Flood control of rivers with Model Predictive Control
– proof of concept based on the river Demer in Belgium –

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Abstract— **This paper shows that Model Predictive Control (MPC) can more effectively be used for flood prevention in comparison with a three position controller. Because MPC takes rain predictions into account, it uses the buffer capacity of the available flood basins in a more optimal way. Simulation results for historical data for the Demer, a river in Belgium with a history of large floodings, show a significant reduction of the number and the impact of floodings when MPC is used. Because the river models behave strongly nonlinear, it is necessary to use a nonlinear model predictive controller. Furthermore, as rainfall predictions tend to be less predictable over longer periods, it is necessary to extend MPC to Multiple MPC in order to cover this uncertainty.**

I. Introduction

To prevent the Demer, a river in Belgium, from flooding, the storage capacity of the river was expanded with flood basins and hydraulic gates were installed at the end of the sixties. These gates are controlled by an advanced three position controller. However, these constructions could not prevent the Demer from flooding in 1998 and 2002.

This paper shows that the number and the impact of these floodings could have been reduced when a Model Predictive Controller (MPC) has been used. Because MPC takes constraints on the inputs and states and the effect of disturbances (e.g. heavy rainfall) explicitly into account, it uses the flood basins in a more optimal way.

So far, only Matlab has been used for MPC implementations for flood control at the K.U.Leuven [1], [2], [3]. However an implementation of this technique should as best practice be based on industrial software. This paper shows that the industrial INCA Software of the company IPCOS[1] is suited for this task. The main reason to use this software instead of own developed software is twofold:

1) INCA is already successfully implemented in MPC applications in the glass industry and the petro chemistry. If we achieve positive results in this case, the effort to build an MPC flood control system will decrease significantly.
2) By avoiding to start from scratch, applying MPC in the domain of flood control, gets full focus right from the start of the project. Other topics like visualization of the simulation, handling of signal drop outs, etc., are already available as well.

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A second extension is the reduction of the influence of uncertainty on the rainfall predictions on the performance of the implemented MPC controllers. In order to cover this uncertainty, MPC is extended to Multiple MPC (MMPC).

The paper is organized as follows. Section II describes the current controller for the Demer and its disadvantages. Section III elaborates how MPC can be used for flood control while Section IV explains how nonlinearities of the river models can be tackled. Section VI explains how robustness can be achieved with respect to uncertainty on the rainfall predictions while Section VII shows results for the controllers discussed in the previous sections.

II. Current control

The current control of the Demer is done by an advanced three position controller, in combination with manual overruling by operators. A standard three position controller is often used to maintain water levels as close as possible to their set points. It changes the heights of the hydraulic structures based on some basic rules [4]:

1) Do not change the position of the gate as long as the water level is within a band around the reference value;
2) Let the gate go down in case the water level is above the upper boundary;
3) Let the gate go up in case the water level is below the lower boundary.

The controller used by the local water authority is more advanced. During periods of no or limited rainfall the main objective is water control. They use the standard three position controller to steer the water levels to their set points. However, during periods of heavy rainfall the focus shifts towards flood prevention and these simple rules are overruled by some advanced rules based on many years of experience in regulating the Demer. The translation of this expertise can best be compared with an if-then logic. These rules determine when and to which upper limit the basins can be filled to prevent flooding. The main disadvantage of the controller is that it does not take the rain forecasts into account as the rules are only based on the current water levels. Therefore the control actions are often suboptimal which forces the operator to overrule. This introduces subjectivity and is dependent of the experience of the operator.

III. Model Predictive Control and Flood Control

This paper investigates whether MPC could outperform the current controller for flood control of the Demer. MPC is a control strategy originating from the process industry

and is nowadays used in various applications going from chemicals and food processing to automotive and aerospace applications [5], [6], [7]. MPC makes use of a process model to predict the future process outputs within a specified prediction horizon. MPC solves an optimization problem over this horizon to determine the optimal inputs for the process taking into account input and output constraints, future disturbances and the process model. Only the first input sample of the complete optimal sequence is applied to the process, new samples are taken and the entire procedure is repeated.

A. Previous work

Several studies can be found in the literature where MPC is used to control water systems [8], [9]. However, these studies are focused on steering the water levels to their set points and not on flood prevention. This simplifies the use of MPC as the nonlinear river dynamics can be approximated sufficiently accurately by a linear model. For flood control MPC can not ignore the nonlinear behaviour as heavy rainfall excites the entire nonlinear dynamics. In [10] MPC is used for flood control but this study does not take the nonlinearity of the gates into account. This is again an oversimplification of the problem because especially the gates are responsible for the nonlinearities.

B. The river models

Because MPC uses a process model, the first task for applying MPC is to find an appropriate model. The models used for the Demer are built, calibrated and validated by the Department of Civil Engineering of the K.U.Leuven. These models are of the reservoir type and are based on the principles described in [11]. Fig. 1 shows the structure of the model for the study scope of the Demer located close to the place where the Mangelbeek pours into the Demer (more information can be found in [12]). This model contains two basins: Schulensmeer and Webbekom. A smaller model is derived for the area around Schulensmeer in the red circle and is visualized in Fig. 2. The hollow rectangles are the controllable hydraulic structures, the full rectangles are fixed spills or overflows. The lines are river reaches with positive flow in the direction of the arrows. The nodes are places where the model simulates the local water levels and water volumes. The symbol k is used for the controllable gates, h for the water levels, v for the water volumes and q for the discharges.

Both models are nonlinear state space models of the form:

$$\begin{cases} \boldsymbol{x}_{k+1} &= \boldsymbol{s}(\boldsymbol{x}_k, \boldsymbol{u}_k, \boldsymbol{d}_k) \\ \boldsymbol{y}_k &= C\boldsymbol{x}_k \end{cases} \tag{1}$$

with $\boldsymbol{x}_k$ the state vector at time k, $\boldsymbol{u}_k$ the input vector at time k, $\boldsymbol{d}_k$ the disturbance vector at time k and $\boldsymbol{y}_k$ the output vector at time k. The state $\boldsymbol{x}$ consists of the water levels, the water volumes and the discharges, the input $\boldsymbol{u}$ of the gates, the disturbances $\boldsymbol{d}$ of the upstream discharges and the output $\boldsymbol{y}$ of the water levels. Details of the state space models can be found in [13]. There one can find the complete description

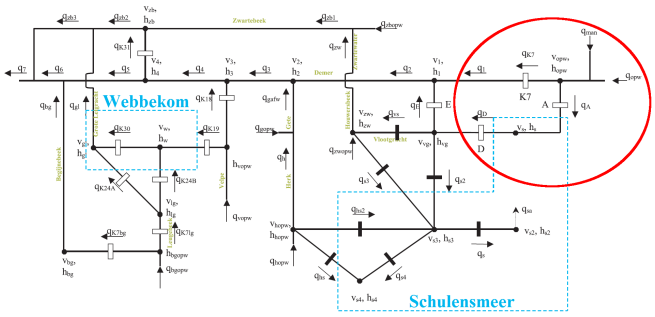

Fig. 1. Model structure of the Demer for the study area with the basins Webbekom and Schulensmeer.

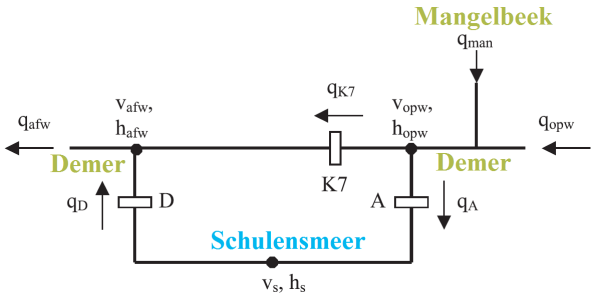

Fig. 2. Model structure of the Demer for the area around Schulensmeer.

of states, inputs and outputs as related to the variables in Fig. 1 and 2.

One difficulty in using MPC for flood control lies in the nonlinear dynamics of the river models. The relation between the discharge over a gate and the height of the gate and the water surrounding the gate is strongly nonlinear. Fig. 3 shows the discharge q over a gate in function of the height of the gate k and the upstream water level h while the downstream water level remains constant. This leads to the conclusion that the nonlinear models cannot be accurately approximated by one linear model over the range of operation. For flood control it is very important that the controller can deal with the nonlinearity of the river models. This brings us to the area of nonlinear model predictive control (NMPC).

IV. Nonlinear Model Predictive Control

A. The influence of nonlinear models on the optimization problem

As explained in the previous Section the models of the Demer are strongly nonlinear. These nonlinearities have a negative influence on the optimization problem that MPC will solve at every time step. When a linear model can be used, the resulting optimization problem is often a quadratic problem (QP) where the model equations are substituted in the objective function or are present as equality constraints. If the Hessian of the quadratic objective function is positive definite, then the corresponding QP is strictly convex and has a unique solution. This solution can efficiently be found with algorithms like active set methods. However, when the model is nonlinear the corresponding optimization problem will not have linear equality constraints: a nonlinear optimization

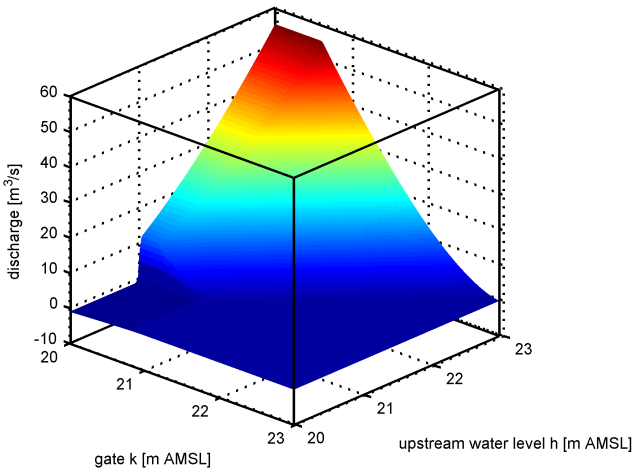

Fig. 3. The nonlinear relation between the discharge q over a gate and the height of a gate k and the upstream water level h while the downstream water level remains constant. The heights are above mean sea level (AMSL).

problem has to be solved (NLP). These are much harder to solve and can have multiple local optima. A solution is described in the next Section.

B. Online linearization

Many algorithms can be found in the literature that process nonlinear models [5]. The algorithm used in this paper is called *online linearization* and can be summarized as follows:

1) Predict the future states within the prediction horizon with the nonlinear model for the optimal inputs found in the previous time step.
2) Derive for every time sample in the prediction horizon a linear state space model based on the future states at that time sample. The linearization used in this paper is implemented with forward differences.
3) Replacing the nonlinear model with these time variant linear models changes the NLP back into a QP. The solution of this QP is a new sequence of optimal inputs.
4) The solution found in step 3 will only be an approximation of the solution of the original NLP because the linear models are only an approximation of the nonlinear model. Therefore, the previous steps have to be repeated until convergence. In every iteration the optimal inputs in step 1 have to be replaced with the solution of the QP in step 3. When the algorithm terminates the first input is applied to the system and the entire strategy is repeated.

It has been proved that this algorithm delivers a local optimum of the original NLP [14].

C. Necessary adaptations for the INCA Software

The INCA Software can handle nonlinear models for only a limited number of applications. For other applications, like flood control, INCA can only work with a linear model. This is however not sufficient for flood control because of the strong nonlinearities of the river models. In order to be able to handle these nonlinearities, the software was expanded with the online linearization algorithm by making a connection between Matlab and INCA. With this connection it is possible to perform step 1 and 2 of the algorithm. Step 3 is performed by the standard solver of INCA. Only the last step could not be implemented: the software can not be adjusted in a straightforward way to solve the QP multiple times for the same time step. However as Section VII shows, good results are achieved even without this last adaptation.

V. Control Objectives and Constraints

A. In general

The control objectives and constraints are situation dependent and are determined by the local water administration. During periods of **no or light rainfall** the controller has to steer the most important water levels to their set points. It is also important to empty the basins and keep them as empty as possible in order to maximize the buffer capacity. This can be achieved by emphasizing the deviations of these water levels and the basins in the objective function of the QP using appropriate weights. The flooding boundaries are not important during these periods as the rainfall is too small to cause flooding.

During periods of **heavy rainfall** the focus shifts from water control towards flood prevention. Because these rainfalls could last more than five days the prediction horizon should have at least the same length to prevent flooding in an optimal way. However this is not possible:

- The rain predictions are only accurate for the first two days.
- The number of optimization variables would be too large to solve the problem in an acceptable amount of time.

Therefore it is necessary for the MPC controller to follow some expert rules:

1) Every water level has a *guard level*. The flood basins should remain empty as long as every water level remains beneath this guard level. To achieve this the guard levels are added as constraints to the optimization problem and a high cost is placed in the objective function on the deviations of the basins from their set points.
2) The basins may be filled when the water levels violate their guard levels over the prediction horizon. However it is not allowed to use the entire buffer capacity. Every basin may only be filled as far as its *safety limit*. After reaching this limit, it is possible that the water levels will violate their guard levels. To allow the basins to be filled, the deviations from their set points are not important any more in the objective function. Also the safety limits are added as constraints and the guard levels are replaced with the flood levels.
3) When the *flood levels* will be violated the last remaining buffer capacity of the basins can be used to prevent flooding. This is achieved by substituting the safety limits with the flood limits of the basins.

4) When it is not possible to prevent a water level from flooding, the corresponding constraint should be removed from the QP. In order to limit the height of the flooding, the weight of this water level in the objective function is increased.

Beside these constraints on the water levels, there are also constraints on the movable gates. Every gate has an upper and a lower limit and can only move over a maximum distance every hour (velocity limit).

B. The area around the basin Schulensmeer

The rules for the area around the basin Schulensmeer are:

- The set point for the upstream water level h_{opw} is 21.5 m. The basin level h_{s} should remain as close as possible to 20.4 m.
- The upper and lower limits for the gates are:
  - $20\,\mathrm{m} \leq k_{\mathrm{A}} \leq 23\,\mathrm{m}$,
  - $20.03\,\mathrm{m} \leq k_{\mathrm{K7}} \leq 23\,\mathrm{m}$
  - and $18.9\,\mathrm{m} \leq k_{\mathrm{D}} \leq 22.9\,\mathrm{m}$.
- The gates are constrained to move not faster than 0.1 m/hour.
- The guard levels for h_{opw} and h_{afw} are 23 m, resp. 22.55 m. The safety limit of h_{s} is 23 m.
- The flood levels are:
  - $h_{\mathrm{opw}} \leq 23.2$ m,
  - $h_{\mathrm{s}} \leq 23.2$ m
  - and $h_{\mathrm{afw}} \leq 22.75$ m.

All the mentioned heights are above mean sea level (AMSL).

C. The area containing the basins Schulensmeer and Webbekom

The rules for the model containing the basins Schulensmeer and Webbekom are here only quantified for the most important water levels:

- The set points for the water levels h_{opw} and hbg_{opw} are 21.5 m and 23.8 m.
- The gates are not allowed to move faster than 0.1 m/hour.
- Every gate has an upper and a lower limit.
- The safety limits for the basins Schulensmeer (h_{s}) and Webbekom (h_{w}) are 23 m and 22 m. Their set points are 20.4 m and 20 m.
- The guard level for h_{opw} is 23 m.
- The flood levels are:
  - $h_{\mathrm{opw}} \leq 23.2$ m,
  - $h_{\mathrm{s}} \leq 23.2$ m,
  - $h_2 \leq 22.737$ m,
  - $hbg_{\mathrm{opw}} \leq 24.84$ m,
  - $h_{\mathrm{w}} \leq 22.4$ m,
  - $h_{\mathrm{gl}} \leq 22$ m,
  - and $h_{\mathrm{afw}} \leq 20.46$ m.

VI. Robustness against uncertainty on rainfall predictions with Multiple MPC

The first MPC implementations (discussed in the next Section) assumed there is no uncertainty on the rainfall predictions. However this is not realistic as the rainfall cannot be predicted with 100% certainty and the uncertainty on the predictions increase with the length of the prediction horizon. The uncertainty can be seen as perturbations superposed on the real rainfall data. These perturbations cannot be modelled as white noise as the uncertainty for time sample i is not independent of the previous time sample: if the prediction for an upstream discharge is too large for time sample $i-1$, the prediction for time sample i is probably also too large. This can be modelled with β-distributions where the position of the (finite) maximum for time sample i coincides with the perturbation for time sample $i-1$. For details, we refer to [13] once again.

One way to deal with this uncertainty is to replace the MPC controller with a Multiple MPC (MMPC) controller [15]. This controller works with three sequences of disturbances:

- **nominal:** This is the sequence of the predicted disturbances over the prediction horizon. The lower the uncertainty the closer the sequence will be to the real data.
- **maximal:** This sequence is an upwards shifted version of the nominal sequence and is in most of the cases an overestimation of the real data.
- **minimal:** This sequence is a downwards shifted version of the nominal sequence and is in most of the cases an underestimation of the real data.

These three sequences are included in the online linearization algorithm:

1) Predict for each disturbance sequence the future states within the prediction horizon with the nonlinear model for the optimal inputs found in the previous time step. This step delivers three sequences of states.
2) Derive for every sequence of states a linear state space model for every time sample in the prediction horizon.
3) At the beginning of this step there are three sequences of disturbances, states and linear models. As there are three sequences of models, there are also three sequences of outputs (water levels). The QP now searches for one input sequence that brings these three sequences as close as possible to their reference values.
4) The previous steps have to be repeated until convergence. When the algorithm terminates the first input is applied to the system and the entire procedure is repeated.

Because step 3 looks for the input sequence that is most optimal for the three output sequences, the controller is protected against uncertainties on the disturbances.

VII. Results

A. The area with Schulensmeer and Webbekom

Before implementing the controllers, the following assumptions were made:

- There is no plant-model mismatch: the model used to simulate the Demer and used by the MPC controller are the same.

- Every state is known at every time sample.
- There is no uncertainty on the rain predictions.

After a successful implementation for the model around Schulensmeer a MPC controller is implemented for the large-scale model. Fig. 5 shows the simulation results for this MPC controller and these should be compared with the results for the current controller (Fig. 4):

- During the first 250 hours MPC steers h_{opw} much closer to its set point of 21.5 m than the current controller. Both controllers keep the basin Webbekom (h_{w}) close to its reference value. This is not the case for Schulensmeer (h_{s}) where only MPC can steer it to 20.4 m. Only the results for hbg_{opw} are a bit better for the current controller.
- The current controller is not capable to prevent many water levels from flooding during the heavy rainfall during the next 150 hours. The results are much better for the MPC controller as it uses the basins and the available information in an optimal way. This can also be concluded from Table I: MPC clearly outperforms the current controller.
- After the heavy rainfall MPC succeeds in completely emptying both basins in a fast way. The current controller can only empty Webbekom completely. MPC also steers h_{opw} much closer to its set point. Only the results for hbg_{opw} are less good.

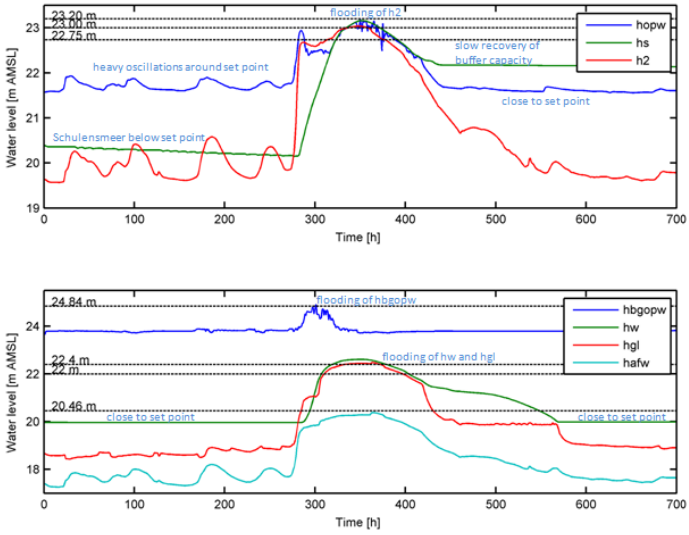

Fig. 4. Control of the model with the basins Schulensmeer (h_{s}) and Webbekom (h_{w}) with the current controller for the historical flood of September 1998.
The controller can not prevent h_2, hbg_{opw}, h_{w} and h_{gl} from flooding. The controller is also not capable to keep h_{opw} close to its set point during the first 250 hours and empties the basin Schulensmeer (h_{s}) very slowly.

B. Robustness against uncertainty on rainfall predictions for the area around Schulensmeer

The previous simulations assumed that there is no uncertainty on the rainfall predictions. This Subsection discusses the simulation results for MMPC for the model around Schulensmeer where the rainfall forecasts are an underestimation

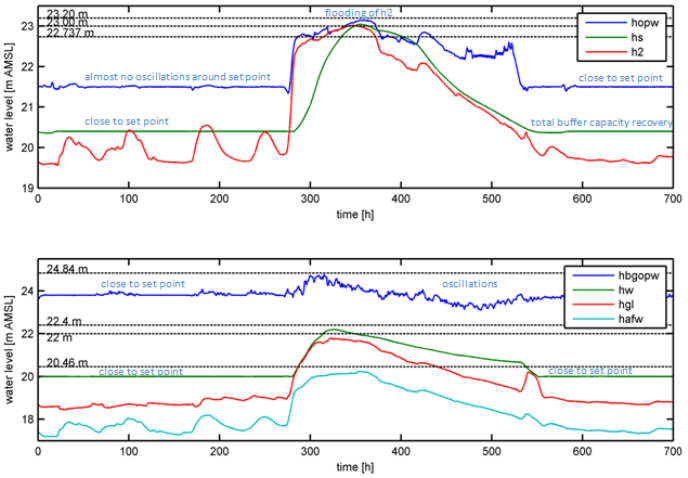

Fig. 5. Control of the model with the basins Schulensmeer (h_{s}) and Webbekom (h_{w}) with MPC for the historical flood of September 1998.
The controller prevent all the important water levels from flooding except for h_2. The controller recovers the full buffer capacity of both basins h_{s} and h_{w} in a very short time compared to the results for the current controller in Fig. 4. MPC also succeeds in keeping h_{opw} very close to its set point before and after the heavy rainfall. Only the results for hbg_{opw} after the heavy rainfall are less good.

TABLE I
COMPARISON OF THE MAXIMAL HEIGHT AND CORRESPONDING FLOOD MARGIN FOR THE MOST IMPORTANT WATER LEVELS BETWEEN THE MPC CONTROLLER AND THE CURRENT CONTROLLER.

A negative margin corresponds with the height of the flooding. Besides the most important water levels mentioned in Section V-C the Table contains one extra water level (h_{vg}) because both controllers can not prevent it from flooding. All the other water levels remain below their flood level.

water level	MPC controller		current controller	
	max. height (m AMSL)	margin (cm)	max. height (m AMSL)	margin (cm)
h_{opw}	23.15	5.4	23.18	1.6
h_{s}	23.04	15.8	23.16	4.3
h_2	23.02	-28.1	23.04	-30.1
hbg_{opw}	24.77	7.4	24.91	-7.2
h_{w}	22.20	20.5	22.61	-20.7
h_{gl}	21.78	22.4	22.48	-48.0
h_{afw}	20.24	22.4	20.38	8.0
h_{vg}	23.03	-50.6	23.10	-58.4

of the historical data of the flood of September 1998. The uncertainty over the prediction horizon of 48 hours lies between -15 % and 2 % for the first 30 hours and between -20 % and 3 % for the remaining 18 hours. Fig. 6 shows the results for the current controller and the MMPC controller:

- The MMPC controller steers h_{opw} much closer to the set point of 21.5 m than the current controller for the first 250 hours. MMPC also keeps the basin h_{s} empty.
- During the heavy rainfall the current controller can not prevent h_{afw} from flooding with 40 cm. Also h_{opw} and h_{s} flooded with 1 cm. The situation is much better with the MMPC controller. It limits the flooding for h_{afw} to only 5 cm. h_{opw} and h_{s} have a flood margin of 13 cm.
- The current controller is not able to empty the basin h_{s}. This is not the case for MMPC which empties the basin

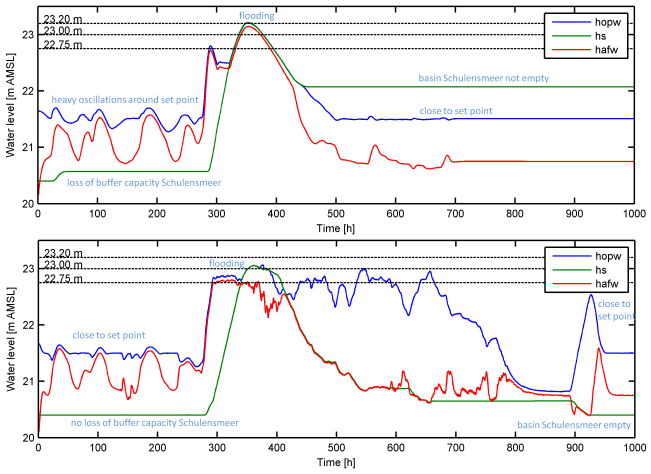

Fig. 6. Control of the reduced area around the basin Schulensmeer (h_{s}) with the current controller (top) and with MMPC (bottom) when the rainfall predictions are an underestimation of the historical flood of September 1998. *During the heavy rainfall the current controller can not prevent any water level from flooding. With MMPC, only h_{afw} floods (with only 5 cm). The current controller can not prevent to lose a small amount of buffer capacity (h_{s}) at the beginning of the simulation and can not completely restore it after the heavy rainfall. This is clearly not the case for MMPC. MMPC also keeps the water level h_{opw} much closer to its set point.*

completely and steers h_{opw} back to its set point.

VIII. Conclusion and Future Work

A. Conclusions

The simulation results for the historical flood event of 1998 clearly show that MPC outperforms the current control structure. Because MPC uses the available buffer capacity of the basins in a more optimal way, it reduces the number and the height of the floodings significantly.

B. Future Works

Before the technique can be applied in practice, future research is necessary:

- The model used for the simulations of the Demer and used by MPC are exactly the same. There is however always a plant-model mismatch as a mathematical model approximates only reality: the model will always contain model errors and uncertainties. A next step is to replace the simulation model by a more accurate one.
- Every simulation assumed that every state is known. In practice however only the most important water levels are measured and the discharges not. Therefore it is necessary to incorporate a state estimator in the control scheme. One possible state estimator is a moving horizon estimator (MHE) [16]. It estimates the states by solving an optimization problem based on the past measurements. Because of the nonlinear dynamics it will be necessary to use a nonlinear moving horizon estimator (NMHE) [17].
- Implement a MMPC controller for the model with the basins Schulensmeer and Webbekom. The challenge will be to limit the impact of the increased number of variables on the computation time.

IX. Acknowledgements

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