then, we focus on the DBP problem and propose DBP algorithms for systems with disjoint and overlapping DS/US bands in Sec. III. Simulation results are provided in Sec. V. Finally the paper is concluded in Sec. VI.

[2]Multiple access channel optimal spectrum balancing

[3]Broadcast channel optimal spectrum balancing

[4]Interference/multiple access channel optimal spectrum balancing

[5]Interference/broadcast channel optimal spectrum balancing

Notation: In this paper, all parameters are typeset in italics except those in Greek letters which are always in regular form. Functions such as $\min\{\cdot\}$ and operators such as $(\cdot)^{\mathrm{H}}$ are in regular form. Text and text abbreviations, such as DS in $\boldsymbol{H}^k_{\mathrm{DS}}$, are in regular form too. Matrices and vectors are bold and italic, where matrices are shown by capital letters and vectors by lower case letters. The element at row n and column m of matrix $\boldsymbol{A}$ is denoted by $a_{n,m}$ or $[\boldsymbol{A}]_{n,m}$ and $\boldsymbol{a}_n$ denotes the n-th column of $\boldsymbol{A}$. Similarly, v_n denotes the n-th element of vector $\boldsymbol{v}$. Matrices $\boldsymbol{I}_N$ and $\boldsymbol{0}_N$ are the identity and all-zero matrix of size N and $\boldsymbol{e}_{N,n} \equiv (\overbrace{0,\ldots,0}^{n-1 \text{ times}},1,\overbrace{0,\ldots,0}^{N-n \text{ times}})^{\mathrm{T}}$ (or simply $\boldsymbol{e}_n$) is the n-th basis vector of size N (or of appropriate size). Integer k denotes the tone index and is always used as a superscript, whereas n and m indicate users and are used as subscripts. The transpose, inverse transpose, and conjugate transpose of a matrix or a vector are denoted by $(\cdot)^{\mathrm{T}}$, $(\cdot)^{-\mathrm{T}}$, and $(\cdot)^{\mathrm{H}}$, respectively. If $\boldsymbol{v}$ is a vector of size N, then $\mathrm{diag}\{\boldsymbol{v}\}$ denotes an $N \times N$ matrix with diagonal elements equal to $\boldsymbol{v}$ and $\|\boldsymbol{v}\|_2$ denotes its ℓ_2 norm. The union and subtraction of the sets A and B are denoted by $A \cup B$ and $A \setminus B$, respectively. Often we make similar statements for parameters defined in DS and US. For brevity, we use "DS/US" to refer to those parameters. For example instead of "$\boldsymbol{x}_{\mathrm{DS}}$ is the DS transmitted vector and $\boldsymbol{x}_{\mathrm{US}}$ is the US transmitted vector" we state "$\boldsymbol{x}_{\mathrm{DS/US}}$ is the DS/US transmitted vector".

II. System Description

We assume discrete multi-tone (DMT) modulation with K tones and synchronous transmission among N users. We also assume that the cyclic prefix and suffix are long enough to ensure orthogonality between the received signals on different tones for any two users in both directions [19]. Due to inconsistency of an optimized bandplan with standard bandplans, DBP cannot be practiced together with local-loop unbundling. Therefore, we assume that all loops in the cable are operated by the same service provider.

A. Channel Model

With disjoint bands, the transmission model depends on the transmission direction. In DS, vectoring is applied at the transmitter side. In US, vectoring is applied at the receiver side. Assuming linear encoding/decoding, the DS and US transmission on tone k are described as follows [6]

$$\boldsymbol{y}^k_{\mathrm{DS}} = \boldsymbol{H}^k_{\mathrm{DS}}\boldsymbol{Q}^k_{\mathrm{DS}}\boldsymbol{x}^k_{\mathrm{DS}} + \boldsymbol{z}^k_{\mathrm{DS}}, \qquad \text{if } k \in \mathcal{K}_{\mathrm{DS}} \text{ and} \quad (1a)$$

$$\boldsymbol{y}^k_{\mathrm{US}} = \left[\boldsymbol{Q}^k_{\mathrm{US}}\right]^{\mathrm{H}}\boldsymbol{H}^k_{\mathrm{US}}\boldsymbol{x}^k_{\mathrm{US}} + \left[\boldsymbol{Q}^k_{\mathrm{US}}\right]^{\mathrm{H}}\boldsymbol{z}^k_{\mathrm{US}}, \text{ if } k \in \mathcal{K}_{\mathrm{US}}, \quad (1b)$$

where $\boldsymbol{x}^k_{\mathrm{DS/US}}$, $\boldsymbol{y}^k_{\mathrm{DS/US}}$, and $\boldsymbol{z}^k_{\mathrm{DS/US}}$ are the $N \times 1$ DS/US transmitted, received, and noise vector respectively, $\boldsymbol{H}^k_{\mathrm{DS/US}}$ is the DS/US (direct and) FEXT channel matrix, $\boldsymbol{Q}^k_{\mathrm{DS/US}}$ is the transmitter/receiver filter matrix, and $\mathcal{K}_{\mathrm{DS/US}}$ is the set of DS/US tones. We assume that the elements of $\boldsymbol{z}^k_{\mathrm{DS/US}}$ are

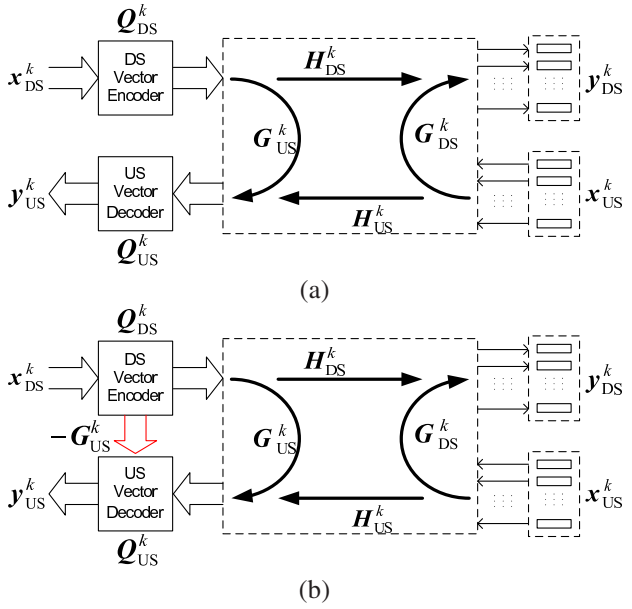


Fig. 2. Block diagram of dynamic bandplanning (a) without US NEXT cancellation and (b) with US NEXT cancellation.

independent between users and $\boldsymbol{H}^k_{\mathrm{DS/US}}$ is normalized such that

$$\mathrm{E}\left\{\boldsymbol{z}^k_{\mathrm{DS/US}}\left[\boldsymbol{z}^k_{\mathrm{DS/US}}\right]^{\mathrm{H}}\right\} = \boldsymbol{I}_N. \tag{2}$$

Assuming overlapping bands, the DS and US transmission on tone k are written as

$$\boldsymbol{y}^k_{\mathrm{DS}} = \boldsymbol{H}^k_{\mathrm{DS}}\boldsymbol{Q}^k_{\mathrm{DS}}\boldsymbol{x}^k_{\mathrm{DS}} + \boldsymbol{G}^k_{\mathrm{DS}}\boldsymbol{x}^k_{\mathrm{US}} + \boldsymbol{z}^k_{\mathrm{DS}}, \quad \text{and} \tag{3a}$$

$$\begin{aligned}\boldsymbol{y}^k_{\mathrm{US}} = &\left[\boldsymbol{Q}^k_{\mathrm{US}}\right]^{\mathrm{H}}\boldsymbol{H}^k_{\mathrm{US}}\boldsymbol{x}^k_{\mathrm{US}} \\ &+ \left[\boldsymbol{Q}^k_{\mathrm{US}}\right]^{\mathrm{H}}\boldsymbol{G}^k_{\mathrm{US}}\boldsymbol{Q}^k_{\mathrm{DS}}\boldsymbol{x}^k_{\mathrm{DS}} + \left[\boldsymbol{Q}^k_{\mathrm{US}}\right]^{\mathrm{H}}\boldsymbol{z}^k_{\mathrm{US}},\end{aligned} \tag{3b}$$

where $\boldsymbol{G}^k_{\mathrm{DS/US}}$ is the DS/US (echo and) NEXT channel matrix with its diagonal and off-diagonal elements denoting the echo and the NEXT couplings respectively. Figure 2.(a) shows a schematic diagram of a DSL network with overlapping bands. Rearranging (3) we obtain

$$\begin{aligned}\begin{bmatrix}\boldsymbol{y}^k_{\mathrm{DS}}\\ \boldsymbol{y}^k_{\mathrm{US}}\end{bmatrix} = &\begin{bmatrix}\boldsymbol{I}_N & \boldsymbol{0}_N\\ \boldsymbol{0}_N & \boldsymbol{Q}^k_{\mathrm{US}}\end{bmatrix}^{\mathrm{H}}\left(\begin{bmatrix}\boldsymbol{H}^k_{\mathrm{DS}} & \boldsymbol{G}^k_{\mathrm{DS}}\\ \boldsymbol{G}^k_{\mathrm{US}} & \boldsymbol{H}^k_{\mathrm{US}}\end{bmatrix}\right.\\ &\left.\times\begin{bmatrix}\boldsymbol{Q}^k_{\mathrm{DS}} & \boldsymbol{0}_N\\ \boldsymbol{0}_N & \boldsymbol{I}_N\end{bmatrix}\begin{bmatrix}\boldsymbol{x}^k_{\mathrm{DS}}\\ \boldsymbol{x}^k_{\mathrm{US}}\end{bmatrix} + \begin{bmatrix}\boldsymbol{z}^k_{\mathrm{DS}}\\ \boldsymbol{z}^k_{\mathrm{US}}\end{bmatrix}\right).\end{aligned} \tag{4}$$

When echo is canceled the diagonal elements of $\boldsymbol{G}^k_{\mathrm{DS/US}}$ are set to the effective echo channel gain. For example, in the case of perfect echo cancellation they are set to zero. When echo and US NEXT are canceled (Fig. 2.(b)) $\boldsymbol{G}^k_{\mathrm{US}}$ and the diagonal elements of $\boldsymbol{G}^k_{\mathrm{DS}}$ are set to the effective echo and NEXT channel gains. In the case of perfect echo and US NEXT cancellation, equations (3b) and (4) are rewritten as

$$\boldsymbol{y}^k_{\mathrm{US}} = \left[\boldsymbol{Q}^k_{\mathrm{US}}\right]^{\mathrm{H}}\boldsymbol{H}^k_{\mathrm{US}}\boldsymbol{x}^k_{\mathrm{US}} + \left[\boldsymbol{Q}^k_{\mathrm{US}}\right]^{\mathrm{H}}\boldsymbol{z}^k_{\mathrm{US}} \tag{5}$$

and

$$\begin{aligned}\begin{bmatrix}\boldsymbol{y}^k_{\mathrm{DS}}\\ \boldsymbol{y}^k_{\mathrm{US}}\end{bmatrix} = &\begin{bmatrix}\boldsymbol{I}_N & \boldsymbol{0}_N\\ \boldsymbol{0}_N & \boldsymbol{Q}^k_{\mathrm{US}}\end{bmatrix}^{\mathrm{H}}\left(\begin{bmatrix}\boldsymbol{H}^k_{\mathrm{DS}} & \boldsymbol{G}^k_{\mathrm{DS}}\\ \boldsymbol{0}_N & \boldsymbol{H}^k_{\mathrm{US}}\end{bmatrix}\right.\\ &\left.\begin{bmatrix}\boldsymbol{Q}^k_{\mathrm{DS}} & \boldsymbol{0}_N\\ \boldsymbol{0}_N & \boldsymbol{I}_N\end{bmatrix}\begin{bmatrix}\boldsymbol{x}^k_{\mathrm{DS}}\\ \boldsymbol{x}^k_{\mathrm{US}}\end{bmatrix} + \begin{bmatrix}\boldsymbol{z}^k_{\mathrm{DS}}\\ \boldsymbol{z}^k_{\mathrm{US}}\end{bmatrix}\right).\end{aligned} \tag{6}$$

B. Power, Bit Rate, and SNR

Let $\boldsymbol{q}^k_{n,\mathrm{DS/US}}$ denote the n-th column of $\boldsymbol{Q}^k_{\mathrm{DS/US}}$, let $s^k_{n,\mathrm{DS/US}}$ and $p^k_{n,\mathrm{DS/US}}$ denote DS/US the transmit power on tone k for user n and the transmit power on tone k for line n, respectively, and let $u^k_{n,\mathrm{DS/US}} \equiv \mathrm{E}\left\{\left|x^k_{n,\mathrm{DS/US}}\right|^2\right\}$, where $x^k_{n,\mathrm{DS/US}}$ is the n-th element of $\boldsymbol{x}^k_{\mathrm{DS/US}}$. In US we have $s^k_{n,\mathrm{US}} = p^k_{n,\mathrm{US}} = u^k_{n,\mathrm{US}}$. However, in DS, we have $s^k_{n,\mathrm{DS}} \equiv u^k_{n,\mathrm{DS}}\left\|\boldsymbol{q}^k_{n,\mathrm{DS}}\right\|_2^2$ and $p^k_{n,\mathrm{DS}} \equiv \left[\boldsymbol{Q}^k_{\mathrm{DS}}\,\mathrm{diag}\left\{\boldsymbol{u}^k_{\mathrm{DS}}\right\}\left[\boldsymbol{Q}^k_{\mathrm{DS}}\right]^{\mathrm{H}}\right]_{n,n}$ where in the general case $s^k_{n,\mathrm{DS}} \neq p^k_{n,\mathrm{DS}} \neq u^k_{n,\mathrm{DS}}$. The total transmit power on line n in DS/US direction is

$$P_{n,\mathrm{DS/US}} = \Delta_{\mathrm{f}} \sum_{k\in\mathcal{K}_{\mathrm{DS/US}}} p^k_{n,\mathrm{DS/US}}, \tag{7}$$

where Δ_{f} is the DMT tone spacing. The bit rate for user n in DS/US direction is

$$R_{n,\mathrm{DS/US}} = f_{\mathrm{s}} \sum_{k\in\mathcal{K}_{\mathrm{DS/US}}} b^k_{n,\mathrm{DS/US}}, \tag{8}$$

where f_{s} is the DMT symbol rate and $b^k_{n,\mathrm{DS/US}}$ is the bit loading defined as

$$b^k_{n,\mathrm{DS/US}} = \begin{cases} 0; & \tilde{b}_{\mathrm{DS/US}} < b_{\min},\\ \min\left\{b_{\max}, \tilde{b}_{\mathrm{DS/US}}\right\}; & \text{otherwise,}\end{cases} \tag{9}$$

where $\tilde{b}_{\mathrm{DS/US}} = \left\lfloor \log_2\left(1 + \frac{1}{\Gamma}\mathrm{SNR}^k_{n,\mathrm{DS/US}}\right)\right\rfloor$, $\Gamma = 9.75$ dB is the SNR gap and $b_{\min} = 2$ and $b_{\max} = 15$ are the minimum and maximum number of bits that can be loaded to a tone [2]. The SNR depends on the transmitter/receiver structure. With linear encoders/decoders and disjoint bands, the SNR is

$$\mathrm{SNR}^k_{n,\mathrm{DS}} = \frac{u^k_{n,\mathrm{DS}}\left|\boldsymbol{e}_n^{\mathrm{T}}\boldsymbol{H}^k_{\mathrm{DS}}\boldsymbol{q}^k_{n,\mathrm{DS}}\right|^2}{1 + \sum_{m\neq n} u^k_{m,\mathrm{DS}}\left|\boldsymbol{e}_n^{\mathrm{T}}\boldsymbol{H}^k_{\mathrm{DS}}\boldsymbol{q}^k_{m,\mathrm{DS}}\right|^2}, \tag{10}$$

if $k \in \mathcal{K}_{\mathrm{DS}}$ and

$$\mathrm{SNR}^k_{n,\mathrm{US}} = \frac{u^k_{n,\mathrm{US}}\left|\left[\boldsymbol{q}^k_{n,\mathrm{US}}\right]^{\mathrm{H}}\boldsymbol{H}^k_{\mathrm{US}}\boldsymbol{e}_n\right|^2}{\left\|\boldsymbol{q}^k_{n,\mathrm{US}}\right\|_2^2 + \sum_{m\neq n} u^k_{m,\mathrm{US}}\left|\left[\boldsymbol{q}^k_{n,\mathrm{US}}\right]^{\mathrm{H}}\boldsymbol{H}^k_{\mathrm{US}}\boldsymbol{e}_m\right|^2}, \tag{11}$$

if $k \in \mathcal{K}_{\mathrm{US}}$.

If $k \in \mathcal{K}_{\mathrm{DS}}$ and a nonlinear vector Tomlinson-Harashima precoder (THP) is used for the DS trasmission, the SNR is

$$\mathrm{SNR}^k_{n,\mathrm{DS}} = \frac{u^k_{n,\mathrm{DS}}\left|\boldsymbol{e}_n^{\mathrm{T}}\boldsymbol{H}^k_{\mathrm{DS}}\boldsymbol{q}^k_{n,\mathrm{DS}}\right|^2}{1 + \sum_{m\in\,\mathcal{I}_{n,\mathrm{DS}}} u^k_{m,\mathrm{DS}}\left|\boldsymbol{e}_n^{\mathrm{T}}\boldsymbol{H}^k_{\mathrm{DS}}\boldsymbol{q}^k_{m,\mathrm{DS}}\right|^2}, \tag{12}$$

where $\mathcal{I}_{n,\mathrm{DS}}$ is the set of interfering users for user n in DS, i.e. the users that are encoded after user n.

Similarly, if $k \in \mathcal{K}_{\mathrm{US}}$ and a generalized decision feedback equalizer (GDFE) is used for the US reception, the SNR is

$$\mathrm{SNR}^k_{n,\mathrm{US}} = \frac{u^k_{n,\mathrm{US}}\left|\left[\boldsymbol{q}^k_{n,\mathrm{US}}\right]^{\mathrm{H}}\boldsymbol{H}^k_{\mathrm{US}}\boldsymbol{e}_n\right|^2}{\left\|\boldsymbol{q}^k_{n,\mathrm{US}}\right\|_2^2 + \sum_{m\in\,\mathcal{I}_{n,\mathrm{US}}} u^k_{m,\mathrm{US}}\left|\left[\boldsymbol{q}^k_{n,\mathrm{US}}\right]^{\mathrm{H}}\boldsymbol{H}^k_{\mathrm{US}}\boldsymbol{e}_m\right|^2}, \tag{13}$$

where $\mathcal{I}_{n,\mathrm{US}}$ is the set of interfering users for user n in US, i.e., the users that are decoded after user n.

When overlapping bands are used, NEXT and echo contribute to the received interference as well. In this case, the following term has to be added to the denominator of (10) and (12):

$$\sum_{m=1}^{N} u_{m,\mathrm{US}}^{k} \left| \boldsymbol{e}_n^{\mathrm{T}} \boldsymbol{G}_{\mathrm{DS}}^{k} \boldsymbol{e}_m \right|^2 \tag{14}$$

and the following term has to be added to the denominator of (11) and (13) (unless NEXT and echo are canceled in the US direction):

$$\sum_{m=1}^{N} u_{m,\mathrm{DS}}^{k} \left| \left[\boldsymbol{q}_{n,\mathrm{US}}^{k}\right]^{\mathrm{H}} \boldsymbol{G}_{\mathrm{US}}^{k} \boldsymbol{q}_{m,\mathrm{DS}}^{k} \right|^2. \tag{15}$$

III. Dynamic Bandplanning Algorithms

In this paper, we consider the DBP problem in the following weighted sum rate maximization form:

$$\begin{aligned}
\text{maximize} \quad & \sum_{n=1}^{N} w_{n,\mathrm{DS}} R_{n,\mathrm{DS}} + \sum_{n=1}^{N} w_{n,\mathrm{US}} R_{n,\mathrm{US}} && \text{(16a)}\\
\text{subject to} \quad & P_{n,\mathrm{DS}} \le P_{n,\mathrm{DS}}^{\max}; \quad 1 \le n \le N, && \text{(16b)}\\
& P_{n,\mathrm{US}} \le P_{n,\mathrm{US}}^{\max}; \quad 1 \le n \le N, && \text{(16c)}\\
& p_{n,\mathrm{DS}}^{k} \le p_{n,\mathrm{DS}}^{k,\mathrm{mask}}; \quad 1 \le n \le N, 1 \le k \le K, && \text{(16d)}\\
\text{and} \quad & p_{n,\mathrm{US}}^{k} \le p_{n,\mathrm{US}}^{k,\mathrm{mask}}; \quad 1 \le n \le N, 1 \le k \le K, && \text{(16e)}
\end{aligned}$$

where $w_{n,\mathrm{DS/US}}$, $R_{n,\mathrm{DS/US}}$, $P_{n,\mathrm{DS/US}}$, and $P_{n,\mathrm{DS/US}}^{\max}$ are the DS/US bit rate weight factor and bit rate for user n, and the total transmit power and total transmit power budget for line n, respectively, and $p_{n,\mathrm{DS/US}}^{k}$ and $p_{n,\mathrm{DS/US}}^{k,\mathrm{mask}}$ are the DS/US transmit power and the nominal PSD mask for line n on tone k.

For a system with disjoint DS and US bands the maximization takes place over $\mathcal{K}_{\mathrm{DS/US}}$, $\boldsymbol{Q}_{\mathrm{DS/US}}^{k}$, and DS/US transmit powers. For a system with overlapping DS and US bands, the maximization takes place over $\boldsymbol{Q}_{\mathrm{DS/US}}^{k}$ and DS/US transmit powers. If a nonlinear receiver/transmitter structure (GDFE or vector THP) is used, the decoding/encoding order of the users is also optimized.

The Lagrange dual decomposition approach has been shown to be optimal for MIMO-OFDM problems when the number of tones is large enough [20]–[23]. Using dual decomposition techniques, (16) is decoupled into the following K independent problems

$$\begin{aligned}
&\text{maximize} \quad \mathcal{L}^{k},\\
&\text{subject to (16d) and (16e),} \qquad \text{for } k = 1 \ldots K
\end{aligned} \tag{17}$$

where

$$\begin{aligned}
\mathcal{L}^{k} \equiv & \textstyle\sum_{n=1}^{N} w_{n,\mathrm{DS}} b_{n,\mathrm{DS}}^{k} + \sum_{n=1}^{N} w_{n,\mathrm{US}} b_{n,\mathrm{US}}^{k}\\
& - \textstyle\sum_{n=1}^{N} \lambda_{n,\mathrm{DS}} p_{n,\mathrm{DS}}^{k} - \sum_{n=1}^{N} \lambda_{n,\mathrm{US}} p_{n,\mathrm{US}}^{k},
\end{aligned} \tag{18}$$

and $\lambda_{n,\mathrm{DS/US}}$ are the Lagrange multipliers corresponding to the DS/US per line total power constraints (16b) and (16c). The Lagrange multipliers should be optimized to satisfy the corresponding constraints. For more details see [24].

A. Disjoint Bands

For a system with disjoint bands, depending on $k \in \mathcal{K}_{\mathrm{DS}}$ or $k \in \mathcal{K}_{\mathrm{US}}$, $\mathcal{L}^{k}$ is simplified to

$$\mathcal{L}_{\mathrm{DS}}^{k} \equiv \sum_{n=1}^{N} w_{n,\mathrm{DS}} b_{n,\mathrm{DS}}^{k} - \sum_{n=1}^{N} \lambda_{n,\mathrm{DS}} p_{n,\mathrm{DS}}^{k} \tag{19}$$

or

$$\mathcal{L}_{\mathrm{US}}^{k} \equiv \sum_{n=1}^{N} w_{n,\mathrm{US}} b_{n,\mathrm{US}}^{k} - \sum_{n=1}^{N} \lambda_{n,\mathrm{US}} p_{n,\mathrm{US}}^{k}. \tag{20}$$

Thus, in order to determine the transmission direction on tone k for a particular set of Lagrange multipliers, we solve the following two problems

$$\text{maximize} \quad \mathcal{L}_{\mathrm{DS}}^{k}, \tag{21}$$

$$\text{maximize} \quad \mathcal{L}_{\mathrm{US}}^{k}. \tag{22}$$

Let $\hat{\mathcal{L}}_{\mathrm{DS}}^{k}$ and $\hat{\mathcal{L}}_{\mathrm{US}}^{k}$ denote the corresponding solutions. If $\hat{\mathcal{L}}_{\mathrm{DS}}^{k} > \hat{\mathcal{L}}_{\mathrm{US}}^{k}$ then $k \in \mathcal{K}_{\mathrm{DS}}$, otherwise $k \in \mathcal{K}_{\mathrm{US}}$. The optimal solutions to (21) and (22) can be found using the BC-OSB and MAC-OSB algorithms proposed in [10] and [9], respectively. A pseudocode for this algorithm is presented in Alg. 1, where we assume that vector THP and GDFE are used in the DS transmitter and US receiver respectively. The calculation of the optimal receiver/transmitter filters and powers for the US and DS is explained in Sec. III-A1 and Sec. III-A2, respectively. The Lagrange multipliers $\lambda_{n,\mathrm{DS/US}}$ can be initialized and updated using many different algorithms such as the bisection search [20], sub-gradient ascend [25], step-adaptive sub-gradient ascent [26], or generalized iterative facet dividing algorithm (IFDA) [27].

1) Optimal Receiver Filters: One-Sided Coordination: For the US, consider the following multiple access channel (MAC) (with simplified notation):

$$\boldsymbol{y} = \boldsymbol{Q}^{\mathrm{H}} \boldsymbol{H} \boldsymbol{x} + \boldsymbol{Q}^{\mathrm{H}} \boldsymbol{z} \tag{23}$$

where the elements of $\boldsymbol{z}$ are Gaussian random variables with covariance matrix $\boldsymbol{Z} \equiv \mathrm{E}\left\{\boldsymbol{z}\boldsymbol{z}^{\mathrm{H}}\right\}$. Note that in DBP with overlapping bands, the contribution of echo and NEXT can be included in $\boldsymbol{z}$. The elements of $\boldsymbol{x}$ are i.i.d Gaussian variables with power $u_n = \mathrm{E}\left\{|x_n|^2\right\}$ and $\boldsymbol{u} \equiv (u_1, u_2, \ldots, u_N)^{\mathrm{T}}$. The n-th column of $\boldsymbol{Q}$, $\boldsymbol{q}_n$, denotes the receiver filter for user n. The SNR maximizing receive filter is the minimum mean square equalizer (MMSE) which is obtained by [28]

$$\boldsymbol{q}_n = \boldsymbol{C}_n^{-1} \boldsymbol{h}_n. \tag{24}$$

where $\boldsymbol{C}_n$ is the covariance matrix of the total noise and interference for user n. For a linear receiver we have

$$\boldsymbol{C}_n = \boldsymbol{Z} + \sum_{m \ne n} u_m \boldsymbol{h}_m \boldsymbol{h}_m^{\mathrm{H}}. \tag{25}$$

where $\boldsymbol{h}_m$ is the m-th column of $\boldsymbol{H}$. For a nonlinear GDFE receiver, we have

$$\boldsymbol{C}_n = \boldsymbol{Z} + \sum_{m \in \mathcal{I}_n} u_m \boldsymbol{h}_m \boldsymbol{h}_m^{\mathrm{H}}. \tag{26}$$

where $\mathcal{I}_n$ is the set of interfering users for user n. Using the above equations the optimal receiver filters can be calculated from the transmit powers. With the MMSE filter calculated

Algorithm 1: DBP with disjoint bands

input : DS/US direct and FEXT channels, DS/US PSD masks, DS/US per-modem total power budgets
output: DS/US bandplans, DS/US bit loadings, DS/US transmit powers, DS transmit and US receive filters, DS encoding and US decoding orders

repeat
  Set/update $\lambda_{n,\mathrm{DS/US}},\ \forall\ n$;
  for $k = 1$ **to** K **do**
    Calculate the maximum achievable Lagrangian in DS direction, $\hat{\mathcal{L}}^k_{\mathrm{DS}}$, by solving (21) using BC-OSB algorithm [10];
    Calculate the maximum achievable Lagrangian in US direction, $\hat{\mathcal{L}}^k_{\mathrm{US}}$, by solving (22) using MAC-OSB algorithm [9];
    if $\hat{\mathcal{L}}^k_{\mathrm{DS}} > \hat{\mathcal{L}}^k_{\mathrm{US}}$ **then**
      $\mathcal{K}_{\mathrm{DS}} \leftarrow \mathcal{K}_{\mathrm{DS}} \cup \{k\}$;
      $\mathcal{K}_{\mathrm{US}} \leftarrow \mathcal{K}_{\mathrm{US}} \backslash \{k\}$;
      Store/update the optimal solution on tone k in DS direction;
    else
      $\mathcal{K}_{\mathrm{US}} \leftarrow \mathcal{K}_{\mathrm{US}} \cup \{k\}$;
      $\mathcal{K}_{\mathrm{DS}} \leftarrow \mathcal{K}_{\mathrm{DS}} \backslash \{k\}$;
      Store/update the optimal solution on tone k in US direction;
  $P_{n,\mathrm{DS/US}} \leftarrow \Delta_{\mathrm{f}} \sum_{k=1}^{K} p^{k,\ \mathrm{opt}}_{n,\mathrm{DS/US}},\ \forall\ n$;
until $P_{n,\mathrm{DS/US}} = P^{\max}_{n,\mathrm{DS/US}}$ or $(P_{n,\mathrm{DS/US}} < P^{\max}_{n,\mathrm{DS/US}}$ and $\lambda_{n,\mathrm{DS/US}} = 0),\ \forall\ n$;

in (24), the SNR for user n for both a linear and a nonlinear receiver as defined in (11) and (13) reduce to

$$\mathrm{SNR}_n = u_n \boldsymbol{q}_n^{\mathrm{H}} \boldsymbol{h}_n. \tag{27}$$

The bit loading then follows from (9).

Conversely, given the bit loadings, it is possible to calculate the required transmit powers and the optimal receiver filters. For the GDFE receiver, these can be explicitly calculated one by one starting from the last decoded user to the first decoded user. For each user n, $\boldsymbol{C}_n$ is calculated using (26) given the transmit power for all users that are decoded later. Then $\boldsymbol{q}_n$ and $u_n = \mathrm{SNR}_n / \left(\boldsymbol{q}_n^{\mathrm{H}} \boldsymbol{h}_n\right)$ are calculated using (24) and (27), respectively. For linear receivers, the transmit powers can be calculated from the bit-loadings using an iterative algorithm. In the first iteration, the transmit power of all users is assumed to be zero. The transmit powers are updated in each iteration by calculating $\boldsymbol{C}_n$ from (25) using the latest values of u_m $(m \neq n)$. Then $\boldsymbol{q}_n$ and u_n are calculated using (24) and (27), respectively. Simulation results show that this algorithm converges to the solution (if feasible) in 3 to 4 iterations [16].

2) Optimal Transmitter Filters: One-Sided Coordination: For the DS, the BC-MAC duality theory [21], [29]–[31] can be used to calculate the optimal transmit filters and powers. Consider the following broadcast channel (BC) (with simplified notation) and assume first that linear encoding is used:

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{z}, \tag{28}$$

where the channel is normalized such that $\boldsymbol{z}$ has unit power i.i.d Gaussian elements and so $\mathrm{E}\left\{\boldsymbol{z}\boldsymbol{z}^{\mathrm{H}}\right\} = \boldsymbol{I}$ (the correlation between the elements of $\boldsymbol{z}$ is ignored because it cannot be exploited to improve the performance [10], [17]). The dual MAC of (28) is defined by

$$\tilde{\boldsymbol{y}} = \boldsymbol{Q}^{\mathrm{H}} \boldsymbol{H}^{\mathrm{H}} \tilde{\boldsymbol{x}} + \boldsymbol{Q}^{\mathrm{H}} \tilde{\boldsymbol{z}}, \tag{29}$$

where $\tilde{\boldsymbol{z}}$ again has unit power i.i.d Gaussian elements. Let $\tilde{u}_n = \mathrm{E}\left\{\left|\tilde{x}_n\right|^2\right\}$ and $\tilde{\boldsymbol{u}} \equiv (\tilde{u}_1, \tilde{u}_2, \ldots, \tilde{u}_N)^{\mathrm{T}}$. The MMSE filters are SNR maximizing in the dual MAC and can be calculated using (24). Let $\boldsymbol{\gamma} \equiv (\gamma_1, \gamma_2, \ldots, \gamma_N)^{\mathrm{T}}$ denote a set of achievable SNR's in the dual MAC corresponding to a set of bit loadings $\boldsymbol{\beta} = (b_1, \ldots, b_N)^{\mathrm{T}}$ based on (9). The BC-MAC duality states that the same set of SNR's (and hence bit loadings) can be achieved in the primal BC. The required transmit powers to reach $\boldsymbol{\gamma}$ in the primal BC is calculated by

$$\boldsymbol{u} = \boldsymbol{A}^{-1} \boldsymbol{1}_N \tag{30}$$

and the required transmit powers to reach $\boldsymbol{\gamma}$ in the dual MAC is calculated by [31]

$$\tilde{\boldsymbol{u}} = \boldsymbol{A}^{-\mathrm{T}} \boldsymbol{\sigma}, \tag{31}$$

where $\boldsymbol{\sigma}$ is a column vector of size N containing the diagonal elements of $\boldsymbol{Q}^{\mathrm{H}}\boldsymbol{Q}$, and $\boldsymbol{A}$ depends on the transmitter structure. For a linear encoder $\boldsymbol{A}$ is defined as

$$a_{n,m} = \begin{cases} \gamma_n^{-1} \left|\hat{h}_{n,n}\right|^2, & \text{if } m = n \\ -\left|\hat{h}_{n,m}\right|^2, & \text{if } m \neq n, \end{cases} \tag{32}$$

where $\hat{\boldsymbol{H}} \equiv \boldsymbol{H}\boldsymbol{Q}$. For a nonlinear encoder, i.e. when vector THP is used, we have to use the dual nonlinear receiver structure, i.e., the GDFE, in the dual MAC [29]. The decoding order in the GDFE must be the reverse of the encoding order in the THP. That is, assuming the decoding order in the GDFE is $1, 2, \ldots, N$, then the encoding order in the THP is $N, N-1, \ldots, 1$. Equation (30) can still be used to calculate the transmit powers from the SNR's where $\boldsymbol{A}$ is redefined as:

$$a_{n,m} = \begin{cases} \gamma_n^{-1} \left|\hat{h}_{n,n}\right|^2, & \text{if } m = n \\ -\left|\hat{h}_{n,m}\right|^2, & \text{if } m \in \mathcal{I}_n \\ 0, & \text{otherwise}, \end{cases} \tag{33}$$

where $\mathcal{I}_n$ is the set of interfering users for user n in the primal BC, i.e., the set of users encoded after user n in the primal BC or equivalently those decoded before user n in the dual MAC.

Obviously, the achievable rates for the GDFE and vector THP depend on the order of decoding and encoding. In [16] it has been shown that setting the decoding order according to the order of US weight factors $\{w_{1,\mathrm{US}}, \ldots, w_{N,\mathrm{US}}\}$ such that the user with the smallest weight factor is decoded first and the user with the largest weight factor is decoded last is near-optimal in US DSL. Also, setting the encoding order in the vector THP such that the user with the largest DS weight factor is encoded first and the user with the smallest DS weight factor is encoded last is near-optimal in DS DSL [17].

From the BC-MAC duality, we also know that

$$\sum s_n = \sum \tilde{u}_n \tag{34a}$$

$$\text{and} \qquad \sum p_n = \sum \tilde{u}_n, \tag{34b}$$

where

$$p_n \equiv \left[\boldsymbol{Q}\text{diag}\left\{\boldsymbol{u}\right\}\boldsymbol{Q}^{\text{H}}\right]_{n,n} \tag{35}$$

is the transmit power on line n. Equation (34a) is very helpful in solving optimization problems for which the utility function and the constraints are functions of the (per-user) bit rates together with the sum transmit power for all the users. For example the weighted sum rate maximization problem with sum power constraint in the BC can be solved in the dual MAC directly using (34a).

However, in DSL, e.g. as in (17), we usually have constraints on the per-line rather than the per-user transmit powers. This problem has been addressed in [31], where the BC-MAC duality is shown to be a special case of Lagrange duality. Briefly, if we scale the noise vector in (29) such that $\text{E}\left\{|\tilde{z}_n|^2\right\} = \lambda_n$ and calculate the optimal filters and the corresponding transmit powers in the primal and the dual channel with scaled noise using (24) to (30), we obtain $\sum \lambda_n p_n = \sum \tilde{u}_n$. This result can be used to solve optimization problems with constraints on per-line transmit powers, where λ_n plays the role of the Lagrange multiplier corresponding to the power constraint on line n.

By increasing λ_n the transmit power on line n decreases and vice versa. The optimal value of λ_1 to λ_N, satisfying all power constraints and leading to the maximum weighted sum rate, is usually calculated in an iterative process. However, in [17] it has been shown that due to the the row-wise diagonal dominance (RWDD) property [6] of the DS channel, setting $\lambda_1 = \lambda_2 = \ldots = \lambda_N = 1$ is near optimal in DSL.

B. Overlapping Bands

In DBP with overlapping bands, full signal coordination is not available. However vectoring is applied at both sides. Consider the following channel with (partial) vectoring at both sides

$$\boldsymbol{y} = \boldsymbol{Q}_{\text{rx}}^{\text{H}}\boldsymbol{H}\boldsymbol{Q}_{\text{tx}}\boldsymbol{x} + \boldsymbol{Q}_{\text{rx}}^{\text{H}}\boldsymbol{z}. \tag{36}$$

Here $\boldsymbol{y}$ is a $2N \times 1$ vector, which is consistent with (4) and (6). In this section, we will then speak of $2N$ users (rather than N users), namely N DS users together with N US users. The dual of (36) is

$$\tilde{\boldsymbol{y}} = \boldsymbol{Q}_{\text{tx}}^{\text{H}}\boldsymbol{H}^{\text{H}}\boldsymbol{Q}_{\text{rx}}\tilde{\boldsymbol{x}} + \boldsymbol{Q}_{\text{tx}}^{\text{H}}\tilde{\boldsymbol{z}}. \tag{37}$$

Using the theory reviewed in Sec. III-A1, from (36), we can find the optimal value of $\boldsymbol{Q}_{\text{rx}}$ (maximizing (18)) provided that $\boldsymbol{Q}_{\text{tx}}$ and the transmit powers are known. On the other hand from Sec. III-A2 and (37), we can find the optimal value of $\boldsymbol{Q}_{\text{tx}}$ provided that $\boldsymbol{Q}_{\text{rx}}$ and the transmit powers in the dual channel are known. However, the optimal transmitter and receiver filters cannot be calculated jointly because the Lagrangian is non-convex in either case. For (36), it is non-convex in terms of $\boldsymbol{Q}_{\text{tx}}$ and for (37) it is non-convex in terms of $\boldsymbol{Q}_{\text{rx}}$. It is known that when full signal coordination exists among the users at both the receiver and transmitter side, waterfilling against the singular values of the channel matrix is capacity achieving [32]. Unfortunately, full signal coordination is not available for the DBP case with overlapping bands as can be seen in (4) and (6). For this reason, we consider iterative techniques for solving (17). In the following, we propose two algorithms to solve this problem.

1) DBP Algorithms for Systems with Overlapping Bands: The **first algorithm** is based on the MAC-OSB [9] and BC-OSB [10] and is essentially an extension of Alg. 1 to systems with overlapping bands where we again assume that vectored THP and GDFE are used in the DS and US respectively. We call this algorithm, DBP with overlapping bands and separate DS/US optimization. A pseudocode for this algorithm is presented in Alg. 2.

The DS and US transmission are iteratively optimized. For US optimization, the DS transmitter filters and transmit powers, i.e., $\boldsymbol{Q}_{\text{DS}}^k$ and $\boldsymbol{u}_{\text{DS}}^k$, are fixed to those of the best solution found so far. Then for all US bit loading combinations, $\boldsymbol{Q}_{\text{US}}^k$ and $\boldsymbol{u}_{\text{US}}^k$ are calculated using the formulas given in Sec. III-A1 with term (15) added in the SNR formula. By changing $\boldsymbol{u}_{\text{US}}^k$, the NEXT power and consequently the SNR for the DS transmission will change (eqn. (14)). Therefore, for each set of US bit loadings, DS bit loadings and the Lagrangian are recalculated. If the Lagrangian is the largest Lagrangian so far, the solution is updated.

Similarly for the DS optimization, $\boldsymbol{u}_{\text{US}}^k$ and $\boldsymbol{Q}_{\text{US}}^k$ are fixed to those of the best solution found so far. DS optimization is carried out in the dual channel. For all DS bit loading vectors in the range, the optimal transmit filters and required transmit powers in the dual channel are calculated. Using (30), the transmit powers are mapped back to the primal channel. Then, the achievable US bit loadings and the Lagrangian are recalculated. If the Lagrangian is the largest Lagrangian so far, the solution is updated.

The order of DS and US optimization can affect the achievable rates for this algorithm. When DS users are not co-located and echo is canceled but US NEXT is not canceled, it can be better to optimize the US receiver filters and powers first and then optimize the DS transmit filters and powers. However, when echo and US NEXT are both canceled, optimizing DS and then US would potentially lead to the optimal solution. The reason is explained in Sec. V.

The DS and US optimization are repeated iteratively until convergence. The convergence is detected when the Lagrangian is within 0.1 % of the Lagrangian in the previous iteration. Usually 3 to 4 iterations are enough for convergence.

The **second algorithm** (Alg. 3) is based on the IF/MAC-OSB [16] and IF/BC-OSB [17] algorithm. Unlike in Alg. 2, the exhaustive search over the DS and US bit loadings is not separated. The algorithm is executed for each combination of DS and US bit loadings. Using an iterative scheme, it is checked if the combination is reachable and if this is the case, the solution is updated provided that the corresponding Lagrangian is larger than the largest Lagrangian found so far.

Similar to Alg. 2, the core of this algorithm consists of two main routines where in the first routine, $\boldsymbol{Q}_{\text{DS}}^k$ is fixed and $\boldsymbol{u}_{\text{DS}}^k$, $\boldsymbol{u}_{\text{US}}^k$, and $\boldsymbol{Q}_{\text{US}}^k$ are optimized. In the second routine, $\boldsymbol{Q}_{\text{US}}^k$ is fixed and $\boldsymbol{u}_{\text{US}}^k$, $\boldsymbol{u}_{\text{DS}}^k$, and $\boldsymbol{Q}_{\text{DS}}^k$ are optimized in the dual channel. However in Alg. 2, the first routine is

Algorithm 2: DBP with overlapping bands and separate DS/US power optimization

input : DS/US direct, echo, FEXT and NEXT channels, DS/US PSD masks, DS/US per-modem total power budgets
output: DS/US bit loadings, DS/US transmit powers, DS transmit and US receive filters, DS encoding and US decoding orders

repeat
  Set/update $\lambda_{n,\mathrm{DS/US}},\ \forall\ n$;
  for $k = 1$ **to** K **do**
    $\boldsymbol{u}^k_{\mathrm{DS/US}} \leftarrow \mathbf{0}$;
    $\boldsymbol{Q}^k_{\mathrm{DS/US}} \leftarrow \boldsymbol{I}_N$;
    repeat
      `/* US Power and Receiver Filter Optimization (at fixed DS power and transmit filters) */`
      Set $\boldsymbol{u}^k_{\mathrm{DS}}$ and $\boldsymbol{Q}^k_{\mathrm{DS}}$ to the so-far optimal solution on tone k;
      for all US bit loading vectors $\boldsymbol{b}^k_{\mathrm{US}}$ in the range **do**
        for $n =$ *last decoded user in US direction* **to** *first decoded user* **do**
          $\gamma^k_{n,\mathrm{US,des}} \leftarrow \Gamma\left(2^{b_{n,\mathrm{US}}} - 1\right)$;
          Update US receiver filter and transmit power for user n, ($\boldsymbol{q}^k_{n,\mathrm{US}}$ and $u^k_{n,\mathrm{US}}$) to reach $\gamma^k_{n,\mathrm{US,des}}$;
        Update achievable SNRs and bit loadings (only in DS direction);
        if $p^k_{n,\mathrm{DS/US}} \le p^{k,\mathrm{mask}}_{n,\mathrm{DS/US}}, \forall n$ and $\mathcal{L}^k > \mathcal{L}^k_{\max}$ **then**
          $\mathcal{L}^k_{\max} \leftarrow \mathcal{L}^k$;
          Store/update the optimal solution on tone k;
      `/* DS Power and Transmit Filter Optimization in Dual Space (at fixed US power and receiver filters) */`
      Set $\boldsymbol{u}^k_{\mathrm{DS/US}}$ and $\boldsymbol{Q}^k_{\mathrm{DS/US}}$ to the so-far optimal solution on tone k;
      Calculate transmit powers in dual channel using (42);
      for all DS bit loading vectors $\boldsymbol{b}^k_{\mathrm{DS}}$ in the range **do**
        for $n =$ *first encoded user in DS direction* **to** *last encoded user* **do**
          $\gamma^k_{n,\mathrm{DS,des}} \leftarrow \Gamma\left(2^{b_{n,\mathrm{DS}}} - 1\right)$;
          Update DS transmit filter and power for user n, ($\boldsymbol{q}^k_{n,\mathrm{DS}}$ and $\tilde{u}^k_{n,\mathrm{DS}}$) to reach $\gamma^k_{n,\mathrm{DS,des}}$;
        Update achievable SNRs and bit loadings (only in US direction);
        Calculate transmit powers in primal channel, $\boldsymbol{u}^k_{\mathrm{DS/US}}$, using (41);
        if $p^k_{n,\mathrm{DS/US}} \le p^{k,\mathrm{mask}}_{n,\mathrm{DS/US}}, \forall n$ and $\mathcal{L}^k > \mathcal{L}^k_{\max}$ **then**
          $\mathcal{L}^k_{\max} \leftarrow \mathcal{L}^k$;
          Store/update the optimal solution on tone k;
    until *convergence/divergence/maximum # of iterations*;
until $P_{n,\mathrm{DS/US}} = P^{\max}_{n,\mathrm{DS/US}}$ or $(P_{n,\mathrm{DS/US}} < P^{\max}_{n,\mathrm{DS/US}}$ and $\lambda_{n,\mathrm{DS/US}} = 0),\ \forall\ n$;

executed to reach the US bit loading vector ($\boldsymbol{b}^k_{\mathrm{US}}$) maximizing the Lagrangian. As the transmit powers in US change, the potentially optimal DS bit loading vector ($\boldsymbol{b}^k_{\mathrm{DS}}$) change as well. Similarly, the second routine is executed to reach the locally optimal DS bit loading vector, which may change the potentially optimal US bit loading vector. However, in Alg. 3, the search is carried out over all DS and US bit loading vectors ($\left(\boldsymbol{b}^k_{\mathrm{DS}}, \boldsymbol{b}^k_{\mathrm{US}}\right)$) maximizing the Lagrangian together.

The two routines are iterated until the procedure converges, diverges, or the maximum number of iterations is reached. Joint calculation of the optimal transmitter/receiver filters and DS and US transmit powers when the receiver/transmitter filters are fixed is explained in more details in Sec. III-B2.

In Alg. 3 the two routines are executed for all DS and US bit loadings combinations (so the size of search space is $(b_{\max} - b_{\min} + 2)^{2N}$), while in Alg. 2, first a search over all US bit loading combinations is executed and then a search over all DS bit loading combinations is executed (i.e., two exhaustive searches of size $(b_{\max} - b_{\min} + 2)^{N}$). The benefit of this extra computational complexity is that the algorithm is less likely to converge to a local optimum compared to Alg. 2.

2) Optimal Transmitter/Receiver Filters: Two-Sided Coordination: In (36), assuming full coordination at the receiver side, the MMSE receiver filter for user n, $\boldsymbol{q}_{n,\mathrm{rx}}$ is obtained by

$$\boldsymbol{q}_{n,\mathrm{rx}} = \hat{\boldsymbol{C}}_n^{-1}\hat{\boldsymbol{h}}_n, \tag{38}$$

where $\hat{\boldsymbol{h}}_n$ is the n-th column of $\hat{\boldsymbol{H}} \equiv \boldsymbol{H}\boldsymbol{Q}_{\mathrm{tx}}$. The noise covariance matrix, $\hat{\boldsymbol{C}}_n$, depends on the transmitter and receiver structure. Assuming linear transmitter and receiver structures

Algorithm 3: DBP with overlapping bands and joint DS/US power optimization

input : DS/US direct, echo, FEXT and NEXT channels, DS/US PSD masks, DS/US per-modem total power budgets
output: DS/US bit loadings, DS/US transmit powers, DS transmit and US receive filters, DS encoding and US decoding orders

repeat
  Set/update $\lambda_{n,\mathrm{DS/US}},\ \forall\ n$;
  for $k = 1$ **to** K **do**
    for all bit loading vectors $\left(\boldsymbol{b}_{\mathrm{DS}}^{k}, \boldsymbol{b}_{\mathrm{US}}^{k}\right)$ in the range **do**
      $\boldsymbol{u}_{\mathrm{DS}}^{k} \leftarrow \mathbf{0}$;
      $\boldsymbol{Q}_{\mathrm{DS}}^{k} \leftarrow \boldsymbol{I}_{N}$;
      $\gamma_{n,\mathrm{DS/US,des}}^{k} \leftarrow \Gamma\left(2^{b_{n,\mathrm{DS/US}}} - 1\right), \forall n$;
      repeat
        /* US Power and receiver Filter Optimization (at fixed DS power and transmit filters) */
        for $n =$ *last decoded user in US direction* **to** *first decoded user* **do**
          Update US receiver filter and transmit power for user n, ($\boldsymbol{q}_{n,\mathrm{US}}^{k}$ and $u_{n,\mathrm{US}}^{k}$) to reach $\gamma_{n,\mathrm{US,des}}^{k}$;
        Update achievable SNRs (only in DS direction);
        /* DS Power and Transmit Filter Optimization in Dual Space (at fixed US power and receiver filters) */
        Calculate transmit powers in dual channel using (42);
        for $n =$ *first encoded user in DS direction* **to** *last encoded user* **do**
          Update DS transmit filter and power for user n, ($\boldsymbol{q}_{n,\mathrm{DS}}^{k}$ and $\tilde{u}_{n,\mathrm{DS}}^{k}$) to reach $\gamma_{n,\mathrm{DS,des}}^{k}$;
        Update achievable SNRs (only in US direction);
        Calculate transmit powers in primal channel, $\boldsymbol{u}_{\mathrm{DS/US}}^{k}$, using (41);
      until *convergence/divergence/maximum # of iterations*;
      if *convergence,* $p_{n,\mathrm{DS/US}}^{k} \le p_{n,\mathrm{DS/US}}^{k,\mathrm{mask}}, \forall n$ and $\mathcal{L}^{k} > \mathcal{L}_{\max}^{k}$ **then**
        $\mathcal{L}_{\max}^{k} \leftarrow \mathcal{L}^{k}$;
        Store/update the optimal solution on tone k;
until $P_{n,\mathrm{DS/US}} = P_{n,\mathrm{DS/US}}^{\max}$ or $(P_{n,\mathrm{DS/US}} < P_{n,\mathrm{DS/US}}^{\max}$ and $\lambda_{n,\mathrm{DS/US}} = 0),\ \forall\ n$;

we have

$$\hat{\boldsymbol{C}}_{n} = \boldsymbol{Z} + \sum_{m \neq n} u_{m} \hat{\boldsymbol{h}}_{m} \hat{\boldsymbol{h}}_{m}^{\mathrm{H}}. \tag{39}$$

For a nonlinear vector THP transmitter and a nonlinear GDFE receiver, we have

$$\hat{\boldsymbol{C}}_{n} = \boldsymbol{Z} + \sum_{m \in \mathcal{I}_{n,\mathrm{tx}} \cap \mathcal{I}_{n,\mathrm{rx}}} u_{m} \hat{\boldsymbol{h}}_{m} \hat{\boldsymbol{h}}_{m}^{\mathrm{H}}. \tag{40}$$

where $\mathcal{I}_{n,\mathrm{tx/rx}}$ is the set of interfering users for user n at the transmitter/receiver side.

The above formulas hold for partial signal coordination among the users too. When two users n and m are not coordinated at the signal level, we have $q_{n,m,\mathrm{rx}} = 0$. However, the non-zero elements of $\boldsymbol{q}_{n,\mathrm{rx}}$ are still calculated optimally by the MMSE criterion using (38) after removing the rows and columns of $\hat{\boldsymbol{C}}_{n}$ and $\hat{\boldsymbol{h}}_{n}$ corresponding to the zero elements of $\boldsymbol{q}_{n,\mathrm{rx}}$ [17].

The transmit powers for the channel described by (36) and its dual (37) are obtained by

$$\boldsymbol{u} = \bar{\boldsymbol{A}}^{-1} \boldsymbol{\sigma}_{\mathrm{rx}}, \tag{41}$$

and

$$\tilde{\boldsymbol{u}} = \bar{\boldsymbol{A}}^{-\mathrm{T}} \boldsymbol{\sigma}_{\mathrm{tx}}, \tag{42}$$

where $u_{n} \equiv \mathrm{E}\left\{|x_{n}|^{2}\right\}$, $\tilde{u}_{n} \equiv \mathrm{E}\left\{|\tilde{x}_{n}|^{2}\right\}$, the column vectors $\boldsymbol{\sigma}_{\mathrm{rx}}$ and $\boldsymbol{\sigma}_{\mathrm{tx}}$ are defined as the diagonal elements of $[\boldsymbol{Q}_{\mathrm{rx}}]^{\mathrm{H}} \boldsymbol{Q}_{\mathrm{rx}}$ and $[\boldsymbol{Q}_{\mathrm{tx}}]^{\mathrm{H}} \boldsymbol{Q}_{\mathrm{tx}}$, respectively, and $\bar{\boldsymbol{A}}$ is a $2N \times 2N$ matrix which depends on the transmitter and receiver structure. Assuming linear transmitter and receiver structure, $\bar{\boldsymbol{A}}$ is defined as

$$\bar{a}_{n,m} = \begin{cases} \gamma_{n}^{-1} \left|\bar{h}_{n,m}\right|^{2}, & \text{if } m = n \\ -\left|\bar{h}_{n,m}\right|^{2}, & \text{otherwise.} \end{cases} \tag{43}$$

where γ_{n} is the SNR for user n and $\bar{\boldsymbol{H}} = \boldsymbol{Q}_{\mathrm{rx}}^{\mathrm{H}} \boldsymbol{H} \boldsymbol{Q}_{\mathrm{tx}}$. For a nonlinear vector THP transmitter and a nonlinear GDFE receiver, $\bar{\boldsymbol{A}}$ is defined as

$$\bar{a}_{n,m} = \begin{cases} \gamma_{n}^{-1} \left|\bar{h}_{n,m}\right|^{2}, & \text{if } m = n \\ -\left|\bar{h}_{n,m}\right|^{2}, & \text{if } m \in \mathcal{I}_{n,\mathrm{rx}} \cap \mathcal{I}_{n,\mathrm{tx}} \\ 0, & \text{otherwise,} \end{cases} \tag{44}$$

where $\mathcal{I}_{n,\mathrm{rx}}$ is the set of users of which the interference is not removed for user n at the receiver side (i.e., the users that are decoded after user n in the GDFE) and $\mathcal{I}_{n,\mathrm{tx}}$ is the set of users of which the interference is not removed for user n at the transmitter side (i.e., the users that are encoded after user n in the vector THP).

Equations (41) and (42) can be directly used for systems with overlapping bands, where

$$\boldsymbol{Q}_{\mathrm{tx}} \equiv \begin{bmatrix} \boldsymbol{Q}_{\mathrm{DS}}^{k} & \boldsymbol{0}_{N} \\ \boldsymbol{0}_{N} & \boldsymbol{I}_{N} \end{bmatrix}, \boldsymbol{Q}_{\mathrm{rx}} \equiv \begin{bmatrix} \boldsymbol{I}_{N} & \boldsymbol{0}_{N} \\ \boldsymbol{0}_{N} & \boldsymbol{Q}_{\mathrm{US}}^{k} \end{bmatrix} \tag{45}$$

and depending on whether (echo and) US NEXT is canceled or not, respectively,

$$\boldsymbol{H} \equiv \begin{bmatrix} \boldsymbol{H}_{\mathrm{DS}}^{k} & \boldsymbol{G}_{\mathrm{DS}}^{k} \\ \boldsymbol{0}_{N} & \boldsymbol{H}_{\mathrm{US}}^{k} \end{bmatrix} \text{ or } \boldsymbol{H} \equiv \begin{bmatrix} \boldsymbol{H}_{\mathrm{DS}}^{k} & \boldsymbol{G}_{\mathrm{DS}}^{k} \\ \boldsymbol{G}_{\mathrm{US}}^{k} & \boldsymbol{H}_{\mathrm{US}}^{k} \end{bmatrix}. \tag{46}$$

The sets $\mathcal{I}_{n,\mathrm{tx}}$ and $\mathcal{I}_{n,\mathrm{rx}}$ are calculated as a function of $\mathcal{I}_{n,\mathrm{DS}}$ and $\mathcal{I}_{n,\mathrm{US}}$ as follows. Based on the definition of $\boldsymbol{Q}_{\mathrm{tx}}$ and $\boldsymbol{Q}_{\mathrm{rx}}$ in (45), if $1 \leq n \leq N$ then user n is the n-th DS user while if $N+1 \leq n \leq 2N$ then it is the $(n-N)$-th US user. For the n-th DS user, the interfering users are $\mathcal{I}_{n,\mathrm{DS}}$ plus all US users (which interfere through the US NEXT channel). Similarly for the n-th US user, the interfering users are $\{m+N \,|m \in \mathcal{I}_{n,\mathrm{US}}\}$ plus DS users (which interfere through the DS NEXT channel). Therefore

$$\mathcal{I}_{n,\mathrm{tx}} = \begin{cases} \mathcal{I}_{n,\mathrm{DS}} \cup \{N+1,\ldots,2N\} & n \leq N, \\ \{1,\ldots,2N\} \setminus \{n\} & n > N \end{cases} \tag{47}$$

and

$$\mathcal{I}_{n,\mathrm{rx}} = \begin{cases} \{1,\ldots,2N\} \setminus \{n\} & n \leq N, \\ \{1,\ldots,N\} \cup \{m+N \,|m \in \mathcal{I}_{n,\mathrm{US}}\} & n > N. \end{cases} \tag{48}$$

IV. Computational Complexity

The computational complexity of Alg. 1 is equal to the sum computational complexities of BC-OSB and MAC-OSB algorithms for N users. The computational complexity of Alg. 2 is equal to the sum computational complexities of BC-OSB and MAC-OSB algorithms for N users multiplied by the number of iterations required for convergence of the loop in steps 6 to 27. Similarly, the computational complexity of Alg. 3 is equal to the sum computational complexities of IF/BC-OSB and IF/MAC-OSB algorithms for a group of size N and N groups of size 1 multiplied by the number of iterations required for convergence of the loop in steps 8 to 17.

Note that the BC-OSB and MAC-OSB algorithms are special cases of IF/BC-OSB and IF/MAC-OSB algorithms where there is only a single group of size N. The computational complexity of IF/BC-OSB and IF/MAC-OSB algorithms has been studied in [17] and [16] and a few techniques to reduce the computational complexity has been proposed. Both of the algorithms involve an exhaustive search. The exhaustive search can be carried out over the space of multi-user bit loadings or power loadings. In Algs. 2 and 3, the search is carried out over the space of bit loadings. Here we only study the computational complexity of Algs. 2 and 3. The computational complexity of Alg. 1 would depend on its implementation which is not stated explicitly in the pseudocode. However, one should be able to estimate the computational complexity of Alg. 1 from the following discussion.

Based on the analyses in [16] and [17], the computational complexity of Alg. 2 is $\mathcal{O}\left(\beta_{\lambda} K \beta_{\mathrm{conv}} b_{\max}^{N} N^{3}\right)$, where β_{λ} is the number of iterations required to find $\left\{\lambda_{n,\mathrm{DS/US}}, \forall n\right\}$, K is the number of tones, $\beta_{\mathrm{conv}} \approx 3,4$ is the average number of iterations of the loop in steps 6 to 27, $b_{\max}^{N}$ is the size of multiuser bit loading space for N DS or N US users, and N^3 is the computational complexity of calculating the receiver/transmit filters and primary and dual channel powers. Parameter β_{λ} depends on the algorithm used for updating the Lagrange multipliers. Using generalized IFDA [27], we would have $\beta_{\lambda} = \mathcal{O}\left(2N \log \frac{1}{\epsilon}\right)$, where ϵ is the desired precision.

Similarly, the computational complexity of Alg. 3 is $\mathcal{O}\left(\beta_{\lambda} K b_{\max}^{2N} \beta_{\mathrm{conv}} \left(2N\right)^{3}\right)$. Here $b_{\max}^{2N}$ is the size of multiuser bit loading space for all DS and US users, and $(2N)^3$ is the computational complexity associated with calculating the transmit powers in primal and dual channels. The rest of the parameters are the same as those for Alg. 2.

The computational complexity of the proposed algorithms is exponential in the number of users, which is because of the exhaustive search over the space of bit loadings as part of the OSB algorithm. Fortunately, several near-optimal spectrum balancing algorithms have been proposed in the literature replacing the exhaustive search with fast search algorithms with polynomial complexity in N. A survey on these algorithms can be found in [4]. More recent algorithms can be found in [33]–[35].

TABLE I
Simulation Parameters

PARAMETER	VALUE
Bandplan (when applicable)	VDSL2E17 [2], [3]
Maximum transmission PSD (all cases)	-60 dBm/Hz
Cable type	26 AWG [36]
Noise	White noise, -140 dBm/Hz
Tone spacing, Δ_{f}	4.3125 kHz
Symbol rate, f_{s}	4 kHz
$b_{\max}$	15
$b_{\min}$	2
SNR gap	9.45 dB
Echo signal's gain	-15 dB
Crosstalk model	ETSI standard 1 % worst-case model [37]

V. Simulation Results

We have simulated the proposed DBP algorithms for the VDSL2 997E17 [3] scenarios depicted in Fig. 3. The scenarios include (a) a 2-line near-far scenario, (b) a 2-line equi-length scenario, (c) a 3-line equi-length scenario, and (d) a 3-line distributed scenario. The simulation parameters are listed in Table I and simulation results are presented in Figs. 4 to 12.

In Fig. 4, the achievable sum bit rates of the users in US direction is plotted vs. the achievable sum bit rate of the users in DS direction for different algorithms and simulation assumptions for the 2-line near-far scenario. The achievable sum bit rates by Alg. 2 when a) echo and b) echo and US NEXT is/are canceled and the achievable sum bit rates by Alg. 3 when a) echo b) US NEXT, and c) echo and US NEXT are canceled and when d) echo and US NEXT are not canceled are plotted in the figure. The achievable sum bit rates for the same vectoring schemes when a fixed VDSL2 997E17

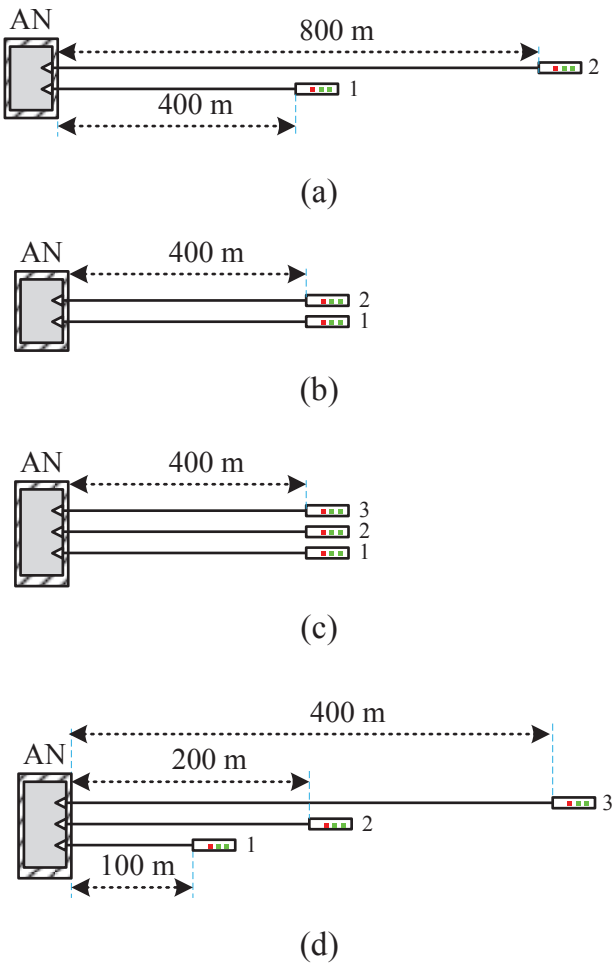


Fig. 3. Simulated scenarios: (a) the 2-line near-far, (b) the 2-line equi-length, (c) the 3-line equi-length, and (d) the 3-line distributed scenario.

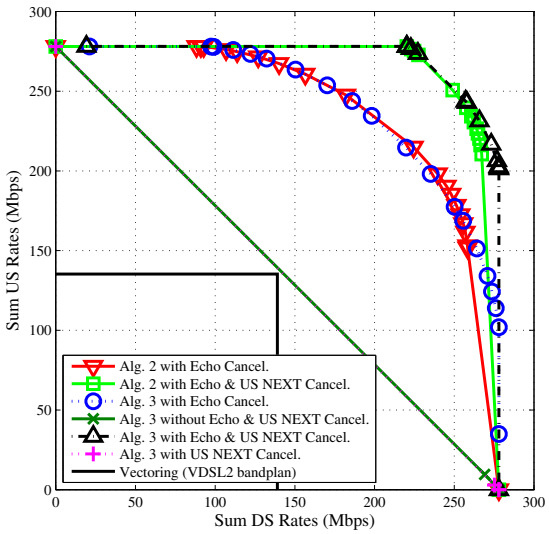


Fig. 4. Achievable sum bit rate of the users in US direction vs. achievable sum bit rate of the users in DS direction for the 2-user near-far scenario in Fig. 3.(a) using different vector DBP schemes.

bandplan is used is also plotted. A few simulation results are not included in Fig. 4 for the sake of higher readability. These include simulation results for Alg. 1 and simulation results for Alg. 2 (i.e., DBP with disjoint bands) when echo is not canceled (regardless of US NEXT being canceled or not). All of these results overlap with the results for Alg. 3 when echo is not canceled. The reason will be discussed shortly.

As expected, the best performance for Algs. 2 and 3 is achieved when echo and US NEXT are both canceled. The achievable sum bit rates in this case are significantly higher than the achievable sum bit rates for the same vectoring schemes [9], [10] using a fixed VDSL2 bandplan. In fact, they are almost equal to the achievable bit rates as if the entire band is fully available to both DS and US transmission (even though DS NEXT is not canceled). When only echo is canceled but US NEXT is not canceled, the achievable sum bit rates are lower but still much higher than the achievable sum bit rates by the same vectoring schemes using a fixed VDSL2 bandplan.

When echo is not canceled (whether or not US NEXT is canceled), the rate region takes a right-triangular shape with its hypotenuse connecting the points at which the entire bandwidth is allocated to either DS or US. This is due to the fact that the echo signal is much stronger than the desired signal such that when transmitting power on a tone in US/DS, the SNR for DS/US is too small to justify DS/US transmission. In fact the performance of Alg. 1 for systems with disjoint bands is exactly the same as Alg. 2 and Alg. 3 when echo is present. Therefore, for higher readability of the figure, we have not plotted the results for Alg. 1 and for Alg. 2 in the presence of echo.

The rate region for vectoring with a fixed bandplan has a rectangular shape. This is an expected behavior as DS and US transmissions are then separated and do not affect each other.

Finally, we compare the performance of Alg. 2 with that of Alg. 3 for the cases with echo cancellation. It can be observed that Alg. 2 performs the same as Alg. 3 when higher bit rates are demanded in US direction. However, Alg. 2 does not achieve the same sum bit rates in DS direction as Alg. 3 when higher bit rates are demanded in the DS direction. This is because the DS and US optimization are separated in Alg. 2. Since the US optimization is carried out first (see Alg. 2) too many bits (requiring too much power) can be loaded to some tones in the US direction. In the subsequent DS optimization, the excessive crosstalk power from the US direction leads to a reduced power and bit loading on these tones in the DS direction. Consequently, the algorithm converges to a local optimum leading to a suboptimal performance. In the rest of this section, we study the performance of Alg. 3 with echo and US NEXT cancellation unless stated explicitly.

From Fig. 4, the maximum sum achievable bit rates in DS and US directions for the 2-user near-far scenario increase by 56 % when echo is canceled and by 80 % when echo and US NEXT are canceled. This improvement in performance is partly due to the topology of the near-far scenario. Note that vectoring is capable of cancelling FEXT almost perfectly. Moreover, DS NEXT coupling is small relative to the desired signal channel transfer function for the 400 m loop. Therefore, when echo and US NEXT are canceled, the only considerable crosstalk term would be the DS NEXT for the 800 m line

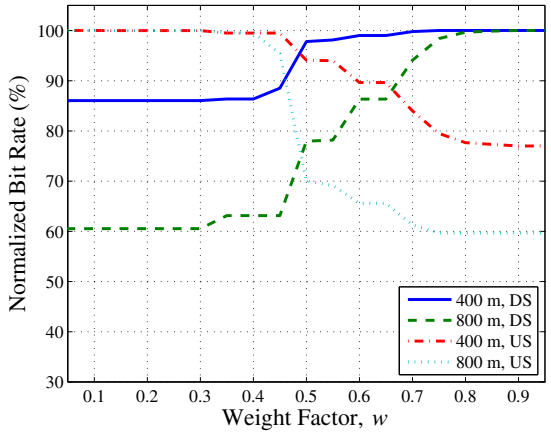


Fig. 5. Normalized bit rate of the users in DS and US directions vs. w for the 2-user near-far scenario in Fig. 3.(a) for Alg. 3 when echo and US NEXT are cancelled.

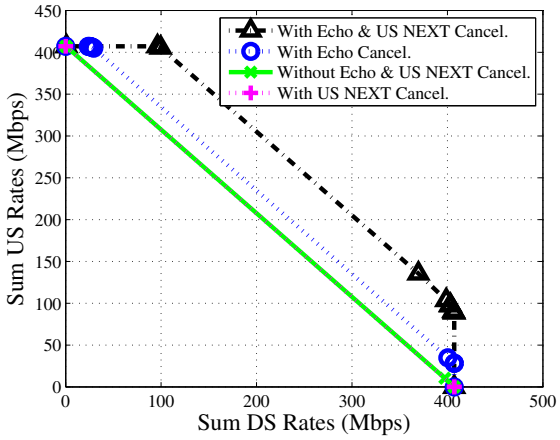


Fig. 6. Achievable sum bit rate of the users in US direction vs. achievable sum bit rate of the users in DS direction for the 2-user equi-length scenario in Fig. 3.(b) using Alg. 3.

resulting in a 80 % increase in the sum bit rate. When echo is canceled but US NEXT is not canceled, the resulting increase in the sum bit rate reduces to 56 %. As we will see later, when the lines have equal lengths and modems at the customer premises (CP) are co-located as in the 2-user and 3-user equi-length scenarios in Fig. 3.(b) and (c), the DS NEXT couplings are much stronger and the increase in the bit rates is much smaller.

To obtain the rate region (RR) in Figs. 4 and 6, we have set the DS weight factors $w_{n,\mathrm{DS}} = w$ $(n = 1, 2)$ and the US weight factors $w_{n,\mathrm{US}} = 1 - w$ $(n = 1, 2)$, where w is an independent variable. Then w is swept from 0 to 1 in 0.05 steps. The normalized achievable bit rates in DS and US by Alg. 3 when echo and US NEXT are canceled is plotted vs. w in Fig. 5. The maximum achievable bit rates for the 800 m and 400 m lines are $R_{1,\mathrm{DS/US}} = 74.40$ and $R_{2,\mathrm{DS/US}} = 203.73$, respectively. At the sum rate maximization point the achievable bit rates are $R_{1,\mathrm{DS}} = 58.00$, $R_{2,\mathrm{DS}} = 199.12$, $R_{1,\mathrm{US}} = 52.08$, and $R_{2,\mathrm{US}} = 191.67$ and the sum bit rate is 500.87 Mbps. That is the bit rate of the longer line decreases by 22 % (16.40 Mbps) and 30 % (22.32 Mbps) in DS and US directions compared to their maximum, respectively, while the bit rate of the shorter line decreases by 2.2 % (4.48 Mbps) and 9.4 % (11.93 Mbps) in DS and US directions, respectively. As expected the longer line has a smaller capacity. Moreover, the reduction of the bit rate at the sum rate maximization point is larger for this line. To overcome, we can assign larger weight factors to the longer line. For example by choosing $w_{1,\mathrm{DS/US}} = 0.9$ and $w_{2,\mathrm{DS/US}} = 0.1$, we obtain $R_{1,\mathrm{DS/US}} = 74.40$, $R_{2,\mathrm{DS}} = 175.20$, and $R_{2,\mathrm{US}} = 156.93$. That is the longer line achieves 100 % of its maximum bit rates in both directions and the shorter line achieves 86 % and 77 % of its maximum bit rates in DS and US directions, respectively. The sum bit rate is 480.93 Mbps, i.e., 96 % of the maximum achievable sum bit rate.

The transmit PSD and loaded bits per tone for the near-far scenario at the maximum sum bit rate point is plotted in Fig.7 . The figure shows that both lines can transmit in both directions from 0 to 8 MHz. In this part of the spectrum, both DS

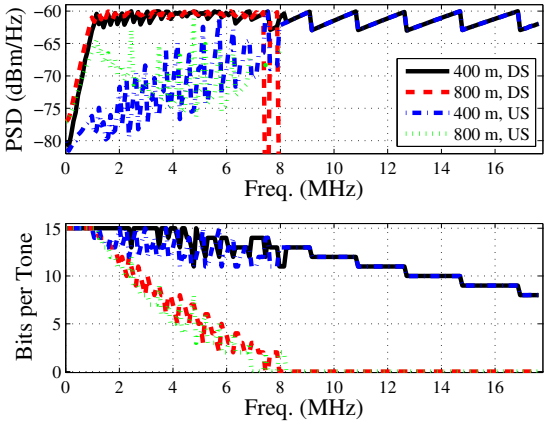


Fig. 7. The PSD and loaded bits per tone obtained by Alg. 3 for the 2-user near-far scenario at the sum bit rate maximization point.

transmit PSDs are at the PSD mask level (i.e., -60 dBm/Hz). That is because the FEXT and NEXT signals associated to the DS signals are perfectly canceled. However, the US PSDs are smaller because the associated NEXT may harm users in DS. For frequencies above 8 MHz, transmission takes place on the 400 m line at the PSD mask level in both directions. This is expected as shorter line can achieve much higher capacity than the longer line with perfect echo cancellation.

The sum US bit rates vs. the sum DS bit rates for the 2-user equi-length scenario is plotted in Fig. 6. In this scenario, the DS NEXT couplings are much stronger. As a result the achievable sum bit rates are smaller. Nevertheless, an increase of about 6.9 % and 24.1 % in the sum bit rate is achieved using Alg. 3 when echo is canceled and when echo and US NEXT are canceled, respectively.

The transmit PSD and loaded bits per tone for this scenario at the maximum sum bit rate point is plotted in Fig. 8. It can be seen that full-duplex transmission takes place for up to about 8 MHz, which is similar to the result obtained for the near-far scenario. The optimal transmit power in DS is again at the PSD mask level, however, the one for US can

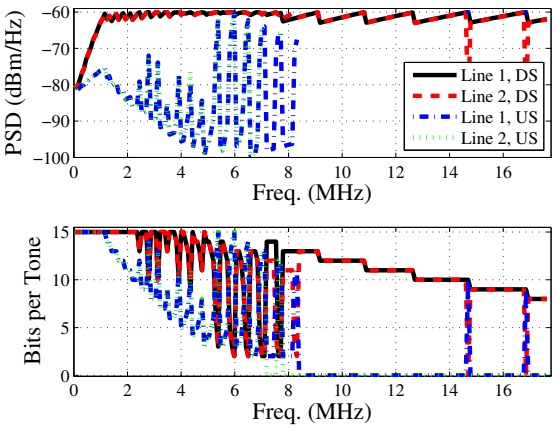


Fig. 8. The PSD and loaded bits per tone obtained by Alg. 3 for the 2-line equi-length scenario at the sum bit rate maximization point.

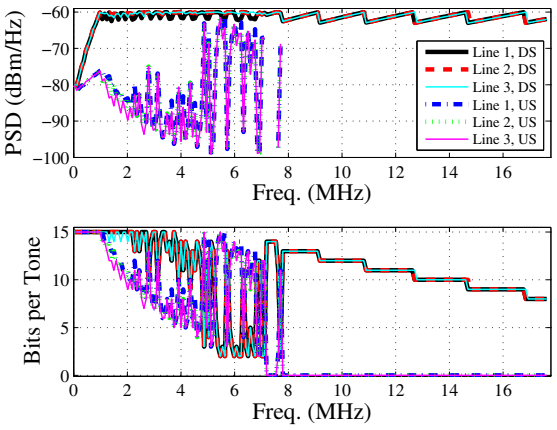


Fig. 9. The PSD and loaded bits per tone obtained by Alg. 3 for the 3-line equi-length scenario at the sum bit rate maximization point.

be up to 40 dB smaller. This is again because the US NEXT is canceled at the central office. For the frequencies above 8 MHz, the transmission is only in the DS direction except for a couple of frequencies at which we have full duplex transmission on line 1. In fact, since the lines are exactly similar, DS only transmission, US only transmission, and full-duplex transmission on either of the lines result in sum bit rate maximization solutions.

The algorithm is also simulated to find the maximum sum rate solution for the 3-user equi-length and the 3-user distributed scenarios in Figs. 3.(a) and (b). The resulting bit rates for the 3-user equi-length scenario are $R_{1,\mathrm{DS}} = 178.24$, $R_{2,\mathrm{DS}} = 178.32$, $R_{3,\mathrm{DS}} = 178.00$, $R_{1,\mathrm{US}} = 70.97$, $R_{2,\mathrm{US}} = 68.89$, and $R_{3,\mathrm{US}} = 68.08$ Mbps and the sum bit rate is 742.50 Mbps. The sum bit rate shows 21.6 % increase compared to the case without echo and US NEXT cancellation. This is only 2.5 % smaller than the increase for the 2-user equi-length scenario. Figure 9 shows the resulting PSD and number of loaded bits per tone. The results are similar to those for the 2-user equi-length scenario. Once again, we see full-duplex transmission for up to about 8 MHz. The achievable rate region for this scenario is plotted in Fig. 10.

The achievable bit rates for the 3-user distributed scenario at the maximum sum rate point are $R_{1,\mathrm{DS}} = 173.52$, $R_{2,\mathrm{DS}} = 186.72$, $R_{3,\mathrm{DS}} = 246.00$, $R_{1,\mathrm{US}} = 179.93$, $R_{2,\mathrm{US}} = 189.40$, and $R_{3,\mathrm{US}} = 243.69$ Mbps and the sum bit rate is 1219.26 Mbps. The sum bit rate increase for this scenario is 75.3 %. Obviously, similar to the first scenario, the reason is the weaker DS NEXT couplings when the CP's are not co-located. The resulting PSD and number of loaded bits per tone for this solution is plotted in Fig. 11. Unlike the first three scenarios, the optimal solution leads to full-duplex transmission for all users for the entire bandwidth (near d.c. to 17.76 MHz). This can be attributed to the shorter loop lengths in this scenario. Figure 12 shows the rate region for this scenario.

The per-tone transmit power plots in Figs. 9 and 11 are helpful in determining the order of DS and US optimizations in Alg. 2. Assume echo and US NEXT are canceled. Figures 9

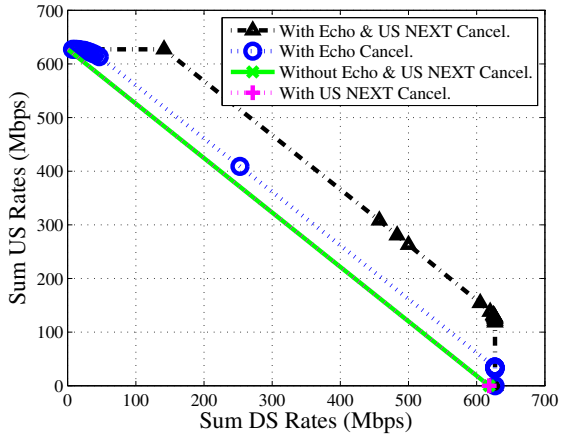


Fig. 10. Achievable sum bit rate of the users in US direction vs. achievable sum bit rate of the users in DS direction for the 3-line equi-length scenario in Fig. 3.(c) using Alg. 2.

and 11 show that the US and DS transmit powers are low and high at the optimal point, respectively. If optimization takes place first in US, the DS rates are zero at the first iteration and will not be affected by DS NEXT. Therefore, a large amount of power is assigned to US tones, particularly at high frequencies where the attenuation is large, to achieve highest possible sum rates. Thus, at the end of the first US optimization, the solution is deviated more from the optimal solution compared to the starting point. However, if the first optimization is carried out in DS direction, DS powers increase and approach the optimal solution particularly in high frequencies. When US optimization is carried out, the users can achieve higher rates in US by increasing their US transmit powers, because US NEXT is canceled. Because of DS NEXT, increasing US powers results in reduced rates in DS. The US rate increase and DS rate decrease are both taken into account in finding the optimal point via the Lagrangian. Therefore, the overall solution gets closer to the optimal solution.

In a distributed scenario, where echo is canceled and US NEXT is not canceled, US NEXT couplings can be much stronger than DS NEXT couplings. Therefore, using the same

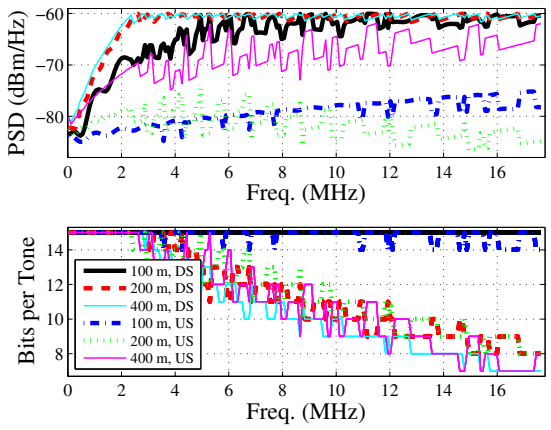


Fig. 11. The PSD and loaded bits per tone obtained by Alg. 3 for the 3-line distributed scenario at the sum bit rate maximization point.

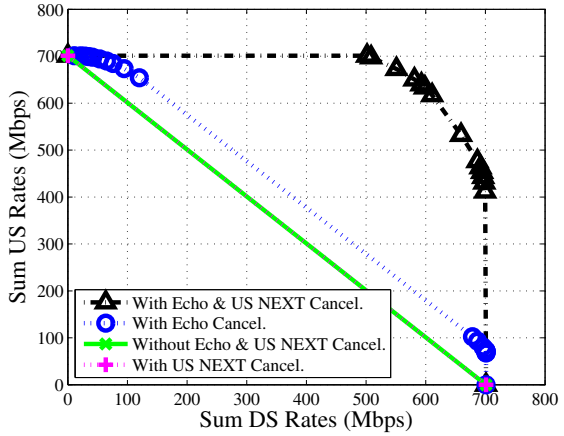


Fig. 12. Achievable sum bit rate of the users in US direction vs. achievable sum bit rate of the users in DS direction for the 3-line distributed scenario in Fig. 3.(d) using Alg. 2.

logic, we conclude that the optimization order should be US and then DS in this case. In scenarios in which DS and US NEXT couplings are not significantly different, we can try both optimization orders for best results.

VI. Conclusion

In this paper, we have studied dynamic bandplanning (DBP) in vectored DSL. We have proposed an optimal algorithm for DBP in DSL with disjoint bands and two iterative sub-optimal algorithms for DBP in DSL with overlapping bands. We have studied the performance of the proposed algorithms by several simulations which show that DBP can significantly increase the achievable bit rates of high speed DSL systems when vectoring is enabled. In systems with disjoint DS and US bands, DBP can increase the achievable bit rates by allocating the available bandwidth to the DS and US transmission much more efficiently, taking into account the topology and instant bit rate requirements. More importantly, the simulation results show that 20 % to 80 % higher bit rates can be achieved by letting DS and US bands overlap and by including echo and US NEXT cancellation. Full-duplex transmission (i.e., overlapping DS and US bands) would be feasible for much higher frequencies (e.g., the entire bandwidth for loop lengths smaller than 400 m, and up to 8 MHz for loop lengths of 400 to 800 m) compared to DSL systems without vectoring in which full duplex transmission is feasible for up to 600 kHz only.

References

[1] ITU-T G.991.2, "Single-pair high-speed digital subscriber line (SHDSL) transceivers," Feb. 2001. Available: http://www.itu.int/rec/T-REC-G.991.2/en

[2] ITU-T G.993.2, "Very high speed digital subscriber line transceivers 2 (VDSL2)," Geneva, Switzerland, Feb. 2006.

[3] ITU-T G.993.2; Amendment 1, "Very high speed digital subscriber line transceivers 2 (VDSL2)," Geneva, Switzerland, Apr. 2007.

[4] S. Huberman, C. Leung, and T. Le-Ngoc, "Dynamic spectrum management (DSM) algorithms for multi-user xDSL," *IEEE Commun. Surveys Tuts.*, vol. 14, no. 1, pp. 109–130, Oct. 2012.

[5] V. Oksman, H. Schenk, A. Clausen, J. Cioffi, M. Mohseni, G. Ginis, C. Nuzman, J. Maes, M. Peeters, K. Fisher, and P.-E. Eriksson, "The ITU-T's new G.vector standard proliferates 100 Mb/s DSL," *IEEE Commun. Mag.*, vol. 48, no. 10, pp. 140–148, Oct. 2010.

[6] G. Ginis and J. M. Cioffi, "Vectored transmission for digital subscriber line systems," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 5, pp. 1085–1104, Jun. 2002.

[7] R. Cendrillon, G. Ginis, E. Van den Bogaert, and M. Moonen, "A near-optimal linear crosstalk canceler for upstream VDSL," *IEEE Trans. Signal Process.*, vol. 54, no. 8, pp. 3136–3146, Aug. 2006.

[8] ——, "A near-optimal linear crosstalk precoder for downstream VDSL," *IEEE Trans. Commun.*, vol. 55, no. 5, pp. 860–863, May 2007.

[9] P. Tsiaflakis, J. Vangorp, J. Verlinden, and M. Moonen, "Multiple access channel optimal spectrum balancing for upstream DSL transmission," *IEEE Commun. Lett.*, vol. 11, no. 4, pp. 398–300, Apr. 2007.

[10] V. Le Nir, M. Moonen, J. Verlinden, and M. Guenach, "Optimal power allocation for downstream xDSL with per-modem total power constraints: broadcast channel optimal spectrum balancing (BC-OSB)," *IEEE Trans. Signal Process.*, vol. 57, no. 2, pp. 690–697, Feb. 2009.

[11] S. Zafaruddin, S. Prakriya, and S. Prasad, "Iterative receiver based on SAGE algorithm for crosstalk cancellation in upstream vectored VDSL," *ISRN Commun. & Networking*, vol. 2011, pp. 586 574_1–586 574_15, 2011.

[12] P. Biyani, S. Prakriya, A. Bagchi, and S. Prasad, "Dynamic programming based multi-user resource allocation for partial crosstalk cancellation in VDSL," *IEEE Commun. Lett.*, vol. 16, no. 3, pp. 420–423, Mar. 2012.

[13] I. Binyamini, I. Bergel, and A. Leshem, "Arbitrary partial FEXT cancellation in adaptive precoding for multichannel downstream VDSL," *IEEE Trans. Signal Process.*, vol. 60, no. 11, pp. 5754–5763, Nov. 2012.

[14] R. Moraes, P. Tsiaflakis, J. Maes, and M. Moonen, "DMT MIMO IC rate maximization in DSL with combined signal and spectrum coordination," *IEEE Trans. Signal Process.*, vol. 61, no. 7, pp. 1756–1769, Jul. 2013.

[15] P. Golden, H. Dedieu, and K. Jacobsen, *Fundamentals of DSL Technology*. Auerbach Publications, 2006.

[16] A. R. Forouzan, M. Moonen, J. Maes, and M. Guenach, "Joint level 2 and 3 dynamic spectrum management for upstream VDSL," *IEEE Trans. Commun.*, vol. 59, no. 10, pp. 2851–2861, Oct. 2011.

[17] ——, "Joint level 2 and 3 dynamic spectrum management for downstream VDSL," *IEEE Trans. Commun.*, vol. 60, no. 10, pp. 3111–3122, Oct. 2012.

[18] R. Lui and W. Yu, "Low-complexity near-optimal spectrum balancing for digital subscriber lines," in *Proc. 2005 IEEE Int. Conf. on Commun.*, vol. 3, pp. 1947–1951.

[19] F. Sjöberg, M. Isaksson, R. Nilsson, P. Ödling, S. Wilson, and P. Borjesson, "Zipper: a duplex method for VDSL based on DMT," *IEEE Trans. Commun.*, vol. 47, no. 8, pp. 1245–1252, Aug. 1999.

[20] R. Cendrillon, W. Yu, M. Moonen, J. Verlinden, and T. Bostoen, "Optimal multiuser spectrum balancing for digital subscriber lines," *IEEE Trans. Commun.*, vol. 54, no. 5, pp. 922–933, May 2006.

[21] W. Yu and J. Cioffi, "Sum capacity of Gaussian vector broadcast channels," *IEEE Trans. Inf. Theory*, vol. 50, no. 9, pp. 1875–1892, Sep. 2004.

[22] Z.-Q. Luo and S. Zhang, "Dynamic spectrum management: complexity and duality," *IEEE J. Sel. Topics Signal Process.*, vol. 2, no. 1, pp. 57–72, Feb. 2008.

[23] ——, "Duality gap estimation and polynomial time approximation for optimal spectrum management," *IEEE Trans. Signal Process.*, vol. 57, no. 7, pp. 2675–2689, Jul. 2009.
[24] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.
[25] W. Yu and R. Lui, "Dual methods for nonconvex spectrum optimization of multicarrier systems," *IEEE Trans. Commun.*, vol. 54, no. 7, pp. 1310–1322, Jul. 2006.
[26] P. Tsiaflakis, J. Vangorp, M. Moonen, and J. Verlinden, "A low complexity optimal spectrum balancing algorithm for digital subscriber lines," *Signal Process*, vol. 87, no. 7, pp. 1735–1753, Jul. 2007.
[27] A. R. Forouzan and M. Moonen, "Lagrange multiplier optimization for optimal spectrum balancing of DSL with logarithmic complexity," in *2011 IEEE Int. Conf. on Commun.*
[28] M. L. Honig, P. Crespo, and K. Steiglitz, "Suppression of near- and far-end crosstalk by linear pre- and post-filtering," *IEEE J. Sel. Areas Commun.*, vol. 10, no. 3, pp. 614–629, Apr. 1992.
[29] P. Viswanath and D. Tse, "Sum capacity of the vector Gaussian broadcast channel and uplink-downlink duality," *IEEE Trans. Inf. Theory*, vol. 49, no. 8, pp. 1912–1921, Aug. 2003.
[30] S. Vishwanath, N. Jindal, and A. Goldsmith, "Duality, achievable rates, and sum-rate capacity of Gaussian MIMO broadcast channels," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2658–2668, Oct. 2003.
[31] W. Yu and T. Lan, "Transmitter optimization for the multi-antenna downlink with per-antenna power constraints," *IEEE Trans. Signal Process.*, vol. 55, no. 6, pp. 2646–2660, Jun. 2007.
[32] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge University Press, 2005.
[33] M. Wolkerstorfer, J. Jalden, and T. Nordstrom, "Column generation for discrete-rate multi-user and multi-carrier power control," *IEEE Trans. Commun.*, vol. 60, no. 9, pp. 2712–2722, Sep. 2012.
[34] D. Wurtz, A. Klein, and M. Kuipers, "Distributed margin optimization using spectrum balancing in multi-user DSL systems," in *Proc. 2012 European Signal Process. Conf.*, pp. 1593–1597.
[35] T. Wang and L. Vandendorpe, "Successive convex approximation based methods for dynamic spectrum management," in *Proc. 2012 IEEE Int. Conf. on Commun.*, pp. 4061–4065.
[36] "Spectrum management for loop transmission systems," ANSI Std. T1.417-2003, Feb. 2003.
[37] "Transmission and Multiplexing (TM); access transmission systems on metallic access cables; Very High Speed Digital Subscriber Line (VDSL); Part I: Functional Requirements," ETSI Std. TS 101 270-1, Rev. V.1.3.1, 2003.



Amir R. Forouzan (S'99–M'04–SM'11) received the B.S. and M.Sc. degrees in electrical engineering from Sharif University of Technology, Tehran, Iran, in 1998 and 2000, respectively, and the Ph.D. degree with highest distinction from the University of Tehran in 2004. From August 1999 to May 2004, he was with the Iran Telecommunication Research Centre as a Research Fellow. From June 2004 to October 2008, he was with the Dept. of Electrical and Computer Engineering, University of Canterbury, Christchurch, New Zealand and from November 2008 to February 2012, he was a Research Scientist with the Electrical Engineering Department (ESAT/SISTA), KU Leuven, Belgium. He is currently an Assistant Professor at the Dept. of Electrical Engineering, Faculty of Engineering, University of Isfahan, Iran. His research interests include dynamic spectrum management in DSL, MIMO, and OFDM communication systems, ultrawideband radio, and wireless and optical CDMA.

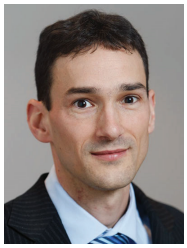


Marc Moonen (M'94–SM'06–F'07) is a Full Professor at the Electrical Engineering Department of KU Leuven, where he is heading a research team working in the area of numerical algorithms and signal processing for digital communications, wireless communications, DSL and audio signal processing. He received the 1994 KU Leuven Research Council Award, the 1997 Alcatel Bell (Belgium) Award (with Piet Vandaele), the 2004 Alcatel Bell (Belgium) Award (with Raphael Cendrillon), and was a 1997 Laureate of the Belgium Royal Academy of Science. He received a journal best paper award from the IEEE TRANSACTIONS ON SIGNAL PROCESSING (with Geert Leus) and from *Elsevier Signal Processing* (with Simon Doclo).

He was chairman of the IEEE Benelux Signal Processing Chapter (1998-2002), a member of the IEEE Signal Processing Society Technical Committee on Signal Processing for Communications, and President of EURASIP (European Association for Signal Processing, 2007-2008 and 2011-2012). He has served as Editor-in-Chief for the *EURASIP Journal on Applied Signal Processing* (2003-2005), and has been a member of the editorial board of IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS II, *IEEE Signal Processing Magazine*, *Integration-the VLSI Journal*, *EURASIP Journal on Wireless Communications and Networking*, and *Signal Processing*. He is currently a member of the editorial board of *EURASIP Journal on Applied Signal Processing* and Area Editor for Feature Articles in *IEEE Signal Processing Magazine*.



Mamoun Guenach is a research engineer with the Alcatel-Lucent Bell Labs. He received the degree of engineer in electronics and communications from the Ecole Mohamadia d'Ingenieurs in Morocco. Following that, he moved to the faculty of applied sciences at the Universite Catholique de Louvain (UCL), Louvain-La-Neuve, Belgium, where he received an M.Ss. degree in electricity and a Ph.D. degree in applied sciences. He served as a post-doctoral researcher in the department of telecommunications and information processing at the Universiteit Gent, Ghent, Belgium prior to joining Alcatel-Lucent.



Jochen Maes (M'10–SM'11) joined Alcatel-Lucent Bell Labs in 2006 where he has been continuously shifting the limits of copper. He innovated in areas such as vectoring and phantom mode transmission, and contributed to the G.vector standard. Today he heads the Bell Labs team that researches transceiver and system design for copper access. The team is currently focused on G.fast that delivers 1 Gb/s over the telephony network. Jochen is senior member of the IEEE, and holds a Masters degree in Physics and a Ph.D. in Science.