COMMITTEE T1 – TELECOMMUNICATIONS
Working Group NAI (T1E1.4 - DSL Access) **T1E1.4/2004-650**
San Antonio, TX: December 13, 2004

CONTRIBUTION

TITLE: Worst-Case Noise in DSLs

SOURCE:

J. Cioffi	**Stanford**	+1-650-723-2150	**cioffi@stanford.edu**
M. Brady			
W. Rhee	**ASSIA Inc.**	+1-650-654-3400	**wrhee@assia-inc.com**

PROJECT: **T1E1.4, DSM**

ABSTRACT

This information-only contribution investigates worst-case victim crosstalk interference from adapting-spectra DSLs within any given maximum-power-spectral-density frequency plan, following the incentive spectrum management discussion in [1]. The often encountered "best effort" rate-adaptive DSLs that run at highest possible speed for a maximum power or spectrum mask create nearly the worst possible crosstalk. A binder of such lines thus mutually reduces attainable rates of all lines. Two solutions are also suggested: (Level 1) polite spectrum loading (use of MAXSNRM parameter in DMT DSLs) and (Level 2) use of preferential band [3],[13] or Optimum Spectral Balancing [2].[13], which can increasingly improve overall binder data rates. This observation motivates some DSM-compliance proposals in related subsequent contributions. An appendix provides a submitted paper that is a detailed investigation into worst-case noise in DSL systems.

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- CONTACT: J.M. Cioffi, Cioffi@stanford.edu, Tel: +1-650-723-2150 ; Fax: +1-650-724-3652

Worst-Case Noise in DSLs

John M. Cioffi and Mark H. Brady	Wonjong Rhee
Dept of EE,	ASSIA-INC.
Stanford University	201 Redwood Shores Parway
363 Packard EE Bldg.	Suite 242
Stanford, CA 94305-9515	Redwood City, CA 94065-1191
Cioffi@stanford.edu,	wrhee@assia-inc.com
+1-650-723-2150 ; F: +1-650-724-3652	1-650-654-3400 F: 1-650-654-3404

ABSTRACT

This information-only contribution investigates worst-case victim crosstalk interference from adapting-spectra DSLs within any given maximum-power-spectral-density frequency plan, following the incentive spectrum management discussion in [1]. The often encountered "best effort" rate-adaptive DSLs that run at highest possible speed for a maximum power or spectrum mask create nearly the worst possible crosstalk. A binder of such lines thus mutually reduces attainable rates of all lines. Two solutions are also suggested: (Level 1) polite spectrum loading (use of MAXSNRM parameter in DMT DSLs) and (Level 2) use of preferential band [3],[13] or Optimum Spectral Balancing [2].[13], which can increasingly improve overall binder data rates. This observation motivates some DSM-compliance proposals in related subsequent contributions. An appendix provides a submitted paper that is a detailed investigation into worst-case noise in DSL systems.

1. Introduction

Incentive-based Spectrum Management [1] encourages the use of better spectra and politeness to improve the overall range and data rates of all DSLs. In particular, difficult or worst-case crosstalk should also be discouraged in all spectrum management. The information theoretic concept of worst-case noise arises in numerous linear cross-talking multi-user-channel situations where the Gaussian noise autocorrelation is selected to minimize the data rate of each of the users [2] [3]. Since DSL noise is often dominated by crosstalk (typically FEXT in FDM systems), the "selection" of noise is dependent on the spectra transmitted by other users. The criteria for a modem's choice of spectra, or "loading" algorithm, is not presently specified in DSL standards (other than some maximum power-spectral-density (PSD) masks and total power constraints). It is however reasonable to expect that most vendors today use approximations of water-filling algorithms[1]. The use of water-filling can be very good in terms of all loops' performance, as has been shown now in a series of contributions on "iterative water-filling" [4],[5],[6]. However, it is also possible to abuse water-filling and lead to very poor performance of all lines, including situations where a DSL deployment (sometimes known as a "best effort" DSL service) attempts to maximize the data rate on all lines simultaneously. This contribution shows this abuse of water-filling and the TSNRM, MAXSNRM, and ATTNDR (see the DSM Report Draft, [13] for a

[1] This assumption may not apply to all modem vendors and indeed some modem vendors have highly questionable methods for selecting spectra.

definition of these parameters) will lead to a noise for each line that is close to the information-theoretic "worst-case noise" thereby reducing the performance of all lines.

Unfortunately, the situation of modems with highest data rate or equivalently maximum margin with high power exists somewhat pervasively in current DSL deployments, despite ADSL and VDSL standards' provisions to avoid such situations. The situation of each user trying to maximize their data rate (or equivalently maximize their margin at maximum power) sometimes causes a "devil's dilemma" where a stable operation point is each player/user selecting a strategy that creates worst-case performance of all other users. There may be many DSL systems presently experiencing the devil's dilemma. This contribution also shows that there are good solutions using DSM and water-filling that DSL systems could be encouraged to use. As such, this contribution is the basis for several specific DSM-Report Annex E proposals that follow [7]-[10].

An Annex to this contribution provides a detailed paper that has been submitted for publication to a EURASIP DSL special issue that may be helpful for those who desire more mathematical rigor in investigating the effects summarized in the body of this paper. Section 2 briefly reviews the difference between rate-adaptive DSLs and fixed-margin or polite DSLS, both with or without PSDMASK settings. Section 3 then provides a few selected DSL results that illustrate the issues.

2. Loading and Water-filling Review

Loading algorithms are typically used in DMT DSL modems (i.e., standardized ADSL and VDSL) to set the number of bits and the power used by each tone. Such choice typically follows a water-fill selection where energy is allocated to tones so that sum of that energy and the equivalent noise[2] is constant on all used tones. A practical choice often used is to add each bit in loading (up to some maximum number of bits) to the tone that requires the least energy to carry that bit reliably. When the maximum number of bits is not limited, and energy is applied up to some fixed energy (power) limit, then the algorithm is said to be "rate adaptive" or RA as in Subsection 2.1. When the maximum number of bits is limited to some fixed number[3], the algorithm is said to be "fixed margin" or FM and minimizes the transmitted power as in Subsection 2.2. FM loading can be more polite. Both RA and FM water-filling can incorporate power-spectral-density masks (PSDMASKs) by imposing an infinite cost for adding a bit to any tone that would cause the loading algorithm to allocate energy that exceeds the PSDMASK at that tone.

The two types of loading can be related and effectively become the same loading procedure if either of the following two equivalences occurs:

1. The data rate chosen in the FM loading is so high that the amount of power needed to support that data rate reliably is the same as the power limit for the RA algorithm, or
2. The power limit chosen in the RA algorithm is same as the power used by FM loading to achieve a given data rate.

Either equivalence causes worst-case noise to occur, and thus both situations should be avoided. A more desirable situation is that the data rate on most lines is not set at the highest possible for maximum power, and then FM water-filling is used within the applicable PSD constraints. Selection of the data rates is accomplished either by DSM spectrum-compatibility rule adherence or by an SMC.

2.1 RA Water-fill

Rate adaptive loading causes a modem to consider only its own performance and load energy to the maximum levels allowed by either the total power constraint or the PSDMASK (or both). As such the

[2] The equivalent noise is the noise of the channel normalized (divided) by the channel gain at any frequency (tone).
[3] The desired data rate including all overhead bits is divided by the symbol rate (4000 Hz in ADSL and VDSL DMT systems) to get the maximum number of bits used in loading.

modem does believe it is running at the highest possible data rate because it has no control of the data rates or of the spectra of the DSL modems on the lines sharing a common binder. If all such modems operated with RA loading, the situation is such that they then essentially compete with one another for the use of the transmission channel spectra. The process is stable and will converge [14] if all the modems are independently executing water-filling; however, the maximum rate of each is less than what might be achieved with a more prudent overall strategy.[4] Figure 1 illustrates a downstream remote-terminal/central-office mixed ADSL situation in which all the modems execute RA loading, with varying levels of RT line power reduction. The performance of a particular modem experiencing the "worst case noise" – that is the noise that would lead to lowest data rate is also shown.

As is evident in Figure 1, there is very small difference between the RA result and the "worst-case" result. In this case, the remote-terminal at distance 4000 meters from a central office serves 25 lines of length 2000 meters. The data rates of these lines correspond to those labeled 6000 meters in the figure. Five more lines are older ADSL lines served from a central office and are of length 5000 meters. The remote-terminal total power (relative to the maximum of 20.4 dBm) is shown on the horizontal axis. The data rates are shown on the vertical axis for both groups of modems using RA water-filling individually. The information-theoretic worst-case noise performance (see Annex for worst-case noise calculation) is also shown in Figure 1. Clearly for any transmit power level, the RA and worst-case data rates are almost the same at any given maximum power level. However, holding the remote-terminal lines at a much lower power level allows the long lines to function at significantly higher rates, while allowing the remote terminals full power is destructive to the performance of the long lines. The horizontal line corresponds to the ADSL PSDMASK set at -40dBm/Hz. As the remote-terminal downstream transmit power increases, the situation becomes FEXT limited at a data rate of 6.3 Mbps at power levels well below 20.4 dBm. Clearly, universal RA loading may be far from best overall DSL deployment range. Unfortunately, most ADSL remote terminals do not reduce transmit downstream power to levels of -10 dBm or less without strong indication from the service provider that this is necessary (and even then the reduction is often not implemented anyway).

[4] "more" prudent does not necessarily imply "central control or coordination." Most prudent would imply some central coordination, but there are many levels between all modems running RA and the best possible use of the binder.

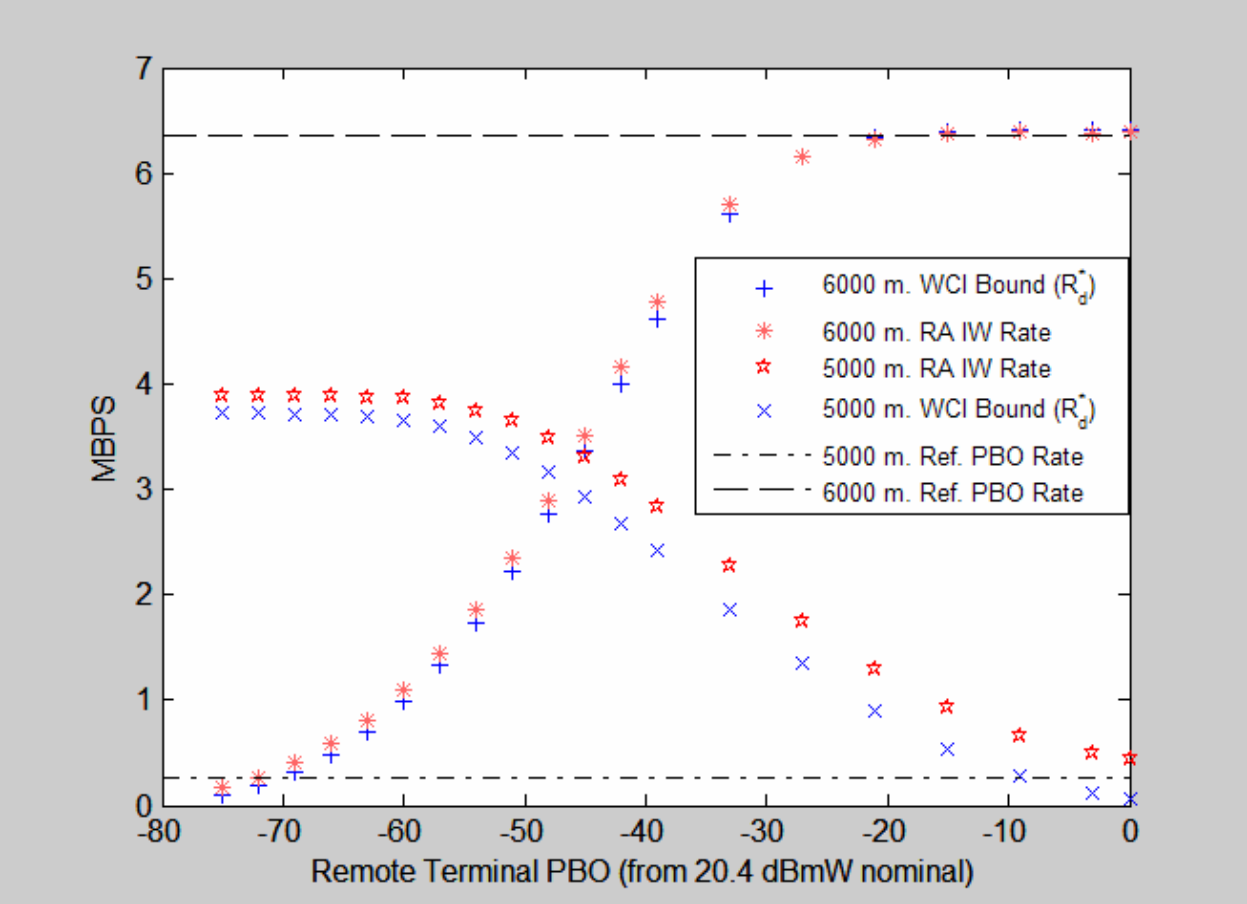


Figure 1 – Illustration of RA IW in comparison to worst-case noise.

2.2 FM Water-fill

Fixed-margin water-filling would exploit the trends of results illustrated in Figure 1 by noting that the long lines all seem to function at 1.5 Mbps if the short lines are held to a fixed data rate and operated at lower power (with very little loss in short-line data rate).

Fixed-margin loading would determine a fixed number of bits to load at some reasonable data rate with the margin not to exceed a certain specified level, often called MAXSNRM in DSL standards. The extra margin is intended to guard against line changes and unforeseen noise disruptions (like a new DSL subscriber increased crosstalk as it initializes). A level of 11 dB allows noise to increase by more than a factor of 10 in power before the line would exhibit bit errors with higher probability than the desired 1e-7. A modem retrain would likely occur if the noise increased by more than a factor of 10. Figure 1 suggests that operation with MAXSNRM=11 dB at a short-line data rate of 5 Mbps would largely preserve the long line data rates at 1.5 Mbps and certainly avoid the near annihilation of DSL service that occurs with RA loading by the remote terminals.

Thus, imposition of a MAXSNRM with a data rate near maximum can greatly improve overall DSL deployment range and rates. A second similar example for upstream VDSL appears in the paper in the Annex, and the conclusion is the same for VDSL.

2.3 Band Preference with PSDMASKs

While FM water filling significantly improves performance, more precise methods that exploit potential coordination and go beyond simple setting of data rate and MAXSNRM can be achieved by contouring spectral masks, that is by using non-flat PSDMASKs. For the situation of Figure 1, a better solution is to steer the spectra of the short lines to the high frequencies where they will not at all affect the long lines.

Presuming an SMC can run some type of algorithm that would produce results close to the OSM of [11], non-flat PSDMASKs can be imposed on the DSL modems so that water-filling is implemented below these masks. The DSM report enables such operation through turning band preference on, so that any MAXSNRM will be observed at all tones (and not just the worst case as is often unfortunately done in many vendors modems today, so they do not observe band preference correctly). Then [12] shows that the data rates of OSM can be closely approximated while the modems continue to bit swap (or water-fill) under the imposed PSDMASK. An example rate region appears in Figure 2.

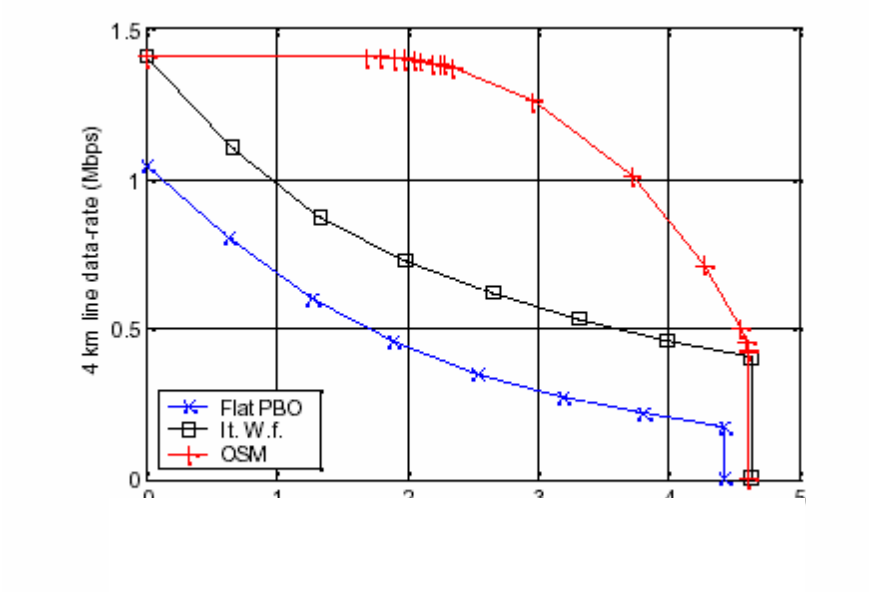


Figure 2 – reproduced from [11] of rate regions for 4 and 3 km ADSL lines.

The outer red region represents an enlarged trade-off of rates that is possible with band preference by imposing essentially 2-level PSDMASKs and observing band preference.

3. Conclusion

RA "best effort" DSL deployments are somewhat self-defeating in that all or most of the DSL lines get lower data rates than could be achieved with more polite FM operation. The FM operation requires Level 1 DSM in that some mild level of service provider choice is necessary for the selected data rate and MAXSNRM for the DSL customers. Nonetheless, the improvements are substantial.

RA "best effort" DSL systems are readily allowed within existing spectrum-management standards and ADSL and VDSL standards. Better spectrum management is possible with less restriction and partitioning into band plans and more observation of polite prudent operation. DSM can help impose new rules for spectrum management that can be more tuned to actual operation and practice than the current static imposition of presumed offending spectra that never occur, while indeed other more realistic worst-case spectra are ignored. The authors are convinced that the proposals in the contributions [7]-[10] improve actual DSL deployment range with respect to existing T1.417 spectrum management rules, and those proposals follow the sentiment of this contribution.

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5. Annex: Courtesy copy of EURASIP DSL submission

The Worst-Case Interference In DSL Systems Employing Dynamic Spectral Management

Mark H. Brady and John M. Cioffi
Department of Electrical Engineering
Stanford University
Stanford, CA 94305-9515
Email: {mhbrady, cioffi}@stanford.edu

Abstract

Dynamic Spectrum Management (DSM) has been proposed to achieve next-generation rates on digital subscriber lines (DSL). Because the copper twisted-pair plant is an interference-constrained environment, the multiuser performance and spectral compatibility of DSM schemes are of primary concern. While the analysis of multiuser interference has been standardized for current *static* spectrum management (SSM) techniques, at present no corresponding standard DSM analysis has been established. Similarly, techniques for upstream power backoff (UPBO) have been extensively studied and standardized for SSM systems, while their role in DSM systems remains relatively open.

This paper examines a multiuser spectrum-allocation problem and formulates a lower bound to the achievable rate of a DSL modem that is tight in the presence of the worst-case interference. A game-theoretic analysis shows that the rate-maximizing strategy under the worst-case interference (WCI) corresponds to a Nash equilibrium in pure strategies of a certain *strictly competitive* game. A Nash equilibrium is shown to exist under very mild conditions, and the rate-adaptive waterfilling algorithm is demonstrated to give the optimal strategy in response to the WCI under an FDM condition.

Numerical results are presented for two important scenarios: an upstream VDSL deployment exhibiting the near-far effect, and an ADSL RT deployment with long CO lines. The WCI rate bound shows that the performance improvement of DSM over SSM techniques can be preserved by appropriate distributed power control, even in worst-case interference environments.

I. Introduction

In recent years, increased demands on data rates and competition from other services have lead to the development of new high-speed transmission standards for digital subscriber line (DSL) modems. Dynamic Spectral Management (DSM) is emerging as a key component in next-generation DSL standards. In DSM, spectrum is allocated adaptively in response to channel and interference conditions, allowing mitigation of interference and best use of the channel. As multiuser interference is the primary limiting factor to DSL performance, the potential for rate improvement by exploiting its structure is substantial.

DSM contrasts with current DSL practice known as static spectrum management (SSM). In SSM, masks are imposed on transmit PSD's to bound the amount of crosstalk induced in other lines sharing the same

binder group [1]. As SSM masks are fixed for all loop configurations, they can often be far from optimal or even prudent spectrum usage in typical deployments. Standardized tests for "spectral compatibility" [1] assess "new technology" by defining PSD masks and examining the impact to standardized systems using the 99^{th} percentile crosstalk scenario. Such methods are useful when a reasonable estimate of spectrum of all users can be assumed *à priori*. However, if spectrum is instead allocated dynamically, not only is this knowledge is not available *à priori*, but because of loop unbundling, other users' spectrum may not even be known even during operation. Spectral compatibility between different operators using DSM is a primary concern because new pathologies may arise with adaptive operation. Moreover, it is not unreasonable to suspect that each competing service provider sharing a binder would perform DSM in a greedy fashion, at the possible expense of other providers' users. However, a worst-case interference analysis based on maximum allowable PSD masks is overly pessimistic. Since the behavior of all DSL modems is dynamic and cannot be assumed fixed *à priori*, the analysis of DSM is inherently different from that of SSM systems. A new paradigm is needed to assess the impact of DSM on multiuser performance of the overall system.

A. Prior Results

The capacity region of the AWGN interference channel (IC) is in general unknown, even for the 2-user case [2]. Communication in the presence of hostile interference has been studied from a game-theoretic perspective in numerous applications, e.g. [3] [4]. A simple and relevant IC achievable region is that attained by treating interference as noise [5]. Capacity results for frequency-selective interference channels satisfying the *strong interference* condition are also known [6].

DSM algorithms have been proposed for the cases of distributed and centralized control scenarios. This paper considers what has been termed "Level 0-2 DSM" [7], wherein cooperation may be allowed to manage spectrum, but not for multiuser encoding and decoding. A centralized DSM center controlling multiple lines offers both higher potential performance and improved management capabilities [8]. Distributed DSM schemes based on the iterative waterfilling (IW) algorithm [9] have been presented. IW has also been studied from a game-theoretic viewpoint [10]. Numerous algorithms for centralized DSM have been proposed. [11] presents a technique to maximize users' weighted sum-rate. Rate maximization subject to frequency-division and fixed rate proportions between users has been considered [12]. Optimal [13] and suboptimal [14] algorithms to minimize transmit power have been studied.

An extensive literature on upstream power-backoff techniques to mitigate the 'near-far' problem have been developed for static spectrum-management systems [15] [16] [13] [17]. A power-backoff algorithm for DSM systems implementing iterative waterfilling has been proposed [18].

In current DSL standards, upstream and downstream transmission use either distinct frequency bands or shared bands. In the latter case, "echo" is created between upstream and downstream transmission [9]. As analog hybrid circuits do not provide sufficient isolation, echo mitigation is essential in practical systems [19]. Numerous echo cancellation structures have been proposed for DSL transceivers [20] [21] [22].

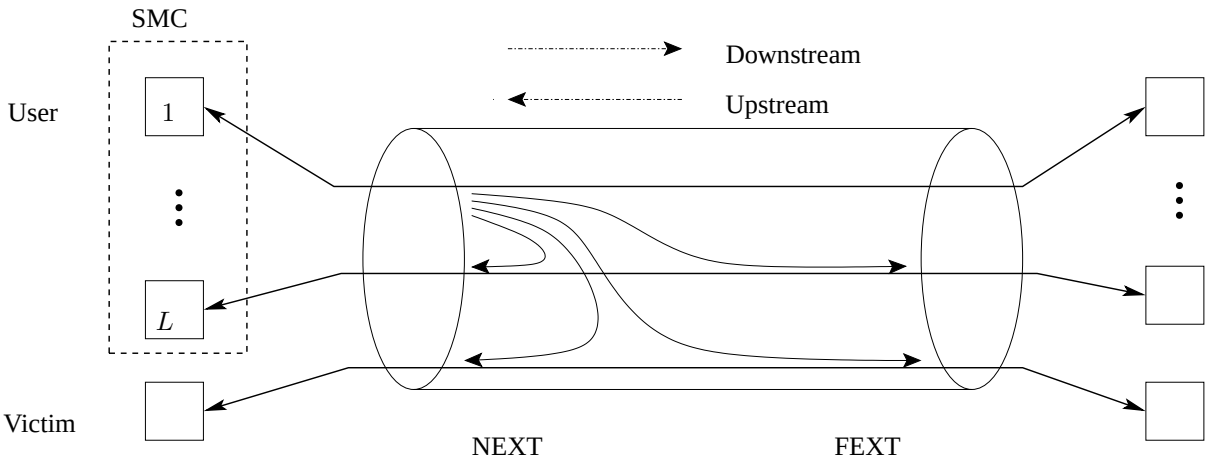


Fig. 1. Illustration of loop plant environment showing downstream FEXT and NEXT from User 1. The victim user is shown at the bottom.

B. Outline

This paper formulates the achievable rate of a single "victim" modem in the presence of the worst-case interference from other interfering lines in the same binder group. The performance under the WCI is a guaranteed-achievable rate that can be used, for example, in studying multiuser performance of DSM strategies and establishing spectral compatibility of DSM systems.

Section II defines the channel and system model. The WCI problem is formalized and studied in Section III from a game-theoretic viewpoint. Certain properties of the Nash equilibrium of this game are explored. Section IV considers numerical examples in VDSL and ADSL systems. Concluding remarks are made in Section V.

A word on notation: $(\mathbf{v})_k$ denotes the k^{th} element of the vector $\mathbf{v}$, and $\mathbf{v} \succeq 0$ denotes that each element is nonnegative. For the symmetric matrix X, $X \succeq 0$ denotes that X is positive semidefinite. $\mathbf{1}$ is a column vector with each element equal to 1. $\mathbf{int}(X)$ denotes the (topological) interior, $\mathbf{cl}(X)$ the closure, and ∂X the boundary of the set X.

II. System Model

A. Channel Model

A copper twisted-pair DSL binder is modelled as a frequency-selective multiuser Gaussian interference channel [9] [23]. The binder contains a total of $L+1$ twisted pairs, with one DSL line per twisted pair, as shown in Figure 1. The effect of NEXT and FEXT interference generated by L "interfering" users that generate crosstalk into one "victim" user is considered. This coupling is illustrated for downstream transmission in Figure 1.

B. DSL Modem Model

1) Modem Architecture: The standardized [24] DMT-based modulation scheme is employed, so that transmission over the frequency-selective channel may be decoupled into N independent subcarriers or tones. Both FDM

and overlapping bandplans are considered. As overlapping bandplans require echo cancellation that is imperfect in practice, error that is introduced acts as a form of interference and is of concern. Echo cancellation error is modelled presuming a prevalent echo cancellation structure utilizing a joint time-frequency LMS algorithm [19] is employed[1]. Using the terminology of [19], let μ denote the LMS adaptive step size parameter. The "excess MSE" for a given tone is modelled [25, Eqn. 12.74] as proportional to the product of the LMS adaptive step size parameter μ and the transmit power on that tone. The constant of proportionality is absorbed by defining $\hat{\beta}$ as the ratio of excess MSE to transmitted energy on a given tone.

2) Achievable Rate Region: This section discusses an achievable rate region for a DSL modem based on the preceding channel and system model. The following analysis applies to both upstream and downstream transmission. For specificity, the following refers to downstream transmission. First, consider the case where echo cancellation is employed. Denote the victim modem's downstream transmit power on tone n, $n \in \{1, \ldots, N\}$ as $\mathbf{x}_n$. Let element l, $l \in \{1, \ldots, L\}$ of the vector $\mathbf{y}^{(n)} \in \mathbb{R}_+^{2L}$ denote the downstream transmit power of interfering user l on tone n. Similarly, let element l, $l \in \{L+1, \cdots, 2L\}$ of $\mathbf{y}^{(n)}$ denote the upstream transmit power of interfering user $l-L$. Define element l, $l \in \{l, \ldots, L\}$ of the row vector $\mathbf{h}^{(n)} \in \mathbb{R}_+^{2L}$, as the FEXT power gain from interfering user l on tone n (necessarily, $\mathbf{h}^{(n)} \succeq 0$). Similarly, define element l, $l \in \{L+1, \ldots, 2L\}$ of $\mathbf{h}^{(n)}$ to be the NEXT power gain from interfering user $l-L$. Let $\tilde{\mathbf{h}}^{(n)} \in \mathbb{R}_+$ denote the victim line's insertion loss on tone n ($\tilde{\mathbf{h}}^{(n)} \geq 0$).

Independent AWGN (thermal noise) with power $\sigma_n^2 > 0$ is present on tone n. Let $\hat{\beta}_n$ denote the echo cancellation ratio on tone n as described above. Echo-cancellation error is treated as AWGN. Let Γ denote the SNR gap-to-capacity [9]. Then the following rate is achievable on tone n [9]:

$$b_n = \log\left(1 + \frac{\tilde{\mathbf{h}}^{(n)}\mathbf{x}_n}{\Gamma(\mathbf{h}^{(n)}\mathbf{y}^{(n)} + \hat{\beta}\mathbf{x}_n + \sigma_n^2)}\right). \tag{1}$$

Observe that if $\tilde{\mathbf{h}}^{(n)} = 0$, then it is necessarily the case that $b_n = 0$, implying that tone n is never loaded. Thus, in the sequel, $\tilde{\mathbf{h}}^{(n)} > 0$ for all $n \in \{1, \ldots, N\}$ is considered without loss of generality by removing those tones with zero direct gain ($\tilde{\mathbf{h}}^{(n)} = 0$). Defining $\alpha_n = \Gamma/\tilde{\mathbf{h}}^{(n)}$, $\beta_n = \Gamma\hat{\beta}_n/\tilde{\mathbf{h}}^{(n)}$, and $N_n = \Gamma\sigma_n^2/\tilde{\mathbf{h}}^{(n)}$, and substituting:

$$b_n = \log\left(1 + \frac{\mathbf{x}_n}{\alpha_n\mathbf{h}^{(n)}\mathbf{y}^{(n)} + \beta_n\mathbf{x}_n + N_n}\right). \tag{2}$$

Since $\Gamma \geq 1$, it follows that $\alpha_n \geq 0$, $\beta_n \geq 0$, and $N_n > 0$.

3) Achievable Rate Region for FDM: When an FDM scheme is employed, NEXT and echo cancellation are eliminated because transmission and reception occur on distinct frequencies[2]. As a common configuration in ADSL and VDSL standards [9], this represents an important special case of the preceding model. (2) is modified such that element l of $\mathbf{h}^{(n)} \in \mathbb{R}_+^L$ denotes the FEXT power gain between user l and the victim modem. Furthermore, let element l of $\mathbf{y}^{(n)} \in \mathbb{R}_+^L$ denote the downstream power of interfering modem l on tone

[1]Other models may be more applicable to different echo cancellation structures.

[2]Effects arising from implementation issues that may lead to crosstalk between upstream and downstream bands are not explicitly considered.

n. Performing the same substitutions as above, the following rate is then achievable on tone n:

$$b_n = \log\left(1 + \frac{\mathbf{x}_n}{\alpha_n \mathbf{h}^{(n)}\mathbf{y}^{(n)} + N_n}\right). \tag{3}$$

III. The Worst-Case Interference (WCI)

A. Game-Theoretic Characterization of the WCI

This section introduces and motivates the concept of the worst-case interference (WCI). Suppose that a "victim" modem desires to keep its data rate at some level. Such an scenario is commonplace as carriers widely offer DSL service at a fixed data rate. The objective is to bound the impact that multiuser interference can have on this victim modem, thereby determining whether service may be guaranteed. To this end, define the worst-case interference as being that interference that minimizes the maximum achievable data rate of the victim subject to the preceding system model. By construction, this rate is a lower bound to the rate achievable under any other interference profile.

It is fruitful to examine this problem using from the standpoint of game theory. In the worst-case interference game, one player jointly optimizes the spectrum all of the interfering modems so as to cause the most deleterious interference to the victim modem. Thus in the WCI game, all of the interfering modems act as one player, while the victim modem acts as the other player. Although such a configuration may appear pathological, it will be shown numerically such a situation is quite reasonable. Neither is assuming such coordination of the interferers unreasonable in practice as under "Level 2" DSM [7] [8], collocated equipment may be centrally controlled by a competing carrier.

A Nash equilibrium in this game may be interpreted as characterizing the worst-case interference and the optimal response (power allocation policy) when experiencing the WCI. The structure of the Nash equilibrium lends insight into the problem as well as suggesting feasible techniques that may be implemented in practical systems.

B. Formalization of the WCI Game

Consider the following two-player game: let Player 1 control the spectral allocation of victim modem, and Player 2 control the spectral allocation of all of the interfering modems. Referring again to downstream transmission for specificity, let the total (sum) downstream power of the victim modem be upper-bounded by P_x, so that if $\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N]^T$, the power constraint is $\mathbf{1}^T\mathbf{x} \leq P_x$. For echo-cancelled systems, the total downstream power of the l^{th} interfering modem $l \in \{1, \ldots, L\}$ is upper bounded by the l^{th} element of $P_y \in \mathbb{R}^{2L}_{++}$, and the total upstream power of interfering modem l is upper bounded by element $l + L$ of P_y. For FDM systems, the total downstream power of the l^{th} interfering modem $l \in \{1, \ldots, L\}$ is upper bounded by the l^{th} element of $P_y \in \mathbb{R}^{L}_{++}$. Without loss of generality, the nontrivial cases where $P_x > 0$, and $P_y \succ 0$ are considered.

The *strategy set* of Player 1 is the set of all feasible power allocations for the victim modem, $\mathcal{S}_1 = \{\mathbf{x} : \mathbf{x} \succeq 0, \mathbf{1}^T\mathbf{x} \leq P_x\}$, and the *strategy set* of Player 2 is the set of all feasible power allocations for the interfering modems, $\mathcal{S}_2 = \{[\mathbf{y}^{(1)}, \mathbf{y}^{(2)}, \ldots, \mathbf{y}^{(N)}] : \mathbf{y}^{(1)} \succeq 0, \mathbf{y}^{(2)} \succeq 0, \ldots, \mathbf{y}^{(n)} \succeq 0, [\mathbf{y}^{(1)}, \mathbf{y}^{(2)}, \ldots, \mathbf{y}^{(N)}]\mathbf{1} \preceq P_y\}$. This

is a *strictly competitive* or *zero sum* two-player game $\{\mathcal{S}_1, \mathcal{S}_2, J\}$ where the objective function $J : \mathcal{S}_1 \times \mathcal{S}_2 \mapsto \mathbb{R}_+$ is defined to be the achievable data rate of the victim user:

$$J(x, [y_1, \ldots, y_N]) = \sum_{n=1}^{N} \log\left(1 + \frac{\mathbf{x}_n}{\alpha_n \mathbf{h}^{(n)} \mathbf{y}^{(n)} + \beta_n \mathbf{x}_n + N_n}\right). \tag{4}$$

The game $G = \{\mathcal{S}_1, \mathcal{S}_2, J\}$ is defined to be the Worst-Case Interference game.

C. Derivation of Nash Equilibrium Conditions

A Nash equilibrium in pure strategies in the game G is defined to be any $(\mathbf{x}, [\mathbf{y}^{(1)}, \ldots, \mathbf{y}^{(N)}]) \in \mathcal{S}_1 \times \mathcal{S}_2$ that satisfies:

$$\begin{aligned} J(\widetilde{\mathbf{x}}, [\mathbf{y}^{(1)}, \ldots, \mathbf{y}^{(N)}]) &\leq J(\mathbf{x}, [\mathbf{y}^{(1)}, \ldots, \mathbf{y}^{(N)}]) \\ &\leq J(\mathbf{x}, [\widetilde{\mathbf{y}}^{(1)}, \ldots, \widetilde{\mathbf{y}}^{(N)}]). \end{aligned} \tag{5}$$

For every $\widetilde{\mathbf{x}} \in \mathcal{S}_1$, $[\widetilde{\mathbf{y}}^{(1)}, \ldots, \widetilde{\mathbf{y}}^{(N)}] \in \mathcal{S}_2$.

Having so defined Nash equilbria of G, it is useful to characterize under what conditions a Nash equilibrium exists, and in which case, how it may be computed numerically. The subsequent two theorems address the former question, while the next section considers the latter. It is helpful to first establish a technical result for the general case (the argument for FDM is identical):

Theorem 1: If $\alpha \geq 0$, $\beta \geq 0$, $\gamma > 0$, $h \succeq 0$, and α, β, γ, h are bounded, then the function $g : \mathbb{R}_+ \times \mathbb{R}_+^{2L} \mapsto \mathbb{R}_+$ defined by:

$$g(x, y) = \log\left(1 + \frac{x}{\alpha h^T y + \beta x + \gamma}\right). \tag{6}$$

Is concave in x and convex in y.

Proof: It is first shown that $f : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$, $f(x, \eta) = \log\left((1+\beta)x + \alpha\eta + \gamma\right) - \log(\alpha\eta + \beta x + \gamma)$ is convex in η and concave in x. Observe that f is continuous and twice differentiable, thus it is sufficient [26] to show that for all $(x, \eta) \in \mathbb{R}_+ \times \mathbb{R}_+$, $\frac{\partial^2 f}{\partial \eta^2} \geq 0$ and $\frac{\partial^2 f}{\partial x^2} \leq 0$. By differentiating and simplifying:

$$\frac{\partial f}{\partial x} = \frac{\alpha\eta + \gamma}{(\alpha\eta + \beta x + \gamma)\left((\beta+1)x + \alpha\eta + \gamma\right)}, \tag{7}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{(\alpha\eta + \gamma)\left(2\beta(\beta+1)x + (2\beta+1)(\alpha\eta+\gamma)\right)}{-(\alpha\eta + \beta x + \gamma)^2\left(\alpha\eta + (\beta+1)x + \gamma\right)^2} \leq 0. \tag{8}$$

$$\frac{\partial f}{\partial \eta} = -\frac{\alpha x}{(\alpha\eta + \beta x + \gamma)\left(\alpha\eta + (\beta+1)x + \gamma\right)}, \tag{9}$$

$$\frac{\partial^2 f}{\partial \eta^2} = \frac{\alpha^2\left(2\alpha\eta + (2\beta+1)x + 2\gamma\right)x}{(\alpha\eta + \beta x + \gamma)^2\left(\alpha\eta + (\beta+1)x + \gamma\right)^2} \geq 0. \tag{10}$$

For all $(x, y) \in \mathbb{R}_+ \times \mathbb{R}_+^{2L}$, it must be that $h^T y \geq 0$. Thus $g(x, y) = f(x, h^T y)$. By the affine mapping composition property [26], it follows that $g(x, y)$ is convex in y and concave in x. ■

It is now possible to give very general conditions under which a Nash equilibrium exists. Moreover, the structure of the Nash equilibrium takes on an intuitive form: waterfilling is always optimal for the victim player when the FDM condition is satisfied.

Theorem 2: In the WCI game, a Nash equilibrium exists in pure strategies provided that P_x and P_y are bounded. Under the FDM criterion, the strategy of Player 1 is given by the waterfilling algorithm using a modified water level. Moreover, Player 1's strategy is identical at each Nash equilibrium.

Proof: Since $\mathcal{S}_1 \subset \mathbb{R}^N$ and $\mathcal{S}_2 \subset \mathbb{R}^{2LN}$ (or $\mathbb{R}^{LN}$ for FDM) are closed and bounded, by Heine-Borel they are both compact. Also, the objective is a composition of continuous functions, and hence continuous. It follows from Theorem 1 and the separability of J over tones that J is convex in $\mathbf{x}$ and concave in $[\mathbf{y}^{(1)}, \ldots, \mathbf{y}^{(N)}]$. The result [27, Thm. 4.4] may therefore be applied to conclude that a pure-strategy saddle point exists. Note that the saddle-point *need not* be unique, in general. As a saddle point exists in pure strategies, the game has a *value* [27, Thm. 4.1], which shall be denoted as R^*. Thus:

$$\sup_{\mathbf{x}\in\mathcal{S}_1} \inf_{[\mathbf{y}^{(1)},\mathbf{y}^{(2)},\ldots,\mathbf{y}^{(N)}]\in\mathcal{S}_2} J = \inf_{[\mathbf{y}^{(1)},\mathbf{y}^{(2)},\ldots,\mathbf{y}^{(N)}]\in\mathcal{S}_2} \sup_{\mathbf{x}\in\mathcal{S}_1} J = R^*. \tag{11}$$

Since the supremum and infemum are achieved:

$$\max_{\mathbf{x}\in\mathcal{S}_1} \min_{[\mathbf{y}^{(1)},\mathbf{y}^{(2)},\ldots,\mathbf{y}^{(N)}]\in\mathcal{S}_2} J = \min_{[\mathbf{y}^{(1)},\mathbf{y}^{(2)},\ldots,\mathbf{y}^{(N)}]\in\mathcal{S}_2} \max_{\mathbf{x}\in\mathcal{S}_1} J = R^*. \tag{12}$$

Let $(\mathbf{x}, [\mathbf{y}^{(1)} \ldots \mathbf{y}^{(n)}])$ be any saddle point. Evaluating the right-hand side of (12), if $\beta_n = 0$ (from FDM assumption) then:

$$R^* = \max_{x\in\mathcal{S}_1} \sum_{n=1}^{N} \log\left(1 + \frac{\tilde{\mathbf{h}}^{(n)}\mathbf{x}_n}{\alpha_n \mathbf{h}^{(n)}\mathbf{y}^{(n)} + N_n}\right). \tag{13}$$

The optimization problem (13) is seen to be precisely the same as single-user rate maximization with parallel Gaussian channels [23], and hence the (modified) waterfilling spectrum is optimal and unique (for fixed $[\overline{\mathbf{y}}^{(1)} \ldots \overline{\mathbf{y}}^{(n)}]$). In particular, the modified AWGN noise level on tone n is seen to be $\alpha_n \mathbf{h}^{(n)}\overline{\mathbf{y}}^{(n)} + N_n$. This is the same modified noise level used in the rate-adaptive IW algorithm [9].

To show uniqueness of Player 1's strategy for all Nash equilibria, suppose that $(\overline{\mathbf{x}}, [\overline{\mathbf{y}}^{(1)}, \ldots, \overline{\mathbf{y}}^{(N)}])$ and $(\widetilde{\mathbf{x}}, [\widetilde{\mathbf{y}}^{(1)}, \ldots, \widetilde{\mathbf{y}}^{(N)}])$ are saddle points. Then $\widetilde{\mathbf{x}}$ uniquely maximizes (13) for $[\mathbf{y}^{(1)}, \ldots, \mathbf{y}^{(N)}] = [\widetilde{\mathbf{y}}^{(1)}, \ldots, \widetilde{\mathbf{y}}^{(N)}]$. Observe that $(\overline{\mathbf{x}}, [\widetilde{\mathbf{y}}^{(1)}, \ldots, \overline{\mathbf{y}}^{(N)}])$ is also a saddle point by the exchangibility property of saddle points [27]. Consequently, $\overline{\mathbf{x}}$ is also the unique maximizer of (13) for $[\mathbf{y}^{(1)}, \ldots, \mathbf{y}^{(N)}] = [\widetilde{\mathbf{y}}^{(1)}, \ldots, \widetilde{\mathbf{y}}^{(N)}]$. This implies that $\overline{\mathbf{x}} = \widetilde{\mathbf{x}}$. ■

The following simple example shows that Player 2's strategy need not be unique even under FDM, and hence the saddle point need not be unique in general. Consider $N = 2$, $L = 2$, $\mathbf{h}^{(1)} = \mathbf{h}^{(2)} = [1\ 1]$, $P_x = 1$, $P_y = [1\ 1]^T$, $N_1 = N_2 > 0$, $\alpha_1 = \alpha_2 = 1$. Then it may be readily verified by symmetry arguments that for $\mathbf{x} = [1/2\ 1/2]^T$, both $\mathbf{y}^{(1)} = [1\ 0]^T$, $\mathbf{y}^{(2)} = [0\ 1]^T$ and $\mathbf{y}^{(1)} = \mathbf{y}^{(2)} = [1/2\ 1/2]^T$ form saddle points $(\mathbf{x}, [\mathbf{y}^{(1)}\ \mathbf{y}^{(2)}])$.

D. Numerical Computation of the Saddle Point

In order to numerically evaluate the achievable rate under worst case interference (R^*), it is necessary to numerically solve for the saddle point (12). While solving for Player 1's strategy given Player 2's can be a simple waterfilling (Theorem 2), computing the Nash equilibrium is nontrivial. However, this optimization problem can be solved using standard optimization techniques such as the "Infeasible Start Newton method" [26, §10.3]. We

now cast the problem (12) in optimization terminology and show that it satisfies certain necessary properties. Logarithmic barrier functions are employed to enforce the positivity and power constraints, and a central path algorithm is used to compute R^* to arbitrary accuracy [26]. Let the central path parameter be denoted by $t \in \mathbb{R}_{++}$ and define $\tilde{\mathcal{S}}_1 = \mathbf{int}(\mathcal{S}_1)$, $\tilde{\mathcal{S}}_2 = \mathbf{int}(\mathcal{S}_2)$, and $\tilde{J} : \tilde{\mathcal{S}}_1 \times \tilde{\mathcal{S}}_2 \mapsto \mathbb{R}_+$, where for the echo-cancelled case[3]:

$$\begin{aligned}\tilde{J}(x,[y_1,\ldots,\mathbf{y}^{(n)}]) &= t^{-1}\log\left(\sum_{n=1}^{N} P_x - \mathbf{x}_n\right)\\ &+ \sum_{n=1}^{N}\left\{\log\left(1+\frac{\mathbf{x}_n}{\alpha_n\mathbf{h}^{(n)}\mathbf{y}^{(n)}+\beta_n\mathbf{x}_n+N_n}\right)+t^{-1}\log(\mathbf{x}_n)\right.\\ &\left.- t^{-1}\sum_{l=1}^{L}\left[\log((\mathbf{y}^{(n)})_l)+\log((\mathbf{y}^{(n)})_{L+l})\right]\right\}\\ &- t^{-1}\sum_{l=1}^{2L}\log\left((P_y)_l-\sum_{n=1}^{N}(\mathbf{y}^{(n)})_l\right). \end{aligned} \tag{14}$$

It is now shown that $\tilde{J}$ satisfies the sufficient conditions [26, §10.3.4] for convergence of this technique: that the sublevel sets of $||\nabla\tilde{J}||_2$ are closed, and that the Hessian of $\tilde{J}$ is Lipschitz continuous with bounded inverse. The partial derivatives of $\tilde{J}$:

$$\begin{aligned}\frac{\partial\tilde{J}}{\partial\mathbf{x}_n} &= \frac{1}{\mathbf{x}_n+\alpha_n\mathbf{h}^{(n)}\mathbf{y}^{(n)}+N_n}+\frac{1}{t\mathbf{x}_n}-\frac{1}{t(P_x-\mathbf{1}^T\mathbf{x})},\\ \frac{\partial\tilde{J}}{\partial(\mathbf{y}^{(n)})_m} &= \frac{\alpha_n\mathbf{h}_m^{(n)}}{\mathbf{x}_n+\alpha_n\mathbf{h}^{(n)}\mathbf{y}^{(n)}+N_n}-\frac{\alpha_n\mathbf{h}_m^{(n)}}{\alpha_n\mathbf{h}^{(n)}\mathbf{y}^{(n)}+N_n}\\ &\quad-\frac{1}{t(\mathbf{y}^{(n)})_m}+\frac{1}{t\left((P_y)_m-\sum_{n=1}^{N}\mathbf{y}^{(n)})_m\right)}.\end{aligned} \tag{15}$$

Are continuous on $\tilde{\mathcal{S}}_1 \times \tilde{\mathcal{S}}_2$, implying by continuity of the norm that $||\nabla\tilde{J}||_2$ is continuous on $\tilde{\mathcal{S}}_1 \times \tilde{\mathcal{S}}_2$. Consequently, the sublevel sets S_α for each $\alpha \in \mathbb{R}$:

$$\begin{aligned}S_\alpha = \{(\mathbf{x},[\mathbf{y}^{(1)},\ldots,&\mathbf{y}^{(N)}]) \in \tilde{\mathcal{S}}_1 \times \tilde{\mathcal{S}}_2 :\\ &\tilde{J}((\mathbf{x},[\mathbf{y}^{(1)},\ldots,\mathbf{y}^{(N)}])) \le \alpha\}\end{aligned} \tag{16}$$

Are closed relative to $\tilde{\mathcal{S}}_1 \times \tilde{\mathcal{S}}_2$. To show that S_α is closed, suppose that $\{z_n\}$ is any sequence in S_α with $z_n \longrightarrow z$. If $z \in \tilde{\mathcal{S}}_1 \times \tilde{\mathcal{S}}_2 = \mathbf{int}(\tilde{\mathcal{S}}_1 \times \tilde{\mathcal{S}}_2)$, then $z \in S_\alpha$ by relative closure. Therefore it remains only to observe that there does not exist any $z_n \longrightarrow z$ with $z \in \partial\,\mathbf{cl}(\tilde{\mathcal{S}}_1 \times \tilde{\mathcal{S}}_2)$. This follows from examining (15), where it can be seen that $||\nabla\tilde{J}(z_n)||_2$ increases without bound for any such $z_n \longrightarrow z$. This contradicts the assumption that $\{z_n\}$ is a sequence in S_α.

In order to show for arbitrary $\alpha \in \mathbb{R}$ that the Hessian is Lipschitz continuous on S_α, it is enough to show that each element of $\nabla^2\tilde{J}$ is continuously differentiable on S_α. The partial derivatives of (15) may be readily computed[4] and seen to be continuous functions on $S_\alpha \subset \mathcal{S}_1 \times \mathcal{S}_2$. However $\mathcal{S}_1 \times \mathcal{S}_2$ is bounded, therefore S_α is also bounded (and closed), hence compact. Therefore each partial derivative of $\nabla^2\tilde{J}$, as a continuous function on a compact set, is bounded. Finally, the bounded inverse condition on the Hessian follows from the fact that

[3]The FDM case is directly analogous.

[4]The expressions are lengthy and omitted for space.

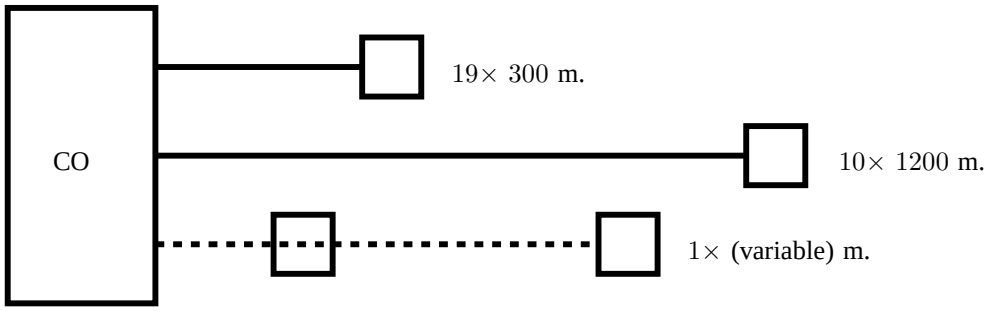


Fig. 2. Binder configuration for upstream VDSL simulations (not to scale). The dashed line is of varying lengths.

the barrier functions are strictly concave in $\mathbf{x}$ and strictly convex in $[\mathbf{y}^{(1)}, \ldots, \mathbf{y}^{(N)}]$. In particular, computation of the Hessian reveals that $\nabla^2_{\mathbf{x}} \tilde{J} \preceq \frac{-t^{-1}}{P_x^2} I$ and $\nabla^2_{[\mathbf{y}^{(1)},\ldots,\mathbf{y}^{(N)}]} \succeq \frac{t^{-1}}{\max_i (P_y)_i^2} I$ on $\tilde{\mathcal{S}}_1 \times \tilde{\mathcal{S}}_2$, and hence S_α.

IV. Simulation Results

The scope of the WCI analysis extends generally to DMT-based DSL systems. This section examines two particular cases that are deployed prevalently: VDSL and ADSL. In VDSL, a prominent interference issue is the upstream near-far effect, which is caused by crosstalk from short ('near') lines FEXT-coupling into longer ('far') lines. In ADSL, the issue of RT FEXT injection into longer CO lines is similarly of concern. Numerical results for these sample deployments demonstrate the practicality of the WCI analysis and show surprising commonalities between the different scenarios.

A. VDSL Upstream

The WCI rate bound is first applied to two different upstream VDSL scenarios exhibiting the near-far effect. The binder configuration is illustrated in Figure 2. For all simulations, 19×300 m. lines, 10×1200 m. lines, and one line of varying length occupy the binder of 24 AWG twisted-pairs. The FTTEx M2 (998 FDM) bandplan is employed with HAM bands notched, and the usual PSD constraints removed. Tones below 138 kHz are disabled for ADSL compatibility. The FDM condition is satisfied for this configuration, hence $\beta_n = 0$. For 10^{-7} BER, assume coding gain of 3 dB, with 6 dB margin, thus $\Gamma = 12.5$ dB. Each line is limited to 14.5 dBm power ($P_x = 14.5$ dBm, $P_y = \mathbf{1} \cdot 14.5$ dBm), unless otherwise stated.

1) WCI rate as a function of line length: First, consider the WCI rate bound when the variable-length line is the victim line (Player 1). Numerical results are shown in Figure 4, where a rate lower bound as well as the rate obtained when *all* lines execute full-power rate-adaptive (RA) IW are plotted as a function of victim line length. Note that full-power RA IW is quite different from fixed-margin (FM) IW, where power is minimized while achieving a fixed rate and margin [18]. To numerically investigate practical bitloading constraints, RA IW with discrete bit constraints [9] is executed on the victim modem assuming the WCI (12). The victim modem's

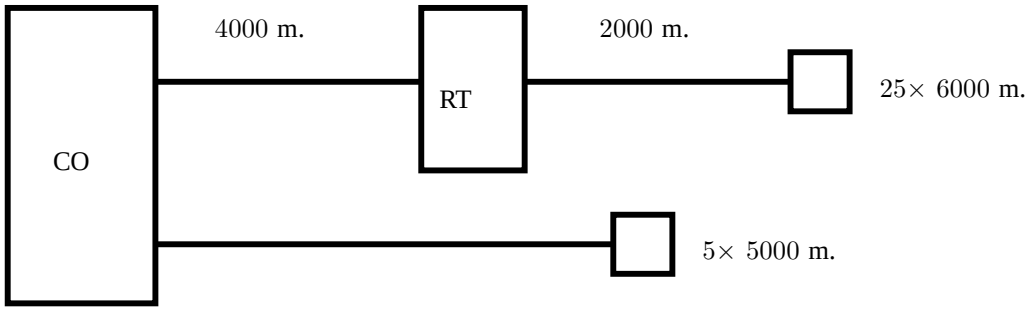


Fig. 3. Binder configuration for downstream RT ADSL simulations (not to scale). A common RT is used for each line.

achieved rate is plotted as R_d^*. Evidently, $R_d^* \leq R^*$. Thus, R_d^* is also a lower bound to the achievable rate under the WCI.

Observe that for most line lengths, the rate achieved by RA IW is fairly close to the WCI bound, particularly near 200 m. and 900 m. For intermediate lengths (≈ 650 m.), rate-adaptive IW can perform up to $\approx 75\%$ better than the WCI bound, though the absolute difference is small. As a corollary, the interference generated by IW in this configuration is deleterious in the sense that it is close in rate to the WCI saddle-point. This finding is consistent with results [11] showing that other *centralized* DSM strategies can significantly outperform IW in such cases. Furthermore, fixed-margin (FM) IW can also be seen to perform significantly better than the WCI bound when rates are adjudicated reasonably [18].

2) WCI rate as a function of PBO: Motivated by the results of the previous section showing that the full-power WCI rate bound can decrease precipitously as loop length increases, the efficacy of UPBO at mitigating this effect is considered. This section examines a simple *power backoff* strategy in the form of power-constrained RA IW. Though the use of RA IW is retained, an effect similar to fixed-margin (rate-constrained) IW [18] is induced by imposing various *tighter* power constraints. In particular, the variable-length line is set to length 300 m., and power back-off is imposed on all (20) 300 m. lines with full power retained on the (10) 1200 m. lines. By taking the victim line to be one of the 300 m. lines, the 300 m. WCI curve in Figure 5 is generated, yielding a lower bound to the achievable data rate for all 300 m. lines in the binder. The 1200 m. WCI curve represents the case where the victim modem is instead taken to be one of the 1200 m. lines. To compare standardized SSM techniques to DSM, the rates achieved using the SSM VDSL UBPO strategy defined for the Noise A environment [28] are illustrated by dashed horizontal lines.

For power constraints of less than approximately -13 dBm, the WCI bound for the 1200 m. loops is higher than that achieved using the standardized SSM technique. The marginal decrease in the maximum 300 m. line rate as compared to SSM can be mitigated by e.g. allowing PBO on the 1200 m. lines while keeping the 300 m. lines at full power.

Examining the 1200 m. lines, DSM with PBO improves both the RA IW-achieved and WCI bounds signifi-

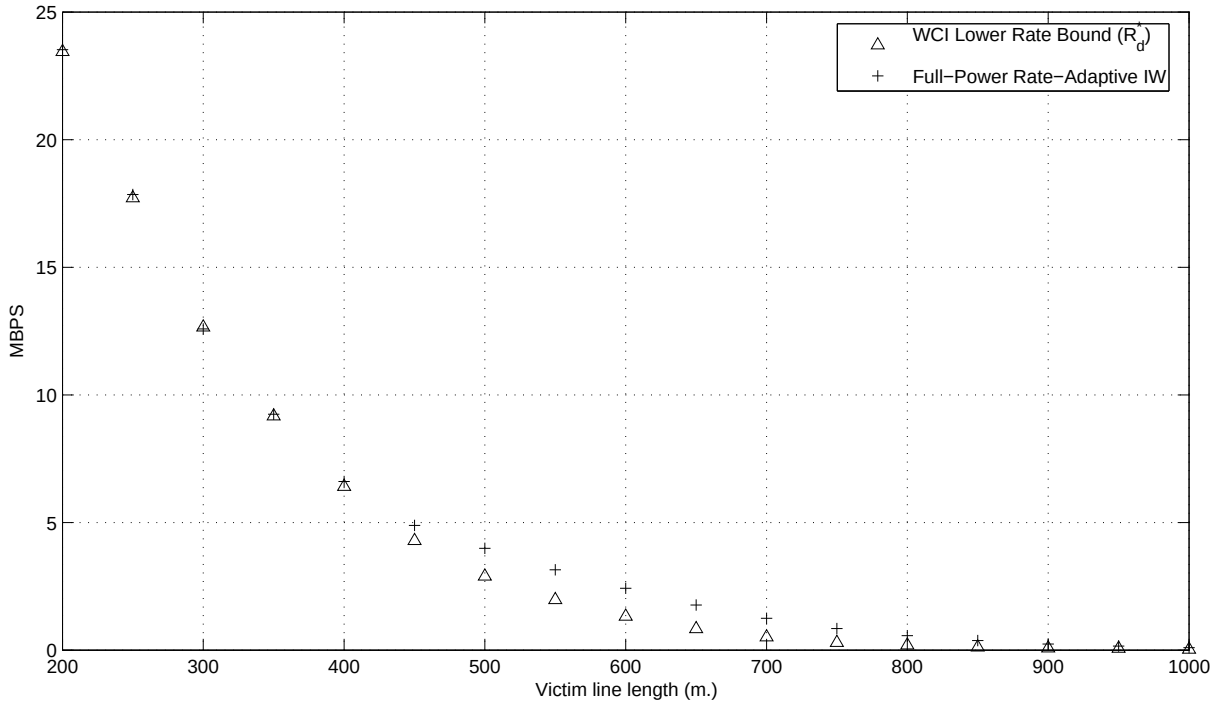


Fig. 4. Achievable rates in upstream VDSL as a function of victim line length (200-1000 m.).

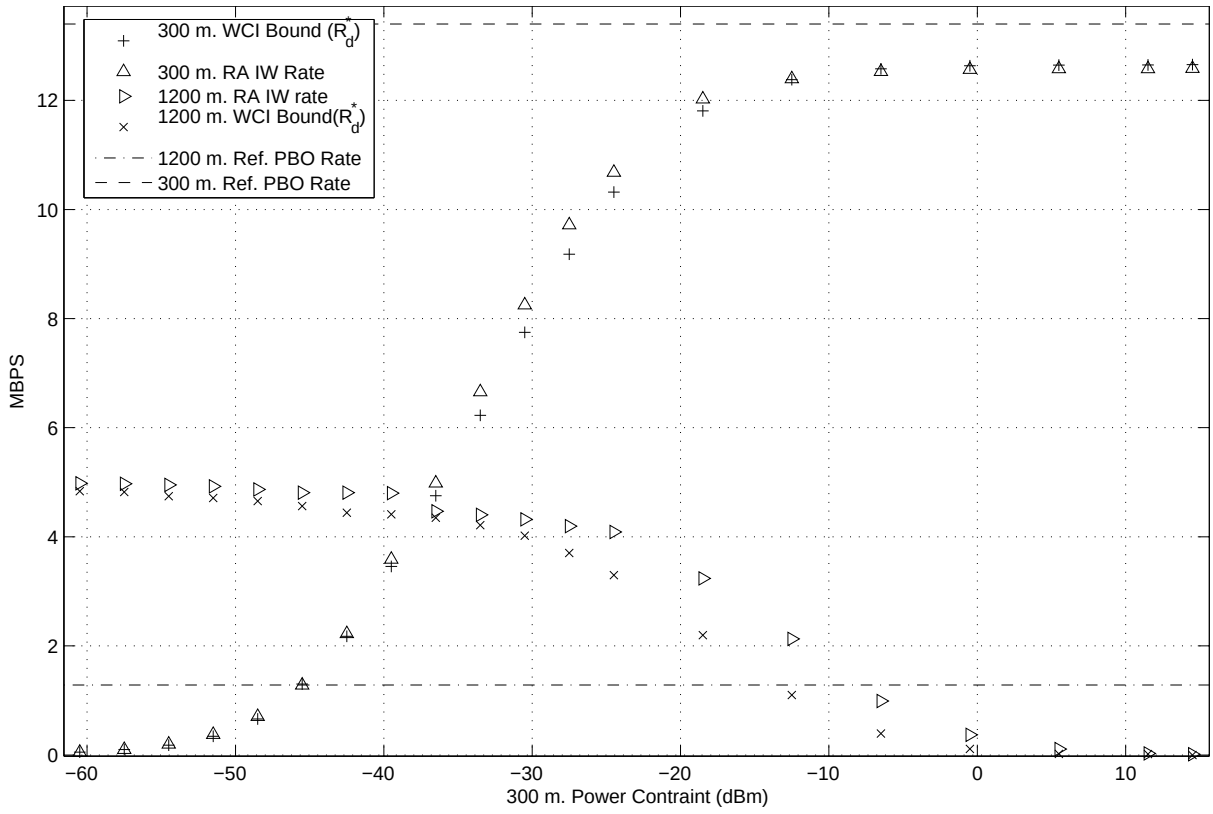


Fig. 5. Achievable rates in upstream VDSL as a function of short line (300 m.) power back-off.

cantly up to approximately -30 dBm, with diminishing returns for further PBO as the 300 m. line FEXT no longer dominates the interference profile. However, further PBO serves to continue to decrease the achievable rates of the 300 m. lines, as expected. The WCI bound is again fairly tight. Thus a DSL modem could safely operate with *guaranteed* ≈ 4 MBPS on the 1200 m. lines and ≈ 8 MBPS on the 300 m. lines, which is not feasible under standardized SSM UBPO.

B. ADSL Downstream with Remote Terminals (RT's)

The WCI rate bound is also applicable to ADSL. This section considers an RT ADSL configuration as illustrated in Figure 3. For all simulations, 25 ADSL lines are located 2000 m. from a fiber-fed RT 4000 m. from the CO. Additonally, 5×5000 m. lines are present in the binder. The FDM ADSL standard [29] parameters are assumed. As in the VDSL simulations, $\Gamma = 12.5$ dB. Each line is limited to 20.4 dBm downstream power ($P_x = 20.4$ dBm, $P_y = \mathbf{1} \cdot 20.4$ dBm).

A common problem of such configurations is that the signal from the CO to the non-RT (7000 m.) modems will be swamped by FEXT from the RT lines. This simulation examines the worst-case efficacy of power-backoff for the RT lines as a means of improving the rate of the CO lines. Figure 6 shows the dependance of rates on the level of power backoff (relative to 20.4 dBm) for the RT lines. The WCI bound is very tight to actual power-controlled RA IW performance on the RT lines, and reasonably close for the CO lines as well. As in VDSL, substantial improvements over SSM can be guaranteed; for example, it is possible (through proper power control) to guarantee 4 MBPS service on all lines, whereas this rate point was far from feasible with SSM or with full-power rate-adaptive IW. However without any power backoff, the performance of RA IW and the WCI bound are near that of SSM, showing the key role of power control in obtaining DSM gains in this setting.

V. CONCLUSION

This paper has studied the worst-case interference encountered when deploying Level 0-2 DSM techniques for next-generation DSL. A game-theoretic analysis has shown that under mild conditions, a pure-strategy Nash equilibrium exists in the WCI game, and can be computed using standard optimization techniques. The Nash equilibrium provides a useful lower bound to the achievable rate for a DSL modem employing DSM under any power-constrained interference profile. Furthermore, the structure of the Nash equilibrium reveals that for FDM systems, IW is optimal in a maximin sense.

The WCI bound was applied to an upstream near-far VDSL scenario and found to be numerically tight. Also, the utility of a simple DSM UPBO strategy employing RA IW was compared to SSM UPBO, were it was found that significantly preferable rate trade-offs are possible with DSM. A similar tradeoff was observed in RT ADSL systems, where RA IW performance benefits significantly from proper power control. These results suggest that the parameter of transmit power is critical to DSM performance, and with proper control can generate large performance gains.

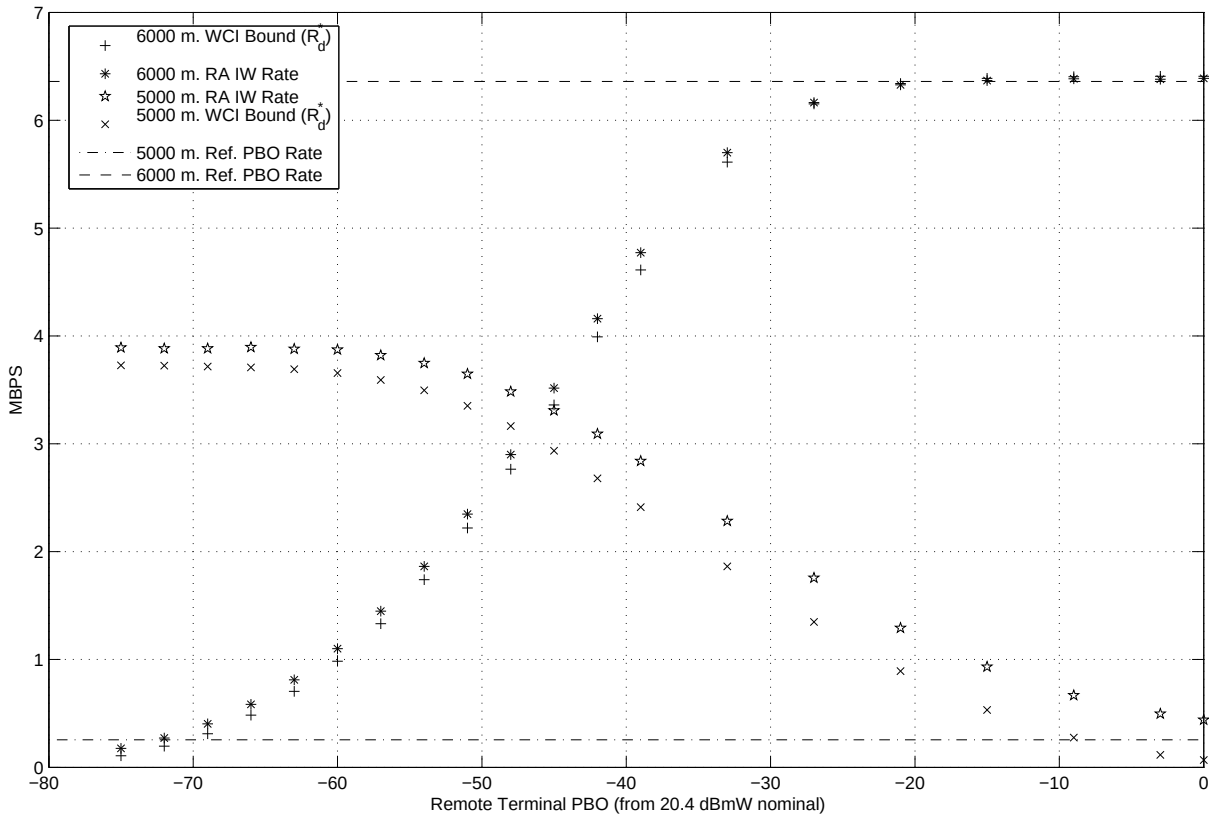


Fig. 6. Achievable rates in downstream ADSL as a function of RT line power backoff (relative to 20.4 dBm nominal TX power).

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