COMMITTEE T1 – TELECOMMUNICATIONS
Working Group T1E1.4 (DSL Access) **T1E1.4/2001-198R1**
Greensboro, NC, November 5, 2001

CONTRIBUTION

TITLE: **SNR Computation from a DFE**

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PROJECT: **T1E1.4, Dyanmic Spectrum Management**

ABSTRACT

The frequency spectra of Signal to Noise Ratio, SNR(f), for a channel can be computed simply from the coefficients of a Decision-Feedback Equalizer (DFE) as illustrated in this short note. The intent of this document is to illustrate that supply of SNR(f) information from any pulse-amplitude-modulation (PAM)- or quadrature-amplitude-modulation (QAM)-based DSL modem is easy, presuming that modem uses the DFE. This is a response to specific DSM inquiry at the May 2001 meeting and subsequent June 15 conference call.

NOTICE

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SNR Computation from A DFE

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Abstract:

The frequency spectra of Signal to Noise Ratio, SNR(f), for a channel can be computed simply from the coefficients of a Decision-Feedback Equalizer (DFE) as illustrated in this short note. The intent of this document is to illustrate that supply of SNR(f) information from any pulse-amplitude-modulation (PAM)- or quadrature-amplitude-modulation (QAM)-based DSL modem is easy, presuming that modem uses the DFE. This is a response to specific DSM inquiry at the May 2001 meeting and subsequent June 15 conference call.

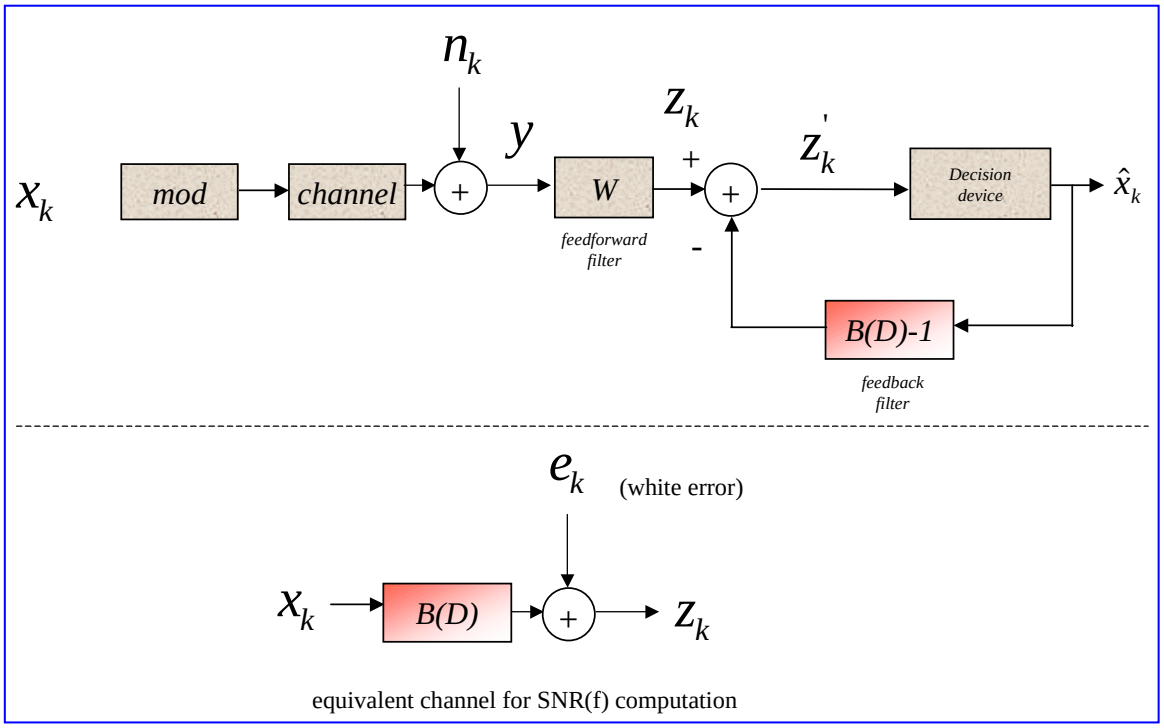


Figure 1 – SNR(f) computation with DFE.

The DFE-Equivalent Channel:

Figure 1 illustrates the DFE and the equivalent channel. No carrier modulation, or center-frequency equivalent, is shown so the system is presumed complex-baseband in the case of QAM modulation and applies directly to PAM transmission. The PAM or QAM data symbol is x_k, and the decision device in the receiver is designed based on the constellation point possibilities for x_k. One symbol is transmitted every T seconds through a transmit filter (including digital-to-analog conversion and all filtering), a linear dispersive channel with additive noise, and the feedforward filter (including analog-to-digital conversion and all filtering) of the DFE. The output of the feedforward filter enters the feedback section where the decision device and its past values are used to construct and subtract previous intersymbol interference. The output of the feedforward section z_k is known [1] to be the convolution of x_k with a monic, causal, and minimum-phase channel equivalent, call it b_k, specifically

$$z_k = \sum_{i=0}^{\nu} b_i x_{k-i} + e_k$$

where e_k is an error signal that represents the error in a best minimum-mean-square-error (MMSE) estimate of x_k produced by the DFE at the decision-device input. The parameter ν is the length of the feedback section and is here assumed to be large or essentially infinite. The first coefficient of the feedback polynomial is $b_0 = 1$, essentially scaling the channel so the decision device thresholds are correctly placed. The DFE uses a best MMSE fit to determine the other values in b_k to estimate the trailing intersymbol interference for removal, which leaves

$$z_k^{'} = x_k + e_k$$

when previous decisions are correct. For sufficiently long filters in the DFE, e_k is white, and the DFE has a detection SNR given by $SNR_{dfe} = E_x / \sigma_e^2$ where E_x is the average symbol variance, and σ_e^2 is the MMSE for the DFE. The equivalent white-noise/error channel just before subtraction of the intersymbol interference is also shown in Figure 1. This channel can be constructed from the feedback coefficients b_k and from the white error sequence e_k, which is computed by subtracting the decision input from the decision output. This equivalent channel contains the information desired for computation of the SNR(f). Defining the function

$$B(f) = \sum_{i=0}^{\nu} b_i \cdot e^{-j2\pi fT} ,$$

the SNR at any frequency in the range $f \in [-1/2T \quad 1/2T]$ is computed by

$$SNR(f) = \frac{E_x}{\sigma_e^2} \cdot |B(f)|^2 \quad .$$

If the feedforward or feedback section is finite and too short, a simple correction is to replace σ_e^2 in the above equation by $\sigma_e^2(f)$, the computed/estimated spectrum of the error sequence (which is no longer white when filters are too short).

Conclusion:

The SNR(f) can be computed from the DFE, assuming that feedforward and feedback filters are sufficiently long, with straightforward computation essentially of the Fourier transform of the feedback section at any frequency within the passband of transmission.

Reference:

[1] Cioffi, J., Dudevoir, D., Eyboglu, M., and Forney, G., "MMSE Decision Feedback Equalizers and Coding: Parts I and II," *IEEE Transactions on Communications*, Vol. 43, No. 10, October 1995, pp. 2582-2604.