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UNEQUAL ERROR PROTECTION IN RATE ADAPTIVE SPECTRUM MANAGEMENT FOR DIGITAL SUBSCRIBER LINE SYSTEMS

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ABSTRACT

Crosstalk between different lines in a cable bundle is the major source of performance degradation in DSL systems. Spectrum coordination techniques have been shown to substantially alleviate the crosstalk problem. The equal level of error protection that these techniques provide can however be excessive for some applications. Many applications with diverse error protection requirements can be sharing the same connection. In this paper, two novel rate adaptive spectrum management algorithms are presented that enable a different level of error protection for different applications. The algorithms are generalizations of the globally optimal OSB and the locally optimal DSB algorithms for systems that incorporate unequal error protection. Through simulation, it is shown that unequal error protection can lead to significant performance gains.

Index Terms— DSL, DSM, Unequal error protection

1. INTRODUCTION

Digital Subscriber Line (DSL) technology remains the most popular broadband access technology. Increasing demand for higher data rates forces telcos to operate at higher frequencies. At these high frequencies, electromagnetic coupling between different wires in a cable bundle causes interference, also called crosstalk. This crosstalk is the major cause of performance degradation in DSL systems.

Dynamic spectrum management (DSM) is a common term used for a set of solutions to the crosstalk problem. DSM techniques can be split into two major categories: signal coordination and spectrum coordination. The focus of this paper will be on spectrum coordination which consists of optimally designing the transmit spectrum of each modem in order to cause minimal disturbance to other modems. Spectrum coordination can be formulated as an optimization problem, referred to as the spectrum management problem (SMP). This optimization problem is non-convex and can have many local optima. Several algorithms have already been developed to solve this problem [1–3].

DSL allows for the simultaneous transmission of several applications. These applications may have different quality of service (QoS) requirements, more specifically, they may require a different level of error protection or average target bit error rate (BER). Current SMP solutions consider the case where every application is assigned the same SNR gap in the calculation of the bitloading, i.e. the same level of error protection. By accommodating unequal error protection through applying different SNR gaps for different applications, further performance gains can be achieved.

Transmission with a different SNR gap for different applications has already been considered [4–6]. The single user case is examined in [4]. A MIMO system where users are separated through OFDMA is considered in [5], and [6] studies the same problem as this paper, but proposes an algorithm with higher complexity.

2. SYSTEM MODEL

Most DSL systems make use of discrete multi-tone (DMT) modulation. DMT splits the spectrum into a large number of sub carriers or tones. For a system with sufficiently long cyclic prefix and perfect tone synchronization, the transmission in an N-user cable bundle is modeled for each tone as

$$\boldsymbol{y}_k = \boldsymbol{H}_k \boldsymbol{x}_k + \boldsymbol{z}_k, \quad \forall k \in \mathcal{K},$$

where $\mathcal{K}$ denotes the set of K tones, $\boldsymbol{x}_k = \left[x_k^1, x_k^2, \ldots, x_k^N\right]^T$ contains the transmitted symbols of all N modems on tone k, and $[\boldsymbol{H}_k]_{n,m} = h_k^{n,m}$ is the $N \times N$ channel matrix that contains the transfer function between every transmitter m and receiver n, evaluated on tone k. Diagonal elements of $\boldsymbol{H}_k$ represent the direct channels, whereas the off-diagonal elements represent the crosstalk channels. Furthermore, $\boldsymbol{z}_k$ is a

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vector of additive zero-mean Gaussian noise on tone k and $\boldsymbol{y}_k$ contains the received signal for all N modems on tone k. Also, let $\mathcal{N}$ denote the set of modems that make use of the same cable bundle.

The transmit power and received noise power of modem n on tone k are given as $s_k^n = \Delta_f \mathcal{E}\{|x_k^n|^2\}$ and $\sigma_k^n = \Delta_f \mathcal{E}\{|z_k^n|^2\}$, where $\mathcal{E}\{\cdot\}$ denotes the expected value operator and Δ_f is the tone spacing. The total power consumption of modem n is given by

$$P^n = \sum_k s_k^n.$$

We will assume no signal coordination between modems. Interference from other modems will thus be treated as noise. When N is large, this interference can be well approximated by a Gaussian distribution. The achievable bit loading for modem n on tone k, given $\boldsymbol{s}_k = \left[s_k^1, s_k^2, \ldots, s_k^N\right]^T$, is then given as

$$b_k^n = \log_2\left(1 + \frac{1}{\Gamma}\frac{|h_k^{n,n}|^2 s_k^n}{\sum_{m\neq n}|h_k^{n,m}|^2 s_k^m + \sigma_k^n}\right). \quad (1)$$

where Γ is the signal-to-noise ratio (SNR) gap to capacity or SNR-Gap for short, which is a function of the coding gain, noise margin and average target BER. The relation between Γ and the BER for QAM and PAM can be found in [7].

A modem n has of a set of different applications $\mathcal{Q}^n$ associated with it. Each of these applications will be offered a different BER. Every tone of each modem will be allocated to one application $q \in \mathcal{Q}^n$. Conversely, each application q of modem n makes use of a set of tones $\mathcal{T}_q^n$. By using $\Gamma = \Gamma_q$ associated with application q in the calculation of the achievable bit loading on each tone $k \in \mathcal{T}_q^n$, a different level of error protection can be offered to each application. The data rate associated with application q of modem n is

$$R_q^n = f_s \sum_{k\in\mathcal{T}_q^n} b_k^n(q),$$

where f_s is the symbol rate and where q in $b_k^n(q)$ indicates that Γ_q should be used in the calculation of the achievable bit loading (1).

3. RATE-ADAPTIVE SPECTRUM MANAGEMENT WITH UNEQUAL ERROR PROTECTION

The problem of choosing the optimal transmit spectrum for each modem in order to maximize the data rate of the DSL network is referred to as the rate-adaptive spectrum management problem [8]. Usually, the objective is to maximize a weighted sum of the per modem data rates, subject to per modem total power constraints. However, here the objective will be to maximize a weighted sum of the per application data rates:

$$\begin{aligned} \max_{\boldsymbol{s},\mathcal{T}} \quad & \sum_{n=1}^{N}\sum_{q\in\mathcal{Q}^n} \omega_q^n R_q^n \\ \text{s.t.} \quad & P^n \leq P^{n,\text{tot}} && \forall n, \\ & 0 \leq s_k^n \leq s_k^{n,mask} && \forall n,k \end{aligned} \quad (2)$$

where $\boldsymbol{s} = \{\boldsymbol{s}_k | k \in \mathcal{K}\}$ and $\mathcal{T} = \left\{\mathcal{T}_q^n | n \in \mathcal{N}, q \in \mathcal{Q}^n\right\}$, $P^{n,\text{tot}}$ is the per modem power budget, and $s_k^{n,\text{mask}}$ denotes the spectral mask of modem n on tone k. The weights ω_q^n can be adjusted in order to satisfy additional rate constraints [9].

Note that this optimization problem has an additional decision variable $\mathcal{T}$ compared to the original rate-adaptive spectrum management problem, which signifies the allocation of tones to specific applications.

4. OPTIMAL SOLUTION

In [1] a dual decomposition algorithm, referred to as optimal spectrum balancing (OSB), has been proposed that, under the assumption that K is large, finds the optimal solution to the original rate-adaptive spectrum management problem. Here, this algorithm will be adapted to solve (2).

We first make an important observation. When transmit spectra are fixed, the choice of allocating a given tone of modem n to a particular application q does not affect the bit loading of any other modem. In other words, when $\boldsymbol{s}$ is fixed, choosing $\mathcal{T}$ is trivial: every tone is to be allocated to the application that brings the largest contribution towards the weighted rate sum. The maximization in (2) is equivalent to

$$\max_{\boldsymbol{s}}\left[f_s \sum_{n=1}^{N}\sum_{k=1}^{K} \max_{q\in\mathcal{Q}^n} \omega_q^n b_k^n(q)\right].$$

In addition to simplifying (2), this formulation of the problem brings something to the surface: if equal weights ω are chosen for all the applications of a modem, then all tones will be allocated to the application with the lowest Γ. This has to be considered when choosing ω. A possible solution is to let ω depend on the queue length of the associated application.

Now define the Lagrangian, which can be decomposed per tone [1], by incorporating the per modem power constraints into the optimization problem:

$$\begin{aligned} \mathcal{L}(\boldsymbol{\lambda},\boldsymbol{s}) &= \sum_{k=1}^{K}\mathcal{L}_k(\boldsymbol{\lambda},\boldsymbol{s}_k) + \sum_{n=1}^{N}\lambda_n P^{n,\text{tot}} \\ \text{with} \quad \mathcal{L}_k(\boldsymbol{\lambda},\boldsymbol{s}_k) &= f_s \sum_{n=1}^{N}\max_{q\in\mathcal{Q}^n}\omega_q^n b_k^n(q) - \sum_{n=1}^{N}\lambda_n s_k^n. \end{aligned} \quad (3)$$

The dual problem is then defined as

$$\min_{\boldsymbol{\lambda}} \quad g(\boldsymbol{\lambda}) \quad (4)$$

$$\text{with} \quad g(\boldsymbol{\lambda}) = \max_{\boldsymbol{s}} \mathcal{L}(\boldsymbol{\lambda},\boldsymbol{s}) \quad (5)$$

where the positivity and masking constraint of (2) have been suppressed to allow for more concise notation. The dual problem consists of a master problem (4) and a slave problem (5).

The slave problem, which has exponential complexity in NK, can be solved independently for each tone, dividing (5) into K sub problems with exponential complexity only in N. As in [1], it is assumed that all modems can control their PSDs with finite accuracy. The maximum of (3) can then be found for each tone by an exhaustive grid search over all possible power combinations.

Since the optimization problem is non-convex, standard optimization theory does not guarentee that the primal (2) and the dual problem (4) have the same solution, the difference between these solutions being the duality gap. However, when K is large, the time sharing property of [10] holds and we can safely assume that the duality gap is zero, and that (2) and (4) have the same solution.

The objective function of the master problem is convex but not differentiable. An efficient subgradient algorithm that solves the master problem is proposed in [9]. The updates of λ are performed as follows:

$$\lambda^{n,l+1} = \left[\lambda^{n,l} + \mu^l \left(\sum_{k=1}^{K} s_k^n - P^{n,\text{tot}}\right)\right]^+, \quad \forall n,$$

where l is the iteration number, μ^l is a scalar step size, and $[\cdot]^+ = \max(\cdot, 0)$. The subgradient method is guaranteed to converge to the optimal $\boldsymbol{\lambda}$ as long as μ^l is sufficiently small [9].

The resulting algorithm is summarized in Algorithm 1, and is referred to as OSB with unequal error protection (OSB-UEP). A similar algorithm was derived in [6], which however has much higher complexity due to an exhaustive search over all bit loading combinations for every possible combination of applications. This leads to polynomial complexity in the number of applications where the complexity of OSB-UEP is only linear in the number of applications.

Algorithm 1 OSB with unequal error protection

while distance $>$ tolerance **do** ▷ Master Problem
  $\mu \leftarrow 1$
  $\lambda \leftarrow$ best λ so far
  $\Delta\lambda \leftarrow \left(\boldsymbol{P}^{\lambda} - \boldsymbol{P}^{\text{tot}}\right)$
  while distance $\leq$ previousDistance **do**
    previousDistance $\leftarrow$ distance
    $\mu \leftarrow \mu \times 2$
    $\left[\boldsymbol{P}^{\lambda+\mu\Delta\lambda}, \boldsymbol{s}\right] \leftarrow$ EXHAUSTIVESEARCH$(\lambda + \mu\Delta\lambda)$
    distance $\leftarrow \left\|\boldsymbol{P}^{\text{tot}} - \boldsymbol{P}^{\lambda+\mu\Delta\lambda}\right\|$
  end while
end while
function EXHAUSTIVESEARCH(λ) ▷ Slave Problem
  for all $k \in \mathcal{K}$ **do**
    $\boldsymbol{s}_k \leftarrow \arg\max_{\boldsymbol{s}_k} \mathcal{L}_k(\lambda, \boldsymbol{s}_k)$
  end for
  $\boldsymbol{P}^{\lambda} \leftarrow \sum_{k\in\mathcal{K}} \boldsymbol{s}_k$
end function

5. DISTRIBUTED SOLUTION

Algorithm 1 works well when the number of modems is small, say $N = 1, 2$, but becomes intractable for more modems because of the exhaustive search with exponential complexity in N. Also, it relies on a spectrum management center (SMC) to optimize all transmit spectra simultaneously. This section contains the derivation of a generalization of the distributed, low complexity, so-called Distributed Spectrum Balancing (DSB) algorithm [2] for rate-adaptive spectrum management with unequal error protection.

Each modem n will locally solve a relaxed version of (2) in order to decide on its own transmit spectrum and tone allocation. This is then repeated in an iterative fashion. Two approximations are made to allow for a simple, distributed solution. It will be assumed that all other modems do not change their transmit spectrum and tone allocation, and the data rate of other modems will be approximated in a point $\boldsymbol{s}'$ with a lower bound hyperplane. Each modem then locally solves the following problem:

$$\begin{aligned}
\max_{\boldsymbol{s}^n} \quad & f_s \sum_{k\in\mathcal{K}} \max_{q\in\mathcal{Q}^n} \omega_q^n b_k^n(q) \\
& + f_s \sum_{m\neq n} \sum_{q\in\mathcal{Q}^m} \omega_q^m \sum_{k\in\mathcal{T}_q^m} s_k^n \frac{\partial b_k^m}{\partial s_k^n}\Big|_{\boldsymbol{s}_k=\boldsymbol{s}_k'} + c \\
\text{s.t.} \quad & P^n \leq P^{n,\text{tot}} \\
& 0 \leq s_k^n \leq s_k^{n,\text{mask}}, \qquad \forall k
\end{aligned} \tag{6}$$

where c is chosen such that the approximation is exact in $\boldsymbol{s}'$. Solving this problem for all modems and using the solution as a new approximation point produces a monotonically increasing objective function value which can be shown to converge to a local optimum of (2).

As before, the total power constraint of (6) can be incorporated into the objective function by defining the Lagrangian, which can again be decomposed per tone.

$$\begin{aligned}
\mathcal{L}(\boldsymbol{s}^n, \lambda^n) &= \sum_{k\in\mathcal{K}} \mathcal{L}_k(s_k^n, \lambda^n) + \lambda^n P^{n,\text{tot}} + c \\
\mathcal{L}_k(s_k^n, \lambda^n) &= s_k^n \Big(f_s \sum_{m\neq n} \omega_{q_k^m}^m \frac{\partial b_k^m}{\partial s_k^n}\Big|_{\boldsymbol{s}_k'} - \lambda^n\Big) \\
&\quad + f_s \max_{q\in\mathcal{Q}^n} \omega_{q_k^n}^n b_k^n(q)
\end{aligned} \tag{7}$$

The dual problem is then defined as

$$\min_{\lambda^n} \quad g(\lambda^n) \tag{8}$$

$$\text{with} \quad g(\lambda^n) = \max_{\boldsymbol{s}^n} \mathcal{L}(\boldsymbol{s}^n, \lambda^n). \tag{9}$$

The objective function of (6) is not concave, but the same case as in section 4 can be made for the duality gap to be zero.

The slave problem (9) can be solved independently for each tone by maximizing the per tone Lagrangian (7). This maximization problem is non-convex due to non-smoothness of the objective function. It can however be transformed into an equivalent problem which is easier to solve:

$$\max_{q \in \mathcal{Q}^n} h(q) \tag{10}$$

$$h(q) = \max_{s_k^n} \left(s_k^n \Big(f_s \sum_{m \neq n} \omega_{q_k^m}^m \frac{\partial b_k^m}{\partial s_k^n}\Big|_{\boldsymbol{s}_{k'}} - \lambda^n \Big) + f_s \omega_q^n b_k^n(q) \right). \tag{11}$$

Optimization problem (11) will thus be solved for every $q \in \mathcal{Q}^n$, before selecting the q that delivers the largest maximum objective function value.

Due to the concavity of the objective function of (11), the Karush-Kuhn-Tucker (KKT) conditions provide a sufficient condition for global optimality. By solving the system of KKT conditions, we obtain a solution to (11) which is almost identical to the DSB update formula [2].

$$s_k^n = \left[\omega_q^n \frac{f_s / \log(2)}{\lambda^n - f_s \sum_{m \neq n} \omega_{q_k^m}^m \frac{\partial b_k^m}{\partial s_k^n}\Big|_{\boldsymbol{s}_{k'}}} - \Gamma_q \frac{\sum_{m \neq n} |h_k^{n,m}|^2 s_k^m + \sigma_k^n}{|h_k^{n,n}|^2} \right]_0^{s_k^{n,\text{mask}}} \tag{12}$$

The objective function of the master problem (8) is convex and not differentiable. Since it has only one variable λ^n, we will employ a bisection search to find its optimal value.

When problem (6) has been solved in every modem n, a new iteration starts and a new approximation of (2) is solved. Setting up the new problem at every modem requires some communication. As can be seen in (12), all required information from other modems is contained within the derivative term. These terms will be calculated at the SMC as

$$\frac{\partial b_k^m}{\partial s_k^n}\Big|_{\boldsymbol{s}_{k'}} = \frac{\Gamma_{q_k^m} |h_k^{m,n}|^2}{\log(2)} \left(\frac{1}{\text{int}_k^{m,n}} - \frac{1}{\text{rec}_k^{m,n}} \right) \tag{13}$$

$$\text{where} \quad \text{int}_k^{m,n} = \Gamma_{q_k^m} \left(\sum_{p \neq m} |h_k^{m,p}|^2 s_k^p + \sigma_k^m \right)$$

$$\text{rec}_k^{m,n} = |h_k^{m,m}|^2 s_k^m + \text{int}_k^{m,n}.$$

For this calculation the SMC has to receive $\mathcal{T}_q^n$, $\forall q \in \mathcal{Q}^n$ and $|h_k^{n,n}|^2 s_k^n$ and $\sum_{m \neq n} |h_k^{n,m}|^2 s_k^m + \sigma_k^n$ from every modem.

The resulting algorithm can be found in Algorithm 2 and is referred to as DSB with unequal error protection (DSB-UEP). As was the case for OSB-UEP, the complexity of DSB-UEP is linear in the number of applications.

Algorithm 2 DSB with unequal error protection

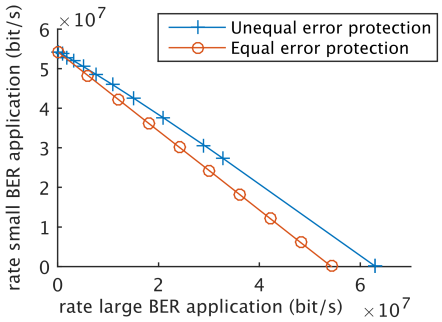
Initialize $\boldsymbol{s}^n$ ▷ Modem n algorithm
repeat
  $\lambda_{\min}^n \leftarrow 0$, $\lambda_{\max}^n \leftarrow \Lambda_{\max}$
  Receive $\sum_{m \neq n} \omega_{q_k^m}^m \frac{\partial b_k^m}{\partial s_k^n}$ from SMC, $\forall k$.
  while $|\sum_k s_k^n - P^{n,\text{tot}}| > \delta$ and $\lambda^n > \gamma$ **do**
    $\lambda^n \leftarrow (\lambda_{\min}^n + \lambda_{\max}^n)/2$
    for all k **do**
      Calculate (s_k^n, q) using (12), $\forall q \in \mathcal{Q}^n$.
      Select (s_k^n, q) that maximizes $h(q)$ in (10).
    end for
    if $\sum_k s_k^n > P^{n,\text{tot}}$ **then**
      $\lambda_{\min}^n \leftarrow \lambda^n$
    else
      $\lambda_{\max}^n \leftarrow \lambda^n$
    end if
  end while
  Measure $\sum_{m \neq n} |h_k^{n,m}|^2 s_k^m + \sigma_k^n$ and $|h_k^{n,n}|^2 s_k^n$, $\forall k$.
  Send these values to SMC, along with $\mathcal{T}_q^n$, $\forall q \in \mathcal{Q}^n$.
until forever
repeat ▷ SMC algorithm
  Receive values from all modems.
  for all $n \in \mathcal{N}$ **do**
    Compute $\sum_{m \neq n} \omega_{q_k^m}^m \frac{\partial b_k^m}{\partial s_k^n}$, $\forall k$, using (13).
    Send these values to modem n.
  end for
until forever

6. SIMULATION RESULTS

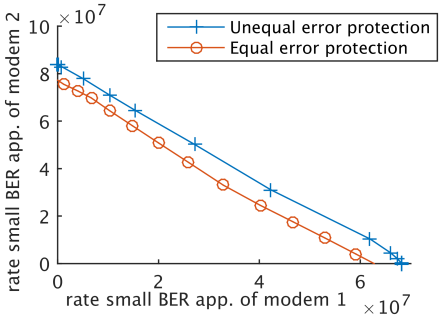
In this section, VDSL simulation results are shown for Algorithms 1 and 2. The following parameter settings are assumed. The twisted pair lines have diameter of 0.5 mm (24AWG). The maximum transimit power is 11.5 dBm. The tone spacing Δ_f is 4.3125 kHz and the DMT symbol rate f_s is 4 kHz. The noise margin and coding gain are respectively set to 6 dB and 3 dB. Modems support two levels of error protection. The high level and low level respectively support an average target BER of 10^{-7} and 10^{-3}. This corresponds to an SNR gap of $\Gamma_{10^{-7}} = 12.8\,\text{dB}$ and $\Gamma_{10^{-3}} = 8.6\,\text{dB}$. Modems that do not support unequal error protection will apply the high level to all applications. All simulations are performed in Matlab.

In Figure 1(a), a single modem rate region is shown. In Figure 1(b), a rate region is drawn for the applications with a high level of protection of two modems, where it is assumed that the applications with a low level of error protection have to achieve a target data rate of 2.10^7 bit/s. This last rate region is generated for a near-far scenario, which is known to exhibit poor performance. Both figures contain the rate region of a VDSL system with and without unequal error protection. Figure 1 clearly shows that the rate region is significantly expanded by applying unequal error protection. The amount by which the complexity of the algorithms grows, compared to algorithms that consider equal error protection, is only linear in the number of error protection levels per modem. This is a small cost for the achieved performance gains.

Figure 2 shows the transmit spectra and bit loading for a 4



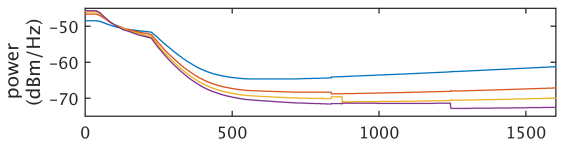
(a) one modem



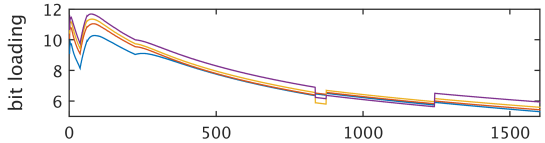
(b) two modems

Fig. 1: Comparison of DSB and DSB-UEP rate region.

user scenario. For all modems, the weights of (2) are chosen as $\omega_{10^{-7}} = 1$ and $\omega_{10^{-3}} = 0.8$. Some discontinuities can be observed on both curves, which originate from a change in application assignment. This figure shows how the gain in performance is obtained. First of all, a low level of error protection allows for higher bit loading. The increase in bit loading would also be present if the transmit spectrum would be calculated for the high level of error protection case, followed by reassigning some tones to the application with a low level of error protection. Secondly, less power is used on tones with a low level of error protection. This decrease in power leads to lower crosstalk and gives the opportunity to use the power elsewhere. Both effects increase the performance.



(a) transmit power spectrum



(b) bit loading

Fig. 2: DSB-UEP solution for scenario with four modems.

7. CONCLUSION

In this paper, we have presented two rate-adaptive spectrum management algorithms that enable a different level of error protection for different applications. The OSB-UEP and DSB-UEP algorithm are generalizations of the OSB and DSB algorithm. Through simulation it has been shown that unequal error protection can lead to significant performance gains.

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