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Author contact	niccolo.antonello@esat.kuleuven.be *+ 32 (0)16 321855*
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IDENTIFICATION OF SURFACE ACOUSTIC IMPEDANCES IN A REVERBERANT ROOM USING THE FDTD METHOD

Niccoló Antonello [†], *Toon van Waterschoot*[†*], *Marc Moonen*[†] *and Patrick A. Naylor*[§]

[†]KU Leuven, Dept. of Electrical Engineering (ESAT), STADIUS Center for Dynamical Systems, Signal Processing and Data Analytics, Kasteelpark Arenberg 10, 3001 Leuven, Belgium

[*]KU Leuven, Dept. of Electrical Engineering (ESAT), ETC, AdvISe Lab, Kleinhoefstraat, 2440 Geel, Belgium

[§]Imperial College London, Dept. of Electrical Engineering, SW7 2AZ, London, United Kingdom

ABSTRACT

In numerical room acoustic simulations the acoustic impedance of walls and boundaries is of fundamental importance for the correct physical and perceptual synthesis of a sound field. Acoustic impedances are usually measured in reverberation chambers as absorption coefficients. In this type of measurements the sound field is assumed to be diffuse, which is an ideal condition usually not met in real rooms. This paper proposes a different approach to obtain the acoustic impedances, where a numerical model of the room is tuned with measurements using nonlinear optimization. The finite difference time domain (FDTD) method is used to simulate a sound field of a room and an optimization problem is solved to recover the acoustic impedances per frequency octave band. Results show that the use of a time-domain numerical method is advantageous as compared to a frequency-domain method, by making the measurement set-up more practical and improving the optimization convergence.

Index Terms— Acoustic impedance, *in-situ* measurement, inverse acoustics, room acoustics, finite difference time domain (FDTD), nonlinear optimization.

1. INTRODUCTION

Recently, numerical wave-based simulations for room acoustics have become possible due to increased available computational power. In particular, the finite difference time domain (FDTD) method is becoming increasingly popular due to its improved boundary model formulation [1] and the possibility to perform FDTD simulations with parallel computing [2, 3]. An identification method for acoustic impedances is also sought for the employment of recent algorithms for source localization and estimation in reverberant environments, where boundary conditions are assumed to be known [4, 5].

Databases of measured acoustic impedances exist in terms of absorption coefficients and are widely employed in geometric acoustic simulations [6]. However, the acoustic impedance is usually complex-valued while the absorption coefficient is real-valued and does not contain any phase information. Moreover, the measurement method to obtain these data assumes that the sound field is *diffuse*, i.e., corresponds to an ideal sound field where uncorrelated plane sound waves are arriving from all possible directions at every position in the room, a condition that is usually not met in real rooms. The measurement method also requires the usage of a reverberation chamber, a large room filled with acoustic diffusers [7]. Since numerical wave-based simulations can provide a realistic model of the sound field, in particular at low frequencies, these can be used as a basis for alternative measurement methods, which would then allow the in-situ identification of acoustic impedances. The main requirement for these methods to be applicable is that the geometry of the room should be known. Algorithms for room geometry estimation exist and are the subject of ongoing research [8, 9]. In [10] a new approach to measure acoustic impedances has been proposed: this approach consists of measuring the sound field by means of a moving microphone in the room under test and matching the measurements with simulations of the acoustic field using the boundary element method (BEM). Solving this inverse problem with nonlinear optimization leads to the simultaneous measurement of acoustic impedances of different surfaces. The approach presented in this paper is related to the approach of [10], but here the FDTD method is used instead of the BEM. This has the advantage that no moving microphone is needed. In addition, the proposed optimization

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algorithm can obtain the acoustic impedances in a narrowband while the method of [10] has to be repeated for each frequency.

2. FINITE DIFFERENCE TIME DOMAIN METHOD

As all numerical methods, the FDTD method aims at finding a solution of a partial differential equation (PDE) by discretizing the domain of interest $\Omega \times \tau$. Here Ω represents the spatial domain, e.g. the geometry of a room, and τ represents the time domain. The PDE governing a sound field is the wave equation with boundary conditions (BCs)

$$\begin{aligned} \text{PDE} \quad & \triangle p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = s \text{ on } \Omega \times \tau \\ \text{BCs} \quad & \frac{\partial p}{\partial t} = -c\xi \nabla p \cdot \mathbf{n} \text{ on } \partial\Omega \times \tau \end{aligned} \tag{1}$$

where $p(\mathbf{x}, t)$ and $s(\mathbf{x}, t)$ are the sound pressure level and source signal, functions of space $\mathbf{x} = (x, y, z) \in \Omega$ and time $t \in \tau$, and c is the speed of sound. The operator $\triangle$ is defined as $\nabla \cdot \nabla = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial}{\partial z^2}$. In the BCs, the *specific acoustic impedance* ξ models the energy transmission between the boundaries and the internal domain and $\mathbf{n}$ is the normal vector to the boundary surface.

The sound pressure level and source signal can be sampled uniformly in time and space, e.g. for $p(\mathbf{x}, t)$

$$p(x, y, z, t) \approx p(lX, mY, iZ, nT) = p^n_{l,m,i}, \tag{2}$$

where X, Y and Z represent the space steps in the x, y and z directions respectively (in the following $X = Y = Z$ is chosen) and T the time sampling interval. This means the domain $\Omega \times \tau$ is discretized into $N_X \cdot N_Y \cdot N_Z \cdot N_T$ grid points. The second-order time derivative in (1) can be approximated as

$$\frac{\partial^2 p}{\partial t^2} \approx \frac{p^{n+1}_{l,m,i} - 2p^n_{l,m,i} + p^{n-1}_{l,m,i}}{T^2}. \tag{3}$$

Similar expressions can be given for the spatial derivatives. By substituting these approximations in the wave equation, the standard leapfrog (SLF) update equation is derived as [11]

$$p^{n+1}_{l,m,i} = \lambda_c^2 P^n_{l,m,i} + 2(1 - 3\lambda_c^3)p^n_{l,m,i} - p^{n-1}_{l,m,i} + s^n_{l,m,i} \tag{4}$$

where λ_c is the Courant number $\lambda_c = cT/X$ (which for the SLF is chosen to be $1/\sqrt{3}$) and

$$P^n_{l,m,i} = \sum_{r=1}^{2} p^n_{l(-1)^r,m,i} + p^n_{l,m(-1)^r,i} + p^n_{l,m,i(-1)^r} \tag{5}$$

is the summation over the neighboring axial grid points of $p^n_{l,m,i}$. The FDTD method converts the PDE into a set of linear equations that can be solved iteratively: the sound pressure at time instant $n+1$ is evaluated using values from the previous two instants. The FDTD method does suffer from numerical errors that result in dispersion: sound waves travel with different speed depending on the direction and frequency. This effect is more prominent at higher frequencies [1].

2.1. Frequency-independent acoustic impedances

In the following all the boundary surfaces are assumed to be *locally reacting*. This means that there is no wave propagation parallel to the boundary surface [11]. When boundaries are considered to be frequency-independent the acoustic impedance ξ is a constant. At the boundaries the update equation (4) will miss some grid points, e.g. for a wall in the positive x direction $P^n_{l,m,i}$ will miss the point $p^n_{l+1,m,i}$. These missing grid points are called *ghost points* and can be employed to model the BCs. The BCs equation in (1) for the case the normal vector is parallel to the x direction can be approximated as

$$(p^{n+1}_{l,m,i} - p^{n-1}_{l,m,i})/T = -c\xi(p^n_{l+1,m,i} - p^n_{l-1,m,i})/X. \tag{6}$$

This equation can be used to obtain the value of the ghost point $p^n_{l+1,m,i}$ which can then be substituted in (4). The resulting modified update equation is

$$\begin{aligned} p^{n+1}_{l,m,i} = [\lambda_c^2(2p^n_{l-1,m,i} + \sum_{r=1}^{2} p^n_{l,m(-1)^r,i} + p^n_{l,m,i(-1)^r}) \\ + (\lambda_c/\xi - 1)p^{n-1}_{l,m,i}](\lambda_c/\xi + 1)^{-1}. \end{aligned} \tag{7}$$

With a similar procedure the update for the corners and edges can be obtained with the elimination of two and four ghost points respectively. For brevity these equations are not shown here and can be found in [11].

2.2. Frequency-dependent acoustic impedances

The acoustic impedance is in general frequency-dependent. One of the simplest approaches to model this frequency dependency is to assume a spring-like behavior:

$$\xi(j\omega) = \xi_R + j\omega\xi_M + \xi_\kappa/j\omega, \tag{8}$$

where $\omega = 2\pi f$ is the angular frequency, ξ_R the specific acoustic damping, $\xi_M = M/\rho c$ the specific acoustic mass, M the acoustic mass, $\xi_\kappa = \kappa/\rho c$ the specific acoustic stiffness, κ the acoustic stiffness and ρ the density of air. The update equation for a wall in the positive x direction is given by [11]:

$$\begin{aligned} p^{n+1}_{l,m,i} = [\lambda_c^2(2p^n_{l-1,m,i} + \sum_{r=1}^{2} p^n_{l,m(-1)^r,i} + p^n_{l,m,i(-1)^r}) \\ + (\lambda_c/a - 1)p^{n-1}_{l,m,i} + g](\lambda_c/a + 1)^{-1} \end{aligned} \tag{9}$$

where $a = \xi_R + a_\kappa + a_M$ and $a_\kappa = T/2 \cdot \xi_\kappa$, $a_M = 2/T \cdot \xi_M$. The equations for obtaining g, which contains information about the *memory* of the impedance are not shown here for brevity and can be found in [11]. This frequency-dependent impedance model is a first-order approximation and does not describe an acoustic impedance in the whole audio frequency range. A more accurate model that employs digital filters at every node exists [1], nevertheless it is not used here due to its higher complexity which would lead to a much more difficult optimization problem.

3. THE OPTIMIZATION ALGORITHM

By solving an optimization problem the boundary surface acoustic impedances can be recovered. It is assumed that the geometry of the room is known. A known source signal $s^n_{l,m,i}$ is placed at a known position in the room and the sound field $p^n_{l_k,m_k,i_k}$ is measured with K microphones where the indices (l_k, m_k, i_k) indicate the known position of the kth microphone. The following optimization problem can be formulated:

$$\begin{aligned} \min_{\boldsymbol{\xi}} \quad & \frac{1}{2}\|\text{FDTD}(\boldsymbol{\xi}) - \tilde{\mathbf{p}}\|_2^2 \\ \text{s. t.} \quad & \boldsymbol{\xi}_{\min} \leq \boldsymbol{\xi} \leq \boldsymbol{\xi}_{\max}. \end{aligned} \tag{10}$$

where $\boldsymbol{\xi}$ is the vector containing the values of the acoustic impedances of the walls, e.g. for a cuboid room with different frequency-independent acoustic impedances at each boundary surface,

$$\boldsymbol{\xi} = [\xi_{x_+}, \xi_{x_-}, \xi_{y_+}, \xi_{y_-}, \xi_{z_+}, \xi_{z_-}]^T \in \mathbb{R}^6. \tag{11}$$

In the case of frequency-dependent acoustic impedances each impedance in $\boldsymbol{\xi}$ becomes a vector containing the three parameters of the impedance, e.g. $\boldsymbol{\xi}_{x_+} = [\xi_{R,x_+}, a_{K,x_+}, a_{M,x_+}]^T$ meaning that $\boldsymbol{\xi} \in \mathbb{R}^{3\cdot 6}$. The function FDTD$(\boldsymbol{\xi})$ returns a vector $\mathbf{p} \in \mathbb{R}^{K \cdot N_T}$ containing the simulated sound pressure level for all time instants at the K microphone positions. The measurement vector $\tilde{\mathbf{p}}$ contains the K microphone signals. Since the unknown parameters in $\boldsymbol{\xi}$ appear nonlinearly in the function FDTD$(\boldsymbol{\xi})$ through the update equation (7) or (9), the cost function in (10) is nonlinear. The inequalities in (10) limit the acoustic impedance values with a lower and upper bound. Since the cost function corresponds to an L2-norm, the Gauss-Newton method can be applied. In Algorithm 1 it can be seen how the Hessian and the gradient of the cost function are approximated. For each iteration a constrained quadratic program is solved. Most of the numerical effort is in the evaluation of the Jacobian $\nabla F(\boldsymbol{\xi}_n)$ which is performed using finite differences and a FDTD simulation for each variable in $\boldsymbol{\xi}$. To obtain global convergence, backtracking with the Armijo condition using a merit function was chosen [12]. Here, a FDTD simulation has to be performed for each correction of the step-size. The stopping tolerance was chosen experimentally to be 10^{-7}.

Figure 1 shows the case of a two-dimensional cost function, corresponding to a cuboid room which has only two different frequency-independent impedances, one for the walls and ceiling, $\xi_{\text{walls}} = 100$, and one for the floor $\xi_{\text{floor}} = 50$. It can be seen that for $\xi = 0$ the cost function goes to infinity, which justifies the lower bound in (10). Moreover, crossing the zero value would lead to a non-physical negative acoustic impedance. The upper bound is also convenient since the optimization path may diverge for an unfortunate choice of the initial condition $\boldsymbol{\xi}_0$. In the following the lower and upper bound will be chosen to be $\xi_{\min} = 1$ and $\xi_{\max} = 10^3$,

Algorithm 1 Gauss-Newton Optimization

Input: $\boldsymbol{\xi}_0$,$\boldsymbol{\xi}_{\max}$, $\boldsymbol{\xi}_{\min}$, $\tilde{\mathbf{p}}$, TOL
Output: $\boldsymbol{\xi}$
  for $n = 0, 1, \ldots, n_{\max}$ **do**
    Evaluate $F(\boldsymbol{\xi}_n) = \text{FDTD}(\boldsymbol{\xi}_n) - \tilde{\mathbf{p}}$
    Evaluate $f(\boldsymbol{\xi}_n) = \frac{1}{2}\|F(\boldsymbol{\xi}_n)\|_2^2$
    if $f(\boldsymbol{\xi}_n) \leq$ TOL **then**
      Acceptable solution found
      Break
    end if
    Evaluate Jacobian $\nabla F(\boldsymbol{\xi}_n)$
    Evaluate GN gradient $\nabla f(\boldsymbol{\xi}_n) \approx \nabla F(\boldsymbol{\xi}_n)^T F(\boldsymbol{\xi}_n)$
    Evaluate GN Hessian $\nabla^2 f(\boldsymbol{\xi}_n) \approx \nabla F(\boldsymbol{\xi}_n)^T \nabla F(\boldsymbol{\xi}_n)$
    Solve: $\boldsymbol{\xi}_{n+1} = \underset{\boldsymbol{\xi}}{\text{argmin}}\ 1/2\boldsymbol{\xi}^T \nabla^2 f(\boldsymbol{\xi}_n)\boldsymbol{\xi} + \nabla f(\boldsymbol{\xi}_n)\boldsymbol{\xi}$
    s.t. $\boldsymbol{\xi}_{\min} \leq \boldsymbol{\xi} \leq \boldsymbol{\xi}_{\max}$
    Find correct step-size $|\boldsymbol{\xi}_{n+1} - \boldsymbol{\xi}_n|$ with line search
  end for

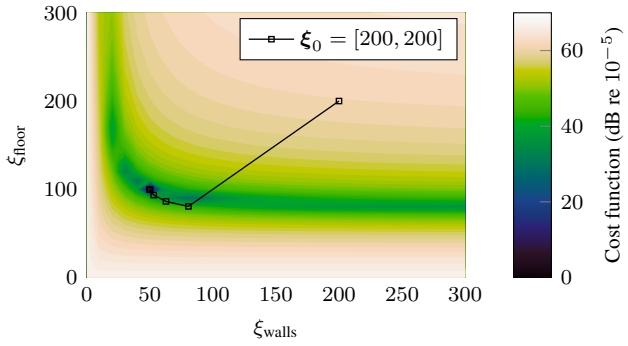

Fig. 1: Surface plot of a two dimensional cost function and optimization path.

which corresponds to a normal reflection coefficient of $R = 0.99$ and $R = 0$, respectively. Negative normal reflection coefficients, $R < 0$ which would correspond to an acoustic impedance of $0 \leq \xi \leq 1$, are not considered since soft walls usually are not found in real rooms. Acoustic specific mass ξ_M is chosen to be restricted in the range between 0 and $56 \cdot 10^{-3}$ ($M_{\max} \approx 23$ kg). The acoustic stiffness ξ_κ is restricted between 0 and $1.8 \cdot 10^5$ ($\kappa_{\max} \approx 7 \cdot 10^7$ N/m). The maximum values are assumed to be much larger than the expected ones. The optimization algorithm is initialized with $\boldsymbol{\xi}_{R,0} = \mathbf{1}$, $\boldsymbol{\xi}_{M,0} = \mathbf{0}$ and $\boldsymbol{\xi}_{\kappa,0} = \mathbf{0}$.

4. SIMULATION RESULTS

Simulations were performed on a cuboid room of $2\times2.3\times2.7$ m^3. The "true" sound field was generated using the FDTD method. K microphone measurements were simulated by adding uncorrelated noise to the FDTD generated sound pressure levels. The optimization algorithm was then used to invert the system and estimate the acoustic impedances. The mi-

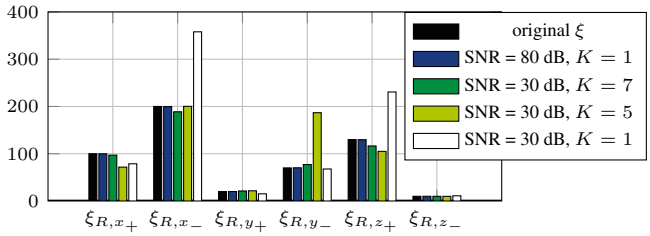

Fig. 2: Estimated frequency-independent impedance parameters using different number of microphones and different SNR.

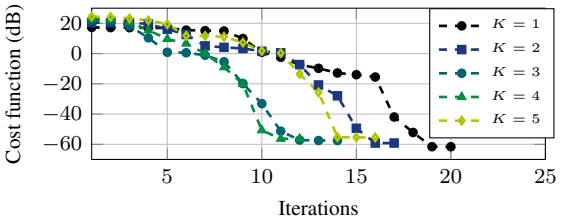

Fig. 3: Convergence curve for frequency-dependent acoustic impedances estimation (SNR = 80 dB).

crophones were placed at fixed random positions. Due to the computational load of the FDTD method, all simulations were performed with a relatively low resolution of $X = 33$ cm for $N_T = 10^3$ time samples (corresponding to 0.55 seconds). It was assumed that the FDTD method approximates accurately the sound field and boundary conditions in one octave band. Therefore the source signal was a white noise bandpass signal, filtered in the octave band of 44-88 Hz and placed in the $(l_s, m_s, i_s) = [2, 5, 3]$ grid point. Other band-limited signals could have been chosen as well. Simulations were performed for both frequency-independent and frequency-dependent impedances. For frequency-independent impedances and high SNRs, the estimation of the true impedance values, which were chosen as $\boldsymbol{\xi} = [100, 200, 20, 70, 130, 10]^T$, was achieved with only one microphone with good accuracy. This is shown in figure 2 where it can also be seen how using more microphones improves the estimation for lower SNRs. In addition, the usage of more microphones can improve the convergence of the algorithm: for $K = 1$ the method converged with 18 iterations while for $K = 5$ with 14 iterations. Similar results were achieved for frequency-dependent impedances: the room was modeled with three acoustic impedances with values similar to the ones used in [13], which are shown in table 1. One of the walls was modeled as a curtain, the floor as a parquet and all the other surfaces as plaster. Figure 3 shows the improvement of the convergence of the algorithms for an increasing number of microphones. It can be noticed that for $K = 5$ more iterations are needed than for $K = 4$ and $K = 3$. This suggests that some microphone positions are better suited for the reconstruction than others. In figure 4 the absolute value of the acoustic impedance of the original materials and the estimated values are plotted in the

	Curtain	Plaster	Parquet
ξ_R	37	50	40
ξ_κ (10^3)	0	21.4	35.6
ξ_M (10^{-3})	12	14.3	14.3

Table 1: True frequency-dependent acoustic impedance values.

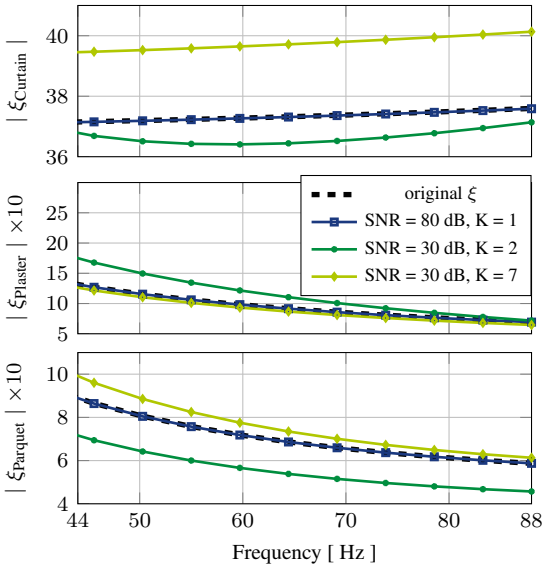

Fig. 4: Original and reconstructed frequency-dependent acoustic impedances.

octave band of interest. Again, a highly accurate estimation can be achieved for high SNRs for $K = 1$. For SNR$= 30$ at least two microphones were needed to achieve meaningful results and the noise robustness could be improved using more microphones.

5. CONCLUSIONS

A method to identify the surface acoustic impedances in rooms has been introduced. Compared to existing work in [10] a time-domain numerical method has been proposed which shows clear advantages over a frequency-domain method such as in [10]. In the frequency domain, spatial sampling is required and a moving microphone has to be used, the position of which has to be estimated with multiple video cameras. The use of the FDTD method simplifies the measurement set-up, particularly for high SNRs where only one microphone is needed. Furthermore the optimization convergence is increased, as only few iterations are needed for the optimization algorithm convergence, typically around 20, while in [10] at least 180 iterations are needed. Future research will focus on the application of the proposed method using real measurements and on understanding the optimal positioning of the microphones in the room.

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