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Joint spectrum management and constrained partial crosstalk cancellation in a multi-user xDSL environment ☆

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Abstract

In modern DSL systems, crosstalk is a major source of performance degradation. Crosstalk cancellation techniques have been proposed to mitigate the effect of crosstalk. However, the run-time complexity of these crosstalk cancellation techniques grows with the square of the number of lines. Therefore one has to be selective in cancelling crosstalk to reduce complexity. Secondly, crosstalk cancellation requires signal-level coordination between transmitters or receivers, which is not always available. Because of accessibility constraints, crosstalk between certain lines cannot be cancelled and so has to be mitigated through spectrum management. After a complexity study, this paper presents a solution for the joint spectrum management and constrained partial crosstalk cancellation problem. The complexity of the partial crosstalk cancellation part of the problem is reduced based on a line selection and user independence observation. However, to fully benefit from these observations, power loading has to be applied in the spectrum management part. We therefore also consider ON/OFF power loading, which has a low complexity and shows only a minor performance degradation compared to normal power loading. The resulting algorithm will be compared to currently available algorithms for independent spectrum management and partial crosstalk cancellation.
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1. Introduction

Current xDSL access networks are evolving into mixtures of various DSL flavours, where for instance traditional ADSL lines provisioning customers over longer distances share binders with VDSL lines deployed from remote terminals (RT's). These network topologies suffer from electromagnetic coupling resulting in crosstalk between lines. As xDSL systems currently under development use higher frequencies to meet the demand for high data rates, crosstalk is indeed becoming particularly harmful. Moreover, significant line length variations and mixed deployments from central offices (CO's) and RT's create a near–far effect in the upstream and downstream direction, respectively. This causes crosstalk to sometimes overpower the direct signals. As a result, crosstalk can be 10–15 dB larger than the background noise, and becomes a major limiting factor in the performance of xDSL systems [2].

One strategy for dealing with this crosstalk is crosstalk cancellation. Several crosstalk cancellation techniques have been proposed to remove crosstalk in different scenarios. Linear pre- and post-filtering [3,4] requires coordination at both the transmitters and receivers. When this level of coordination is not available, successive interference cancellation or precompensation [5,6] can be used if there is only coordination at the receivers or transmitters, respectively. For this level of coordination, it is shown in [7,8] that a simple linear zero-forcing canceller or linear precompensator performs near optimally in an xDSL environment.

However, even for these simple linear cancellers, the run-time complexity grows with the square of the number of lines. For example, in a binder of 8 VDSL lines transmitting on 4096 tones at a block rate of 4000 blocks per second, the run-time complexity of crosstalk cancellation exceeds 1 billion multiplications per second. Because most of the crosstalk originates from a limited number of lines on a limited number of tones, a fraction of this complexity suffices to cancel most of the crosstalk. This is called partial crosstalk cancellation [9,10].

Crosstalk cancellation requires signal-level coordination at either the transmitter or receiver, i.e. the signals transmitted on interfering lines should be known. Oftentimes, not all interfering lines can be cancelled because they are not accessible. This is the case in a mixed CO–RT deployment where CO and RT reside in different geographical locations. Here partial crosstalk cancellation at the CO side has to be done independent of the partial crosstalk cancellation at the RT side. Secondly, accessibility constraints restrict the number of lines that can have signal-level coordination, even if they are at the same location. For example, crosstalk cancellation may not be possible between lines connected to different line cards.

In such situations, spectrum management [11,12] can be used to mitigate the crosstalk originating from lines that are not accessible. This is a second strategy for dealing with crosstalk. Instead of cancelling the crosstalk after it has occurred, transmit spectra are chosen such that the effect of crosstalk is minimized.

Currently available algorithms independently solve the spectrum management and partial crosstalk cancellation problem. A spectrum management algorithm first selects transmit spectra to avoid crosstalk. As an example, Optimal spectrum balancing (OSB) [13,14] can be used to calculate the optimal spectra that minimize the effect of crosstalk. Given these spectra, a partial crosstalk cancellation scheme can be used to cancel the remaining crosstalk. This two-step approach can be very suboptimal. The spectrum management algorithm does not take into account that a certain amount of crosstalk can be cancelled afterwards and hence the resulting spectra will be overly conservative.

A better solution can be obtained if the spectrum management and partial crosstalk cancellation problems are solved jointly. In [15], partial crosstalk cancellation based on resource allocation [9] is combined with Iterative Waterfilling (IW) spectrum management [16] in an iterative fashion. However, IW tends to be highly suboptimal in near–far scenarios. In this paper, the OSB algorithm is extended to include constrained partial crosstalk cancellation, based on [17].

In Section 2 of this paper, the joint spectrum management and constrained partial crosstalk cancellation problem is defined and a solution is proposed based on a dual decomposition. However, the complexity of this solution is found to be too high [17]. Therefore, in Section 3, possibilities to reduce this complexity are explored. In a first subsection the complexity of the partial crosstalk cancellation part of the problem is reduced based on a line selection and a user independence observation. These observations do not affect the optimality of the solution. In a second subsection, the complexity of the spectrum management part of

the problem is reduced by applying ON/OFF power loading, which has a low complexity and shows only a minor performance degradation compared to the optimal solution. Finally, in Section 4, simulation results are presented where the performance of the joint algorithm is compared to the solution obtained by independently solving the spectrum management problem and the partial crosstalk cancellation problem. Finally, Section 5 concludes the paper.

2. Joint spectrum management and constrained partial crosstalk cancellation

2.1. System model

Most current DSL systems use discrete multi-tone (DMT) modulation. The available frequency band is divided in a number of parallel subchannels or tones. Each tone is capable of transmitting data independently from other tones, and so the transmit power and the number of bits can be assigned individually for each tone. This gives a large flexibility in optimally shaping the transmit spectrum to minimize the effect of crosstalk.

Transmission for a binder of N users can be modeled on each tone k by

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{z}_k, \quad k = 1 \ldots K.$$

The vector $\mathbf{x}_k = [x_k^1, x_k^2, \ldots, x_k^N]^{\mathrm{T}}$ contains the transmitted signals on tone k for all N users. $[\mathbf{H}_k]_{n,m} = h_k^{n,m}$ is an $N \times N$ matrix containing the channel transfer functions from transmitter m to receiver n. The diagonal elements are the direct channels, the off-diagonal elements are the crosstalk channels. $\mathbf{z}_k$ is the vector of additive noise on tone k, containing thermal noise, alien crosstalk, RFI, ... The vector $\mathbf{y}_k$ contains the received symbols.

To take crosstalk cancellation into account, an equivalent channel $\tilde{\mathbf{H}}$ is introduced. This is the same channel as the original channel $\mathbf{H}$, but with off-diagonal elements set to 0 where the crosstalk is cancelled. If user n is cancelling crosstalk originating from user m on tone k, then $\tilde{h}_k^{n,m} = 0$. We refer to [9,10] where procedures are explained for cancelling individual crosstalk channels, based on particular DSL channel characteristics (row/column-wise diagonal dominance).

We denote the transmit power as $s_k^n \triangleq \Delta_f E\{|x_k^n|^2\}$, $\mathbf{s}_k = [s_k^1, s_k^2, \ldots, s_k^N]^{\mathrm{T}}$, $\mathbf{s} = [\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_K]$ and the noise power as $\sigma_k^n \triangleq \Delta_f E\{|z_k^n|^2\}$. The DMT symbol rate is denoted as f_s, the tone spacing as Δ_f.

It is assumed that each modem treats interference from other modems as noise. When the number of interfering modems is large, the interference is well approximated by a Gaussian distribution. Under this assumption the achievable bit loading of user n on tone k, given the transmit spectra of all modems in the system, is

$$b_k^n \triangleq \log_2\left(1 + \frac{1}{\Gamma}\frac{|\tilde{h}_k^{n,n}|^2 s_k^n}{\sum_{m \neq n} |\tilde{h}_k^{n,m}|^2 s_k^m + \sigma_k^n}\right), \tag{1}$$

where Γ denotes the SNR-gap to capacity, which is a function of the desired BER, the coding gain and noise margin. Note that taking the same SNR-gap for all the tones can be suboptimal since it is the mean symbol error probability across the tones that should be constrained.

We also define $\mathbf{b}_k = [b_k^1, b_k^2, \ldots, b_k^N]^{\mathrm{T}}$ and $\mathbf{b} = [\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_K]$. The data rate for user n is then

$$R^n = f_s \sum_k b_k^n.$$

When the bit loading b_k^n of the users is given for a specific tone k, the required transmit power $\mathbf{s}_k$ for the modems in the system can be calculated by [14]

$$\mathbf{s}_k = (\mathbf{D}_k - \mathbf{\Lambda}_k \mathbf{A}_k)^{-1} \mathbf{\Lambda}_k \boldsymbol{\sigma}_k, \tag{2}$$

$$\mathbf{D}_k \triangleq \mathrm{diag}\{|\tilde{h}_k^{1,1}|^2, |\tilde{h}_k^{2,2}|^2, \ldots, |\tilde{h}_k^{N,N}|^2\},$$

$$\mathbf{\Lambda}_k \triangleq \mathrm{diag}\{2^{b_k^1} - 1, 2^{b_k^2} - 1, \ldots, 2^{b_k^N} - 1\},$$

$$[\mathbf{A}_k]_{n,m} \triangleq a_k^{n,m} \quad \text{with } a_k^{n,m} \triangleq \begin{cases} 0, & n = m, \\ \Gamma |\tilde{h}_k^{n,m}|^2, & n \neq m, \end{cases}$$

$$\boldsymbol{\sigma}_k \triangleq \Gamma[\sigma_k^1, \sigma_k^2, \ldots, \sigma_k^N]^{\mathrm{T}}.$$

Not all bit loadings lead to feasible power loadings and Eq. (2) can lead to negative powers. To avoid this, a positivity constraint has to be added: $0 < s_k^n$. This positivity constraint allows the detection of infeasible bit loadings.

The total power used by user n is then

$$P^n = \sum_k s_k^n.$$

2.2. Problem statement

The joint spectrum management and constrained partial crosstalk cancellation problem amounts to finding an optimal allocation of transmit power and selection of the crosstalk to cancel, thereby

maximizing the capacity of the network. In doing so, there are a number of constraints.

First of all, there is a *total power constraint* $P^{n,\mathrm{tot}}$ for each user. This constraint ensures the user's total power does not exceed the maximum allowed total transmit power. On top of this constraint there can be a *spectral mask constraint* $s_k^{n,\mathrm{mask}}$ for each tone to guarantee electromagnetic compatibility with other systems.

Secondly, because of the run-time complexity of full crosstalk cancellation, there is a limited amount of resources for crosstalk cancellation. The cancellation of the crosstalk from one crosstalker on a tone is done by one cancellation tap [9,10]. The number of cancellation taps used is constrained by the *cancellation tap constraint* C^{tot} [17]. Furthermore, in a bundle of lines, not all crosstalk can be cancelled. This is the case when receivers are in different geographical locations or when lines are terminating on different line cards. These scenarios can be modeled by multiple cancellation tap constraints $C^{q,\mathrm{tot}}$, i.e. one constraint for each subset q of lines with full signal-level access.

Finally, there is a *rate constraint* $R^{n,\mathrm{target}}$ for each user. Typically, service providers offer a number of profiles to guarantee a certain quality-of-service. The rate constraint then indicates a minimum data rate required by the user.

Joint spectrum management and constrained partial crosstalk cancellation then results in solving the following maximization problem [17]:

$$
\begin{aligned}
&\text{maximize}_{\mathbf{s},\mathbf{c}} && \textstyle\sum_{n=1}^{N} R^n \\
&\text{subject to} && P^n \leqslant P^{n,\mathrm{tot}}, \quad n = 1 \ldots N, \\
& && 0 \leqslant s_k^n \leqslant s_k^{n,\mathrm{mask}}, \quad n = 1 \ldots N, \;\; k = 1 \ldots K, \\
& && R^n \geqslant R^{n,\mathrm{target}}, \quad n = 1 \ldots N, \\
& && C^q = \textstyle\sum_k \sum_n \sum_m c_k^{n,m} \leqslant C^{q,\mathrm{tot}}, \\
& && n, m \in i^q, \quad q = 1 \ldots Q, \\
&\text{with} && [\mathbf{c}_k]_{n,m} = c_k^{n,m}, \quad c_k^{n,m} = \begin{cases} 0 \Rightarrow \tilde{h}_k^{n,m} = h_k^{n,m}, \\ 1 \Rightarrow \tilde{h}_k^{n,m} = 0, \end{cases} \\
& && s_k^n \in \mathscr{S},
\end{aligned}
\tag{3}
$$

where $\mathbf{c} = [\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_K]$. $\mathbf{c}_k$ is a matrix containing the crosstalk cancellation configuration for tone k. $c_k^{n,m} = 1$ indicates that a cancellation tap is assigned on tone k for cancelling crosstalk on line n originating from line m. Because of accessibility constraints, n and m are restricted to the subset of line indices i^q which have full signal-level control. For lines n, m that have no signal-level control, $c_k^{n,m} = 0, \forall k$.

Finally, $\mathscr{S}$ is a set of possible power levels. With (3), all possible transmit power levels (together with all possible cancellation tap configurations) are scanned and so this procedure will from now on be referred to as "power loading".

Alternatively, (3) can be reformulated as an optimization problem in $\{\mathbf{b}, \mathbf{c}\}$ instead of $\{\mathbf{s}, \mathbf{c}\}$, i.e.

$$
\begin{aligned}
&\text{maximize}_{\mathbf{b},\mathbf{c}} && \textstyle\sum_{n=1}^{N} R^n, \\
&\text{subject to} && P^n \leqslant P^{n,\mathrm{tot}}, \quad n = 1 \ldots N, \\
& && 0 \leqslant s_k^n \leqslant s_k^{n,\mathrm{mask}}, \quad n = 1 \ldots N, \;\; k = 1 \ldots K, \\
& && R^n \geqslant R^{n,\mathrm{target}}, \quad n = 1 \ldots N, \\
& && C^q = \textstyle\sum_k \sum_n \sum_m c_k^{n,m} \leqslant C^{q,\mathrm{tot}}, \\
& && n, m \in i^q, \quad q = 1 \ldots Q, \\
&\text{with} && [\mathbf{c}_k]_{n,m} = c_k^{n,m}, \quad c_k^{n,m} = \begin{cases} 0 \Rightarrow \tilde{h}_k^{n,m} = h_k^{n,m}, \\ 1 \Rightarrow \tilde{h}_k^{n,m} = 0, \end{cases} \\
& && b_k^n \in \mathscr{B},
\end{aligned}
\tag{3bis}
$$

where $\mathscr{B}$ is a set of possible bit loadings. This will be referred to as "bit loading". The cardinalities of $\mathscr{S}$ and $\mathscr{B}$ will both be referred to as B.

2.3. Dual decomposition

Optimization problem (3)–(3bis) is a non-convex problem. To find the global optimum of (3)–(3bis), no other solution is known than to exhaustively search through all possible transmit spectra $\mathbf{s}$ (or bit load vectors $\mathbf{b}$) and cancellation tap configurations $\mathbf{c}$. Because some constraints are coupled over the tones, this results in an exponential complexity in the number of tones. In [13,14], the complexity is made linear in K by using a dual decomposition that decouples the problem over the tones. This is done by using Lagrange multipliers to move the constraints coupled over tones into the objective function of the optimization problem. It was shown in [17] that if the number of tones is sufficiently large, the duality gap is zero and therefore the dual decomposition does not affect the solution. Because the cancellation tap constraint is also coupled over the tones, extra Lagrange multipliers are introduced to make the complexity linear. For the power loading case this results in

$$
\begin{aligned}
\mathbf{s}^{\mathrm{opt}}, \mathbf{c}^{\mathrm{opt}} = \underset{\mathbf{s},\mathbf{c}}{\mathrm{argmax}} & \sum_{n=1}^{N} \omega_n R^n \\
& + \sum_{n=1}^{N} \lambda_n \left(P^{n,\mathrm{tot}} - \sum_{k=1}^{K} s_k^n \right)
\end{aligned}
$$

$$+\sum_{q=1}^{Q} v_q\left(C^{q,\mathrm{tot}}-\sum_{k=1}^{K}\sum_{n\in i^q}\sum_{m\in i^q} c_k^{n,m}\right), \tag{4}$$

$$\begin{aligned}\text{subject to}\quad & 0\leqslant s_k^n\leqslant s_k^{n,\mathrm{mask}},\quad n=1\ldots N,\\ & \lambda_n\geqslant 0,\omega_n\geqslant 0,\quad n=1\ldots N,\\ & v_q\geqslant 0,\quad q=1\ldots Q,\\ & s_k^n\in\mathscr{S},\end{aligned}$$

where ω_n, λ_n and v_q are Lagrange multipliers. $\boldsymbol{\lambda}=[\lambda_1,\ldots,\lambda_N]^{\mathrm{T}}$ and $\boldsymbol{v}=[v_1,\ldots,v_Q]^{\mathrm{T}}$ can be seen as a cost for power and crosstalk cancellation taps, respectively, that replace the total power constraint and the cancellation tap constraint. Larger values for these Lagrange multipliers result in less power and allocated cancellation taps. The data rates of the users are weighted by $\boldsymbol{\omega}=[\omega_1,\ldots,\omega_N]^{\mathrm{T}}$, thereby giving more importance to some users and replacing the target rate constraints. In this way, all possible trade offs can be made to enforce the data rate constraints.

For a given set of $\boldsymbol{\omega}$, $\boldsymbol{\lambda}$ and $\boldsymbol{v}$ and by collecting the terms per tone and noting that a sum can be maximized by maximizing the individual terms, this optimization problem can then be solved in a per-tone fashion:

$$\begin{aligned}\text{for } k=1\ldots K:\ \mathbf{s}_k^{\mathrm{opt}},\mathbf{c}_k^{\mathrm{opt}} &= \underset{\mathbf{s}_k,\mathbf{c}_k}{\operatorname{argmax}}\sum_{n=1}^{N}\omega_n f_s b_k^n\\ &-\sum_{n=1}^{N}\lambda_n s_k^n-\sum_{q=1}^{Q}\sum_{n\in i^q}\sum_{m\in i^q} v_q c_k^{n,m},\end{aligned} \tag{5}$$

$$\begin{aligned}\text{subject to}\quad & 0\leqslant s_k^n\leqslant s_k^{n,\mathrm{mask}},\quad n=1\ldots N,\\ & \lambda_n\geqslant 0,\quad \omega_n\geqslant 0,\quad n=1\ldots N,\\ & v_q\geqslant 0,\quad q=1\ldots Q,\\ & s_k^n\in\mathscr{S}.\end{aligned}$$

Similar formulas can be given for the bit loading case.

Maximization of (5) for given Lagrange multipliers $\boldsymbol{\omega}$, $\boldsymbol{\lambda}$ and $\boldsymbol{v}$ can be performed by an exhaustive search. For each tone, the objective function should be evaluated for all possible combinations of the transmit power levels $\mathbf{s}_k$ and cancellation tap configurations $\mathbf{c}_k$ of the users. The combination giving the largest value for this expression is the optimal allocation of transmit power and cancellation taps for this tone and thus the original constraints can be checked.

To solve (3) by (5), $\boldsymbol{\omega},\boldsymbol{\lambda}$ and $\boldsymbol{v}$ should be tuned such that after the per-tone exhaustive search the original constraints are satisfied. In [13], an efficient Lagrange multiplier search procedure for $\boldsymbol{\omega}$ and $\boldsymbol{\lambda}$ is presented. In Appendix A, this procedure is extended to include $\boldsymbol{v}$, resulting in an update formula as follows:

$$\begin{bmatrix}\boldsymbol{\omega}\\ \boldsymbol{\lambda}\\ \boldsymbol{v}\end{bmatrix}^{t+1}=\left(\begin{bmatrix}\boldsymbol{\omega}\\ \boldsymbol{\lambda}\\ \boldsymbol{v}\end{bmatrix}^{t}-\mu\begin{bmatrix}\mathbf{R}-\mathbf{R}^{\mathrm{target}}\\ \mathbf{P}^{\mathrm{tot}}-\mathbf{P}\\ \mathbf{C}^{\mathrm{tot}}-\mathbf{C}\end{bmatrix}\right)^{+}, \tag{6}$$

where t is the iteration index, $(x)^+$ means $\max(0,x)$ and where $\mathbf{R}=[R^1,\ldots,R^N]^{\mathrm{T}}$, $\mathbf{P}=[P^1,\ldots,P^N]^{\mathrm{T}}$ and $\mathbf{C}=[C^1,\ldots,C^Q]^{\mathrm{T}}$ are vectors with the total powers, data rates and number of cancellation taps corresponding to the Lagrange multipliers at hand. This update formula is used in Algorithm 1 adopted from [13]. The non-positive inner product of (A.4) ensures that if the stepsize μ is small, update formula (6) will bring us closer to the optimum where the distance is expressed as the Euclidean distance to satisfying the constraints with equality. If the current stepsize brings us closer to the optimum, we could have taken a larger step to converge faster, e.g. doubling the stepsize. If we detect that the stepsize is too large (distance to the optimum increases), we take the best Lagrange multipliers found so far and start over. If our initial stepsize is too large, we can start halving it until it is small enough.

Algorithm 1. Multi-user Lagrange multiplier search algorithm.

```
μ = 1
While distance > tolerance do
  Θ = [ω, λ, v]^T = best [ω, λ, v]^T so far
  step = 1
  While distance ⩽ previousDistance do
    previousDistance = distance
    ΔΘ = [Δω, Δλ, Δv]^T =update formula (6)
    [R^{Θ+ΔΘ}, P^{Θ+ΔΘ}, C^{Θ+ΔΘ}, s, c]= exhaustiveSearch(Θ + ΔΘ)
    distance=‖[R^{Θ+ΔΘ} − R^target, P^tot − P^{Θ+ΔΘ}, C^tot − C^{Θ+ΔΘ}]^T‖
    if distance > previousDistance & step = 1 then
      μ = μ/2
    else
      μ = μ × 2
      step = 0
    end if
  end while
end while
```

Note that all the Lagrange multipliers are updated in parallel. In [13] it is observed that

adding extra Lagrange multipliers does not increase the number of steps required for convergence. The search procedure typically converges in 50–150 steps. Therefore, the cancellation tap constraint only adds to the complexity of the per-tone exhaustive search.

2.4. Complexity

The joint spectrum management and constrained partial crosstalk cancellation problem (3)–(3bis) is a non-convex constrained optimization problem. Without the dual decomposition, finding the global optimum requires an exhaustive search over all possible solutions. First, assume there are no accessibility constraints, so that all crosstalk can be cancelled. On a certain tone, a user has to decide which crosstalk from $N-1$ other users has to be cancelled. There are 2^{N-1} possibilities to do this. Together with B possibilities for discrete bit or power loading, this results in a total of $B2^{N-1}$ possibilities for each user on each tone and hence a total complexity of $\mathcal{O}((B2^{N-1})^{KN})$.

The dual decomposition decouples the problem over the tones, therefore reducing the exponential complexity in the number of tones K to linear complexity: $\mathcal{O}(K(B2^{N-1})^N)$. This amounts to K exhaustive searches of complexity $\mathcal{O}((B2^{N-1})^N)$. This is an enormous reduction in complexity. However, this solution is still computationally intractable because of the remaining complexity of the per-tone exhaustive search, which is indeed $(2^{N-1})^N$ times more complex than the per-tone search in the spectrum management problem without crosstalk cancellation. In a 4-user upstream VDSL scenario for example, it takes 20 days to calculate optimal spectra with OSB on a Pentium IV. When partial crosstalk cancellation is added, computing the optimal solution would take about 225 years.

The dual decomposition approach is only feasible if the per-tone exhaustive search can be performed with manageable complexity, as was also concluded in [17]. In the next section, methods are introduced to make this possible.

3. Complexity reduction

The complexity of the per-tone exhaustive search for the joint spectrum management and constrained partial crosstalk cancellation problem is $\mathcal{O}(K(B2^{N-1})^N)$, in the case where all crosstalk can be cancelled (Section 2.4). This can be rewritten as $\mathcal{O}(KB^N(2^{N-1})^N)$, clearly showing the complexity due to spectrum management, $\mathcal{O}(B^N)$, and partial crosstalk cancellation, $\mathcal{O}((2^{N-1})^N)$. In this section we focus on reducing the complexity originating from these two individual subproblems.

It will turn out that, unlike in the bit loading case, a significant complexity reduction can be achieved in the power loading case.

3.1. Partial crosstalk cancellation

This subsection again starts with the assumption that there are no accessibility constraints, i.e. that all crosstalk can be cancelled. Later, observations will be extended to the case with accessibility constraints.

To determine the optimal allocation of crosstalk cancellation taps for a given bit or power loading on a certain tone, all of the $(2^{N-1})^N \approx 2^{N^2}$ possible allocations have to be evaluated. Fortunately, in the power loading case many of these possibilities can be eliminated based on two observations: line selection and user independence.

- *Line selection*: A user has to decide whether or not it will cancel the crosstalk originating from $N-1$ other users. This leads to 2^{N-1} possible crosstalk cancellation configurations. However, from (1) it can be seen that to maximize the capacity for given transmit powers s_k^n (power loading case), one should allocate crosstalk cancellation taps to cancel those users that are causing the largest crosstalk. Therefore, if r crosstalk cancellation taps are available, they should be used to cancel the r largest sources of crosstalk.
  As a consequence, *when power loading is applied*, the 2^{N-1} possibilities for crosstalk cancellation are reduced to N possibilities for each user: no crosstalk cancellation, cancel the strongest crosstalker, cancel the two strongest crosstalkers, ..., cancel the $N-1$ strongest crosstalkers. In the N-user case, this observation results in the following complexity reduction:
  $$(2^{N-1})^N \rightarrow N^N. \tag{7}$$

- *User independence*: All users have to decide on a crosstalk cancellation configuration. This leads to an exponential complexity in the number of users

N. However, from (1) it can be seen that if for given transmit powers s_k^n (power loading case) user n allocates a crosstalk cancellation tap to cancel crosstalk caused by user m (i.e. $\tilde{h}_k^{n,m} = 0$) this only has an influence on the capacity of user n. Therefore, *when power loading is applied*, the users are "decoupled", i.e. they can choose a crosstalk cancellation configuration independently. The per-tone optimization problem (5) (without accessibility constraints) can indeed be rewritten as

$$\max_{\mathbf{s}_k,\mathbf{c}_k}\left\{\sum_{n=1}^{N}\omega_n f_s b_k^n - \sum_{n=1}^{N}\lambda_n s_k^n - \sum_{n}^{N}\sum_{m\neq n}^{N} v_q c_k^{n,m}\right\} = \max_{\mathbf{s}_k}\left\{\sum_{n=1}^{N}\max_{c_k^{n,1},c_k^{n,2},\ldots,c_k^{n,N}}\left(\omega_n f_s b_k^n - \lambda_n s_k^n - \sum_{m\neq n}^{N} v_q c_k^{n,m}\right)\right\}. \quad (8)$$

This means that for each selection of the power load vector $\mathbf{s}_k$, the selection of the corresponding optimal crosstalk cancellation configuration (for a given set of Lagrange multipliers) can be done in a per-user fashion.

As a consequence, the exponential complexity in N is reduced to linear complexity. This observation results in the following complexity reduction:

$$(2^{N-1})^N \rightarrow N(2^{N-1}). \quad (9)$$

It is noted that in the case of bit loading (3bis), (2) is used to determine the power necessary to load a certain number of bits on a tone. Cancelling crosstalk for one user now affects all other users. Adding a crosstalk cancellation tap for one user changes the power needed to transmit a certain number of bits for this user, thus also the crosstalk into other users changes. This may again affect the configuration of crosstalk cancellation taps for these other users. Therefore, user independence does not hold when bit loading is applied.

In the power loading case, both observations can be combined. N users then independently have to choose one of N possible crosstalk cancellation configurations. This results in the following total complexity reduction:

$$(2^{N-1})^N \rightarrow NN. \quad (10)$$

These observations can be easily extended to the case where there are accessibility constraints, reducing the number of crosstalkers that can be cancelled. Assume there are Q line cards. Line card q has access to M_q lines, with $\sum_{q=1}^{Q} M_q = N$. The complexity reduction by line selection and user independence is then summarized in Table 1.

In an 8-user case, the above observations reduce the number of crosstalk cancellation configurations from 2^{56} to 2^6. If there are 2 line cards, each having 4 lines, the number of crosstalk cancellation configurations is reduced from 2^{24} to 2^5. Note that despite these complexity reductions, the computed solution is still globally optimal.

3.2. Spectrum management: ON/OFF power loading

In this subsection, the complexity of the spectrum management part of the problem is reduced. Despite the complexity reduction provided by dual decomposition, OSB is still found to be too complex for scenarios with more than 3 users. The reason is the per-tone exhaustive search which still has exponential complexity in the number of users: $\mathcal{O}(B^N)$. In [18,19] an iterative procedure (ISB) is used to make this complexity linear. However, optimality cannot be guaranteed.

In this paper, the complexity is combated by reducing B, the number of possible transmit levels (for power loading). Originally, for OSB, typical values for B are 60 in the case of power loading and 14 in the case of bit loading. Therefore, bit loading would be the most efficient method for OSB. However, as shown in the previous subsection, power loading is necessary to fully benefit from the line selection and user independence observations. By limiting the transmit spectra to ON/OFF power loading, $B = 2$, the complexity is reduced from $\mathcal{O}(B^N)$ to $\mathcal{O}(2^N)$ and the result are simple transmit spectra similar to what is currently used in ADSL systems. This ON/OFF power loading problem equals (5) with the spectral mask constraints replaced by

$$s_k^n \in \{0, s_k^{n,\mathrm{ON}}\} \quad \text{with } s^{n,\mathrm{ON}} \leqslant s_k^{n,\mathrm{mask}}. \quad (11)$$

Table 1
Complexity reduction partial crosstalk cancellation

	1 line card	Q line cards
Full complexity	$\mathcal{O}((2^{N-1})^N)$	$\mathcal{O}(\prod_{q=1}^{Q}(2^{M_q-1})^{M_q})$
Line selection	$\mathcal{O}((N)^N)$	$\mathcal{O}(\prod_{q=1}^{Q}(M_q)^{M_q})$
User independence	$\mathcal{O}(N2^{N-1})$	$\mathcal{O}(\sum_{q=1}^{Q} M_q 2^{M_q-1})$
Line selection and user independence	$\mathcal{O}(NN)$	$\mathcal{O}(\sum_{q=1}^{Q} M_q M_q)$

In order to perform the reduced per-tone exhaustive searches, $s^{n,\mathrm{ON}}$ has to be defined. This ON-level should not violate the spectral mask constraint and the resulting spectra should not violate the total power constraint. An ON-level that automatically enforces the total power constraint is

$$s_k^{n,\mathrm{ON}} = \min\left(\frac{P^n}{K}, s_k^{n,\mathrm{mask}}\right). \tag{12}$$

Table 2
Complexity ON/OFF loading

	OSB	ISB
Full complexity	$\mathcal{O}(UKB^N)$	$\mathcal{O}(UKI_{\mathrm{ISB}}BN)$
Fixed on	$\mathcal{O}(K2^N)$	$\mathcal{O}(KI_{\mathrm{ISB}}2N)$
Adaptive on	$\mathcal{O}(I_1K2^N)$	$\mathcal{O}(KI_1I_{\mathrm{ISB}}2N)$
Adaptive threshold	$\mathcal{O}(I_2K2^N)$	$\mathcal{O}(KI_2I_{\mathrm{ISB}}2N)$

Even when all tones are switched on, the total power constraint is not violated. Therefore, no Lagrange multipliers for the total power constraints have to be searched.

However, with this fixed ON-level, if tones are switched off, not all available power is allocated while it could be redistributed to the tones that are switched on. Because of the lack of this redistribution of power, ON/OFF loading with a fixed ON-level is found to perform worse than IW [16].

The performance of ON/OFF loading can be greatly improved by redistributing the power of tones that are switched off to the other tones. To do this, one can use an iterative procedure, in each iteration updating the ON-level according to the number of active tones:

$$s_k^{n,\mathrm{ON}} = \min\left(\frac{P^n}{(\#\text{ active tones})^n}, s_k^{n,\mathrm{mask}}\right). \tag{13}$$

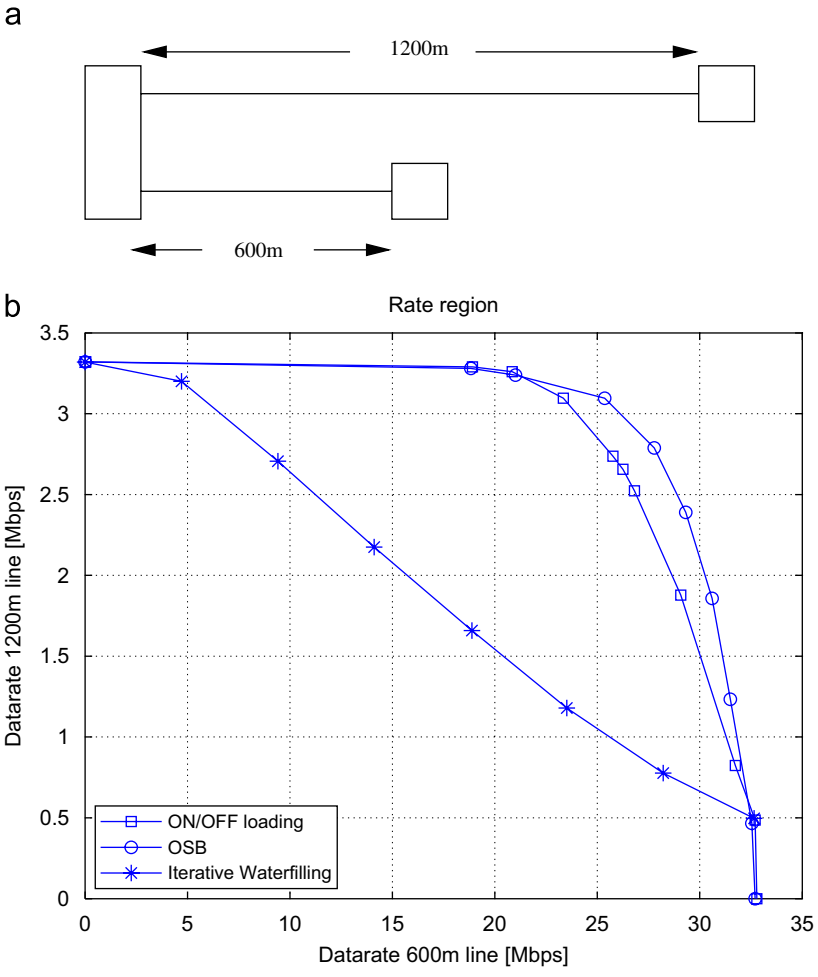


Fig. 1. Two-user VDSL scenario, adaptive threshold.

However, in some cases, the per-tone exhaustive search does not automatically switch off tones. E.g. in a CO/RT scenario, CO lines do not cause significant crosstalk on the RT lines. Thus these tones are not switched off by the per-tone exhaustive search. As these tones carry (very) few bits, the overall performance would then benefit from a redistribution of power from high frequencies to lower frequencies.

This can be accomplished by setting a threshold on the minimum number of bits that a tone should carry. Tones that do not reach this minimum are then switched off. Tones that are switched off once ($t_k^{n,\text{usable}} = 0$) are never activated again, thereby assuring convergence of this iterative procedure. The complete procedure (without crosstalk cancellation) is presented in Algorithm 2.

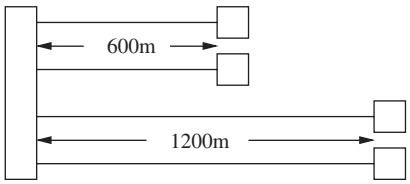


Fig. 2. Four-user VDSL scenario.

Algorithm 2. Multi-user ON/OFF loading with adaptive ON-level.

$t_k^{n,\text{usable}} = 1, \quad n = 1 \ldots N, k = 1 \ldots K$

repeat

$s_k^{n,\text{ON}} = \min\left(\frac{P^n}{(\#\text{ active tones})^n}, s_k^{n,\text{mask}}\right)$

for $k = 1$ to K **do**

$s_k^{1,\text{opt}}, \ldots, s_k^{n,\text{opt}} = \text{argmax}_{s_k^1,\ldots,s_k^n} \omega_1 b_k^1 + \cdots + \omega_n b_k^n$

with $s_k^n \in \{0, s_k^{n,\text{ON}} \cdot t_k^{n,\text{usable}}\}$

$\forall n$: **if** $b_k^n < \textit{threshold}$ **then** $t_k^{n,\text{usable}} = 0, s_k^{n,\text{opt}} = 0$

end for

until convergence

The procedure typically converges in 4–5 iterations. A good value for the threshold depends on the scenario and the target rates. Low thresholds give good performance for high rates on the RT lines, high thresholds give good performance for high rates on the CO lines. To adaptively select a good threshold for a specific scenario and given target rates, a number of different thresholds can be used. The complexity increase is minor when the thresholds are applied in ascending order. For each new threshold, one can then start from the converged transmit spectra of the previous threshold.

It is noted that while ON/OFF power loading in itself drastically reduced the complexity of the per-tone search, the algorithm can still be combined

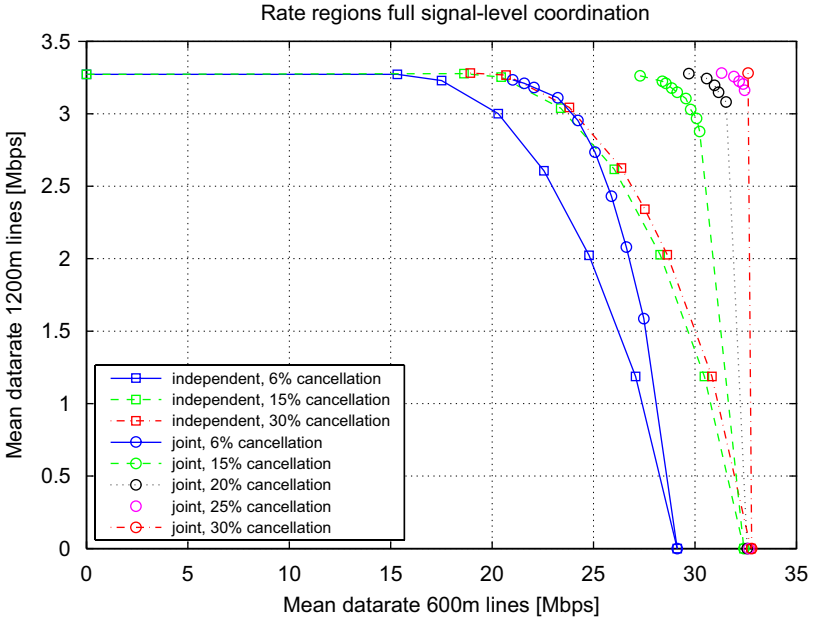


Fig. 3. Rate regions 4-user VDSL scenario.

with the iterative search algorithm (ISB) presented in [18,19]. Instead of performing an exhaustive multi-user search, each user iteratively performs a single-user search to optimize the weighted rate sum, while other users are kept fixed. Because in this case there are only 2 choices for the power loading in the single-user search, this iterative approach performs as good as an exhaustive search.

An overview of the complexities of various algorithms discussed is shown in Table 2. U is the number of updates required to find the Lagrange multipliers when there are more than 2 power levels, typically 50–150 [13]. I_1 is the number of iterations needed to converge to an adaptive ON-level, typically 4–5, I_2 is the number of iterations needed to converge in the adaptive threshold case, typically 10–12 for 7 different thresholds. In the ISB case [18,19], I_{ISB} iterations are performed in the per-tone search, typically 2–3.

In Fig. 1(b) the rate regions are shown for the scenario of Fig. 1(a), without crosstalk cancellation.

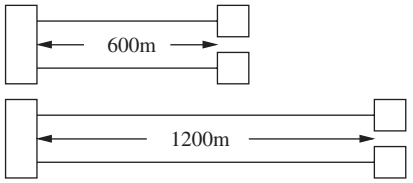


Fig. 5. Four-user VDSL scenario with constrained signal-level coordination: equal line lengths together.

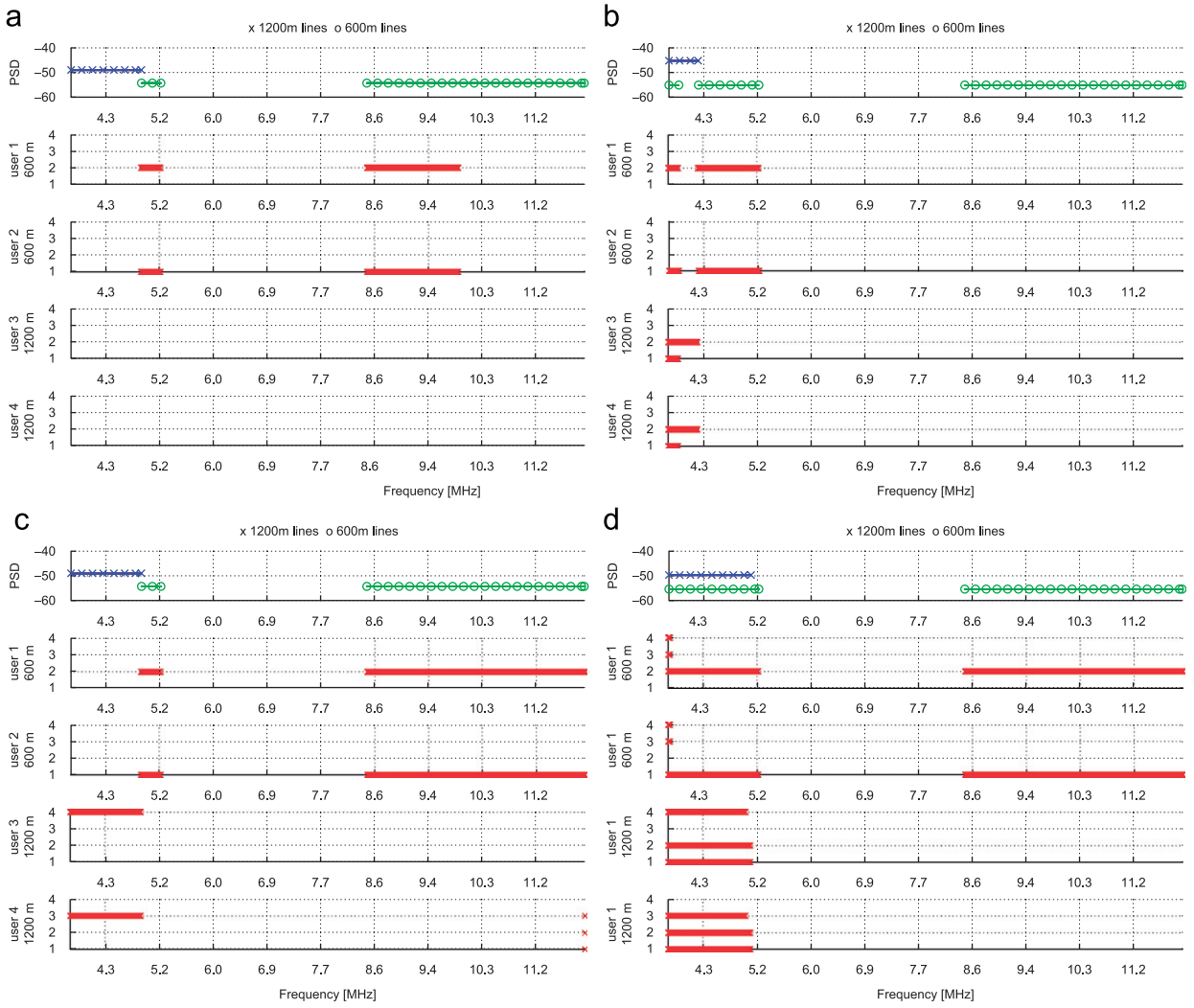


Fig. 4. Spectra and cancellation configurations: (a) independent solution, 6% of full cancellation; (b) joint solution, 6% of full cancellation; (c) independent solution, 30% of full cancellation; (d) joint solution, 30% of full cancellation.

Both OSB and ON/OFF loading with adaptive threshold achieve a significant performance gain over IW. ON/OFF loading introduces only 10% performance loss compared to OSB.

When combining the line selection and user independence observations with ON/OFF power loading, the optimization of (3) for a 4-user upstream scenario can be done in a few minutes instead of the original 225 years (Section 2.4).

4. Simulation results

In this section the performance is analyzed when solving the joint spectrum management and constrained partial crosstalk cancellation problem as opposed to independently solving these two problems. An upstream VDSL scenario is considered as shown in Fig. 2, with full signal-level coordination. A line diameter of 0.5 mm (24 AWG) is used and the maximum transmit power is 11.5 dBm. The SNR gap Γ is set to 12.9 dB, corresponding to a target symbol error probability of 10^{-7}, coding gain of 3 dB and a noise margin of 6 dB. The tone spacing is $\Delta_f = 4.3125$ kHz and the DMT symbol rate $f_s =$ 4 kHz [20].

To manage the complexity of the problem, ON/OFF loading is used for the spectrum management part of the per-tone search. Furthermore, the ISB procedure is used to search the maximum. In Fig. 3 rate regions are shown to compare the performance of the joint solution and the independent solution obtained by independently solving the spectrum management problem (with ON/OFF loading) and the partial crosstalk cancellation problem.

The rate regions show significant performance gains of the joint solution over the independent solution. Because the independent solution first independently solves the spectrum management problem, the transmit spectra are chosen to avoid crosstalk. This can be seen in Fig. 4(a) and (c), where the PSD is shown along with the allocation of cancellation taps for each user and the originating user of the crosstalk that is cancelled. For this strong crosstalk scenario the transmit spectra result in long and short lines occupying different frequency bands. When the partial crosstalk cancellation problem is solved, there is not much crosstalk left to cancel. Therefore, only a limited crosstalk cancellation tap budget can be used effectively. Fig. 3 shows that no performance is gained by increasing the crosstalk cancellation tap budget beyond 15% of full crosstalk cancellation.

When the spectrum management problem and the partial crosstalk cancellation problem are solved jointly, transmit spectra are chosen such that only crosstalk that cannot be cancelled is avoided. This can be seen in Fig. 4(b) (d), where all crosstalk cancellation taps can now be used effectively.

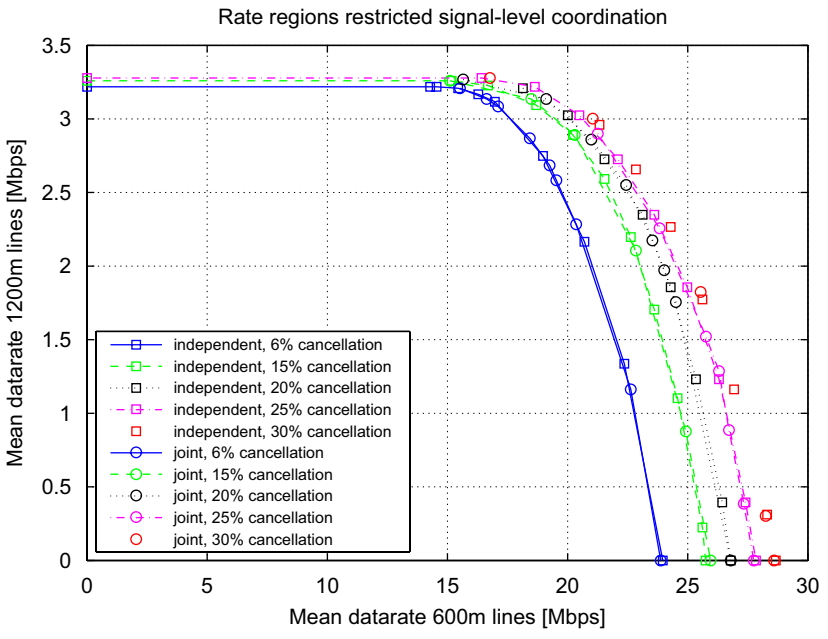


Fig. 6. Rate regions 4-user VDSL scenario with constrained signal-level coordination: equal line lengths together.

Depending on the crosstalk cancellation tap budget, transmit spectra can overlap on frequencies with the highest capacity, resulting in significant performance gains.

When there are restrictions on the signal-level coordination, a choice has to be made as to which lines will be connected to the same line card. In this case, there are two possibilities: connect the lines with similar lengths to the same line card (Fig. 5) or connect lines with different lengths to the same line card (Fig. 8).

Connecting lines with similar lengths to the same line card results in the rate regions of Fig. 6. Only limited performance is gained by increasing the crosstalk cancellation tap budget. This is caused by the fact that the long lines do not have access to the short lines. Therefore, this major source of crosstalk cannot be cancelled. As a result, the spectrum management has to be used to avoid this crosstalk and both groups of lines occupy different frequency bands as shown in Fig. 7. Therefore the joint and independent solutions are similar and only a limited number of crosstalk cancellation taps can be used effectively.

When lines with different lengths are connected to the same line card as shown in Fig. 8, the rate regions of Fig. 9 are obtained. Again, there is no significant difference between the joint and

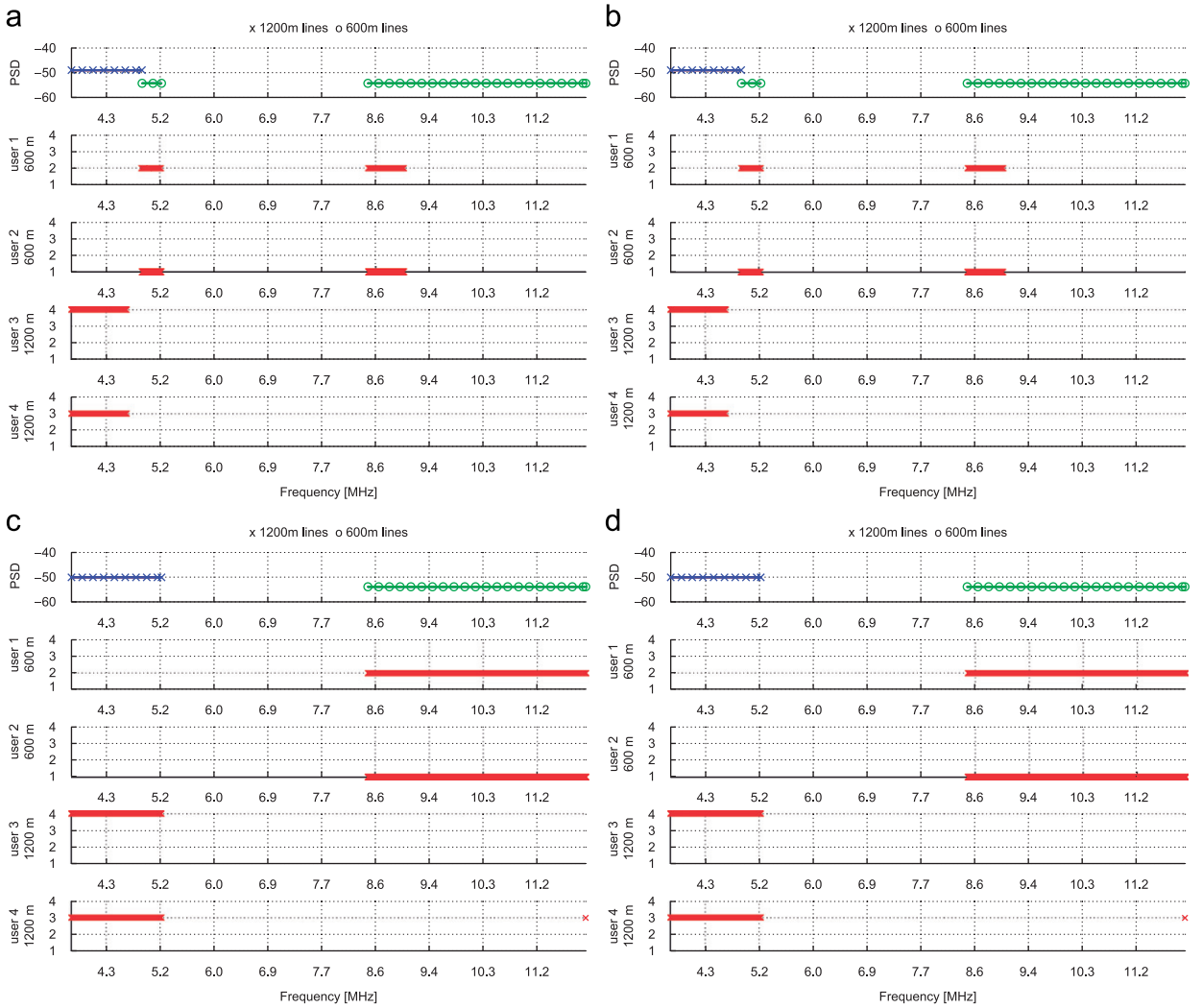


Fig. 7. Spectra and cancellation configurations with signal-level coordination between lines of equal length: (a) independent solution, 6% of full cancellation; (b) joint solution, 6% of full cancellation; (c) independent solution, 30% of full cancellation; (d) joint solution, 30% of full cancellation.

independent solutions. Because the long lines cannot access *all* short lines, there will be severe crosstalk that cannot be cancelled if these lines would use the same frequency band. As a consequence, long and short lines use different frequency bands. Moreover, because lines with similar lengths are on different line cards, no crosstalk cancellation taps can be assigned. This is shown in Fig. 10.

In Fig. 11 the different possibilities for signal-level coordination are summarized for the case where a cancellation tap budget of 30% of full crosstalk cancellation is available. It can be seen that the best performance is obtained when there is full coordination (all lines accessible). Also, in this case there is a significant performance increase when jointly solving the spectrum management and partial crosstalk cancellation problem, reducing the run-time complexity of partial crosstalk cancellation. However, when certain lines are not accessible, the performance of the joint solution is similar to the performance when the spectrum management problem and the partial crosstalk cancellation problem are solved independently. In this case, some performance can be gained by carefully deploying the access network: lines with similar lengths should have signal-level control to maximize performance.

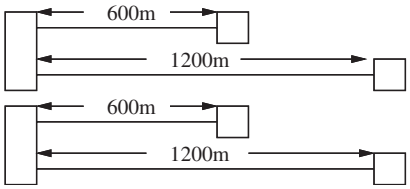


Fig. 8. Four-user VDSL scenario with constrained signal-level coordination: mixed line lengths.

5. Conclusion

In modern DSL systems, crosstalk is a major source of performance degradation. Crosstalk cancellation techniques have been proposed to mitigate the effect of crosstalk. However, the run-time complexity of these crosstalk cancellation techniques grows with the square of the number of lines. Therefore one has to be selective in cancelling crosstalk to reduce complexity. Secondly, crosstalk cancellation requires signal-level coordination between transmitters or receivers, which is not always available. Because of accessibility constraints, crosstalk between certain lines cannot be cancelled and so has to be mitigated through spectrum management.

In this paper, a solution was presented to jointly solve the spectrum management and constrained partial crosstalk cancellation problem based on a

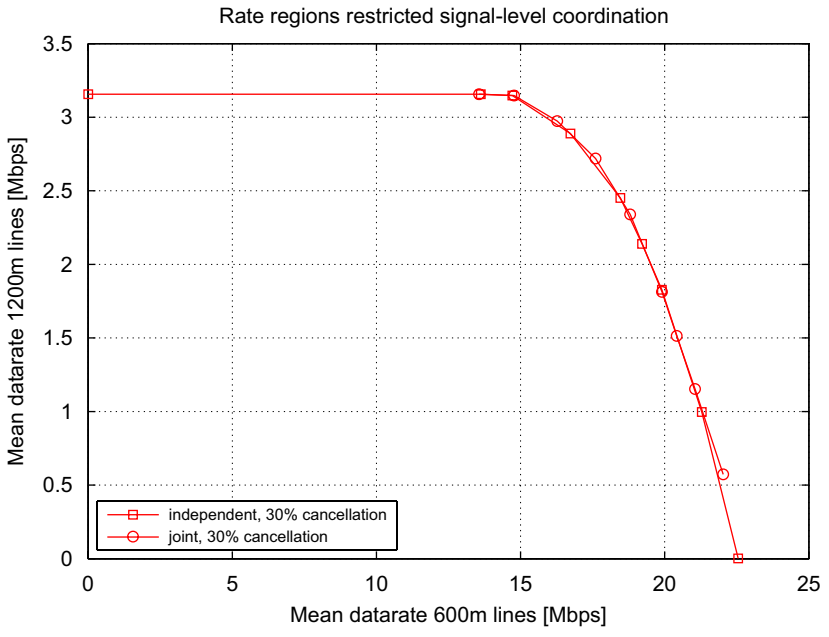


Fig. 9. Rate regions 4-user VDSL scenario with constrained signal-level coordination: mixed line lengths.

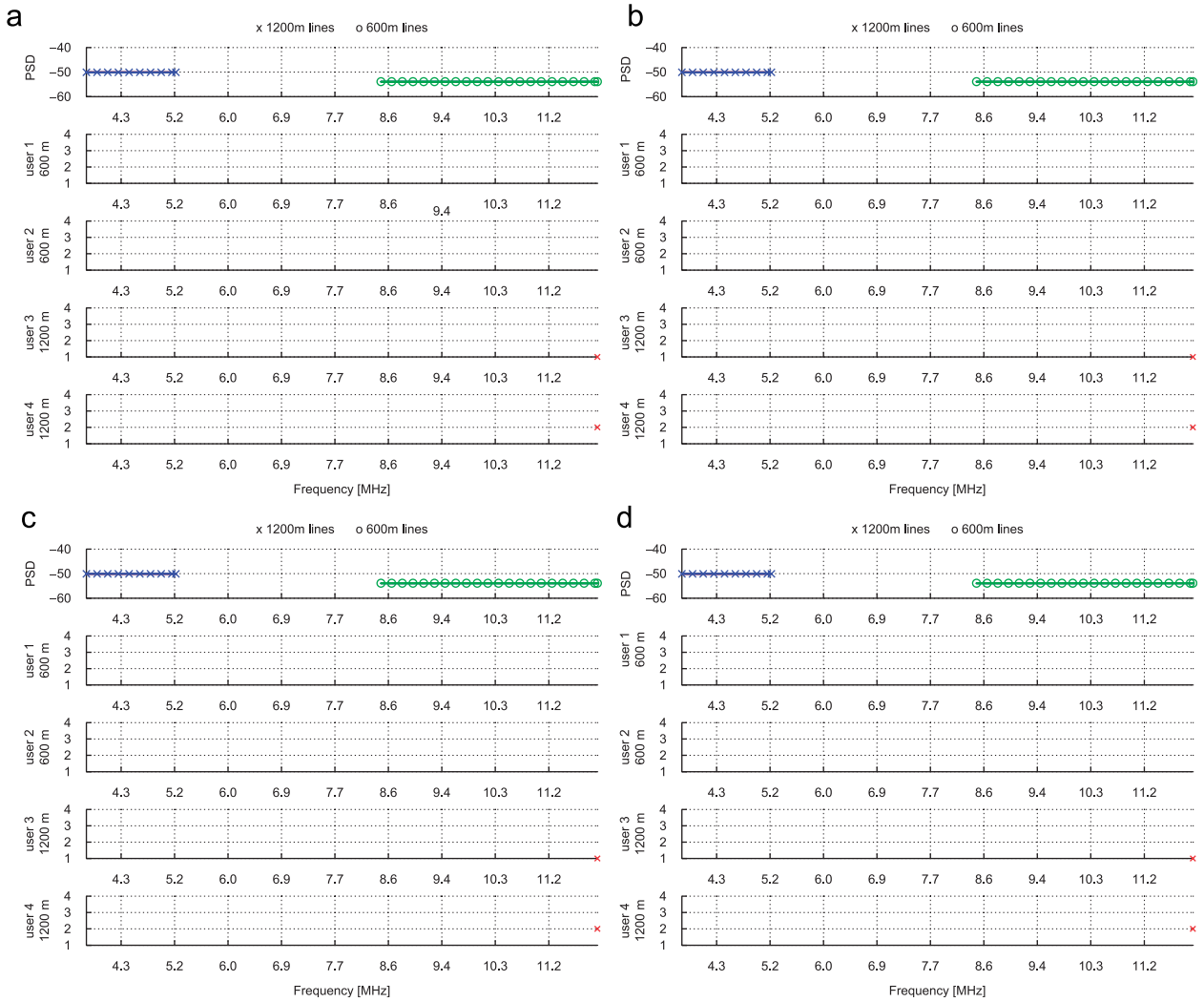


Fig. 10. Spectra and cancellation configurations with signal-level coordination between lines of different length: (a) independent solution, 6% of full cancellation; (b) joint solution, 6% of full cancellation; (c) independent solution, 30% of full cancellation; (d) joint solution, 30% of full cancellation.

dual decomposition approach. The complexity of the partial crosstalk cancellation part of the solution was reduced based on a line selection and a user independence observation. However, to fully benefit from these observations, power loading has to be applied in the spectrum management part. We have therefore considered ON/OFF power loading, which shows only a minor performance degradation compared to the original power loading.

It was shown that when the spectrum management problem and constrained partial crosstalk cancellation problem are solved independently, only a limited number of crosstalk cancellation taps can be used effectively because crosstalk is avoided in the first place by the spectrum management. When jointly solving the problems, only crosstalk that cannot be cancelled is avoided, thereby significantly increasing performance. However, this performance increase is only available when there is full signal-level coordination. When only a limited number of lines is accessible, the joint solution provides only little performance increase. It was shown that in this case lines with similar lengths should be made accessible to each other to maximize performance.

Appendix A. Search algorithm for the Lagrange multipliers

The proof presented in [13] can be easily extended to include $\boldsymbol{v}$. First assume a two-user scenario with

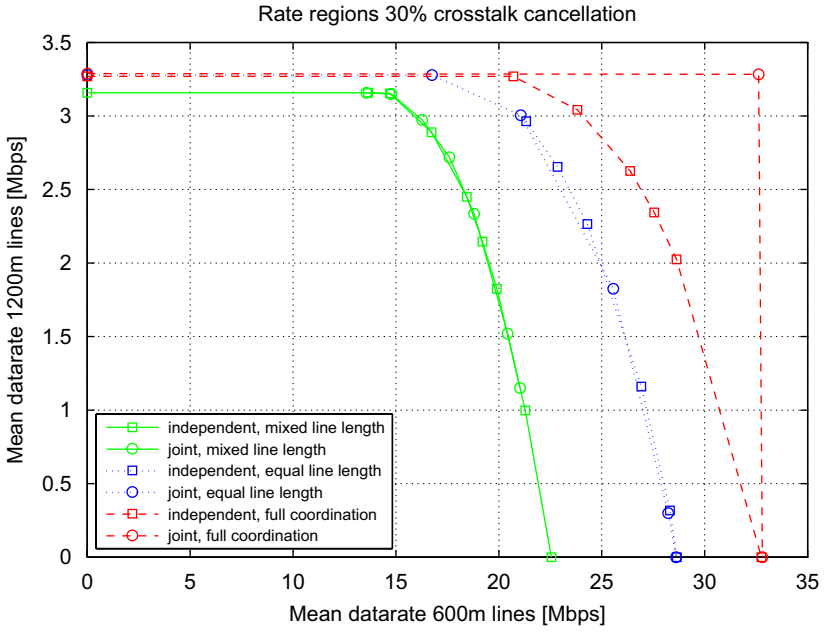


Fig. 11. Rate regions 4-user VDSL scenario: comparison of constraints on signal-level coordination.

signal-level control. Starting from two optimal solutions $(R^{1,\omega_A,\lambda_A,v_A}, P^{1,\omega_A,\lambda_A,v_A}, R^{2,\omega_A,\lambda_A,v_A}, P^{2,\omega_A,\lambda_A,v_A}, C^{\omega_A,\lambda_A,v_A})$ and $(R^{1,\omega_B,\lambda_B,v_B}, P^{1,\omega_B,\lambda_B,v_B}, R^{2,\omega_B,\lambda_B,v_B}, P^{2,\omega_B,\lambda_B,v_B}, C^{\omega_B,\lambda_B,v_B})$ corresponding to $(\boldsymbol{\omega}_A, \boldsymbol{\lambda}_A, \boldsymbol{v}_A)$and $(\boldsymbol{\omega}_B, \boldsymbol{\lambda}_B, \boldsymbol{v}_B)$, respectively, optimality for $(\boldsymbol{\omega}_A, \boldsymbol{\lambda}_A, \boldsymbol{v}_A)$ implies

$$\begin{aligned}&\omega_{1,A}R^{1,\omega_B,\lambda_B,v_B} + \omega_{2,A}R^{2,\omega_B,\lambda_B,v_B} - \lambda_{1,A}P^{1,\omega_B,\lambda_B,v_B}\\ &\quad - \lambda_{2,A}P^{2,\omega_B,\lambda_B,v_B} - v_{1,A}C^{\omega_B,\lambda_B,v_B}\\ &\leqslant \omega_{1,A}R^{1,\omega_A,\lambda_A,v_A} + \omega_{2,A}R^{2,\omega_A,\lambda_A,v_A} - \lambda_{1,A}P^{1,\omega_A,\lambda_A,v_A}\\ &\quad - \lambda_{2,A}P^{2,\omega_A,\lambda_A,v_A} - v_{1,A}C^{\omega_A,\lambda_A,v_A}. \qquad \text{(A.1)}\end{aligned}$$

Optimality for $(\boldsymbol{\omega}_B, \boldsymbol{\lambda}_B, \boldsymbol{v}_B)$ implies

$$\begin{aligned}&\omega_{1,B}R^{1,\omega_A,\lambda_A,v_A} + \omega_{2,B}R^{2,\omega_A,\lambda_A,v_A} - \lambda_{1,B}P^{1,\omega_A,\lambda_A,v_A}\\ &\quad - \lambda_{2,B}P^{2,\omega_A,\lambda_A,v_A} - v_{1,B}C^{\omega_A,\lambda_A,v_A}\\ &\leqslant \omega_{1,B}R^{1,\omega_B,\lambda_B,v_B} + \omega_{2,B}R^{2,\omega_B,\lambda_B,v_B} - \lambda_{1,B}P^{1,\omega_B,\lambda_B,v_B}\\ &\quad - \lambda_{2,B}P^{2,\omega_B,\lambda_B,v_B} - v_{1,B}C^{\omega_B,\lambda_B,v_B}. \qquad \text{(A.2)}\end{aligned}$$

Taking the sum of (A.1) and (A.2) results in

$$\begin{aligned}&-\underbrace{(\omega_{1,B}-\omega_{1,A})}_{\Delta\omega_1}\underbrace{(R^{1,\omega_B,\lambda_B,v_B}-R^{1,\omega_A,\lambda_A,v_A})}_{\Delta R^1}\\ &\quad-\underbrace{(\omega_{2,B}-\omega_{2,A})}_{\Delta\omega_2}\underbrace{(R^{2,\omega_B,\lambda_B,v_B}-R^{2,\omega_A,\lambda_A,v_A})}_{\Delta R^2}\\ &\quad+\underbrace{(\lambda_{1,B}-\lambda_{1,A})}_{\Delta\lambda_1}\underbrace{(P^{1,\omega_B,\lambda_B,v_B}-P^{1,\omega_A,\lambda_A,v_A})}_{\Delta P^1}\\ &\quad+\underbrace{(\lambda_{2,B}-\lambda_{2,A})}_{\Delta\lambda_2}\underbrace{(P^{2,\omega_B,\lambda_B,v_B}-P^{2,\omega_A,\lambda_A,v_A})}_{\Delta P^2}\\ &\quad+\underbrace{(v_{1,B}-v_{1,A})}_{\Delta v_1}\underbrace{(C^{\omega_B,\lambda_B,v_B}-C^{\omega_A,\lambda_A,v_A})}_{\Delta C}\leqslant 0. \qquad \text{(A.3)}\end{aligned}$$

Relation (A.3) can be extended straightforwardly to a multi-user scenario with multiple line cards:

$$\begin{bmatrix}-(\Delta\boldsymbol{\omega})^{\mathrm{T}} & (\Delta\boldsymbol{\lambda})^{\mathrm{T}} & (\Delta\boldsymbol{v})^{\mathrm{T}}\end{bmatrix}\begin{bmatrix}\Delta\mathbf{R}\\ \Delta\mathbf{P}\\ \Delta\mathbf{C}\end{bmatrix}\leqslant 0, \qquad \text{(A.4)}$$

where $\boldsymbol{\omega}=[\omega_1,\ldots,\omega_N]$, $\boldsymbol{\lambda}=[\lambda_1,\ldots,\lambda_N]$ and $\boldsymbol{v}=[v_1,\ldots,v_Q]$ are vectors containing the Lagrange multipliers, $\mathbf{R}=[R^1,\ldots,R^N]^{\mathrm{T}}$, $\mathbf{P}=[P^1,\ldots,P^N]^{\mathrm{T}}$ and $\mathbf{C}=[C^1,\ldots,C^Q]^{\mathrm{T}}$ are vectors with the corresponding data rates, total powers and number of cancellation taps.

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