Katholieke Universiteit Leuven

Departement Elektrotechniek

ESAT-SISTA/TR 08-225

Resource allocation in DMT transmitters with per-tone pulse shaping [1]

Prabin Kumar Pandey[2] and Marc Moonen[2] and Luc Deneire[3]

January 2009
Submitted for publication in IEEE Transactions on Signal Processing

[1]This report is available by anonymous ftp from *ftp.esat.kuleuven.ac.be* in the directory *pub/sista/pkumarpa/reports/08-225.pdf*

[2]K.U.Leuven, Dept. of Electrical Engineering (ESAT), Research group SISTA, Kasteelpark Arenberg 10, 3001 Leuven, Belgium, Tel. 32/16/32 19 24, Fax 32/16/32 19 70, E-mail: *pkumarpa@esat.kuleuven.ac.be*. This research work was carried out at ESAT laboratory of Katholieke Universiteit Leuven, Leuven, Belgium, in the frame of K.U.Leuven Research Council CoE EF/05/006 Optimization in Engineering, Research Project FWO nr. G.0235.07 Design and evaluation of DSL systems with common mode signal exploitation and the Marie-Curie EST-SIGNAL program (http://est-signal.i3s.unice.fr) under contract No. MEST-CT-2005-021175.

[3]Laboratoire I3S Algorithmes/Euclide-B University of Nice, France Phone : +33 492 94 27 38 deneire@i3s.unice.fr

Abstract

Per-tone pulse shaping has been proposed as an alternative to time domain spectral shaping for DMT transmitters, e.g. VDSL modems. It shapes the spectrum of individual tones such that their stop band energy contribution can be minimized. This in particular enables the transmitter to use more tones without violating the PSD mask constraint for data transmission. In per-tone pulse shaping based DMT transmitters a fixed length pulse shaping filter is typically used for every tone. Furthermore no specific power loading scheme is used. However, the contribution of a particular tone to the stop band energy depends on two factors, firstly the amount of power transmitted on the tone and secondly the distance of the tone from the band edges. Tones with sufficiently low power as well as tones sufficiently away from the band edges do not contribute much to the overall stop band energy and therefore could be assigned a lower order pulse shaping filter, which could then help to reduce the overall number of filter taps. In general the combination of power loading and variable length per-tone pulse shaping can be used to achieve a high data rate under given resource constraints. In this paper we present an algorithm to optimally allocate available resources, i.e. power and filter taps, using a dual problem formulation. This algorithm optimally allocates power and filter taps over tones for a given PSD mask constraint, with relatively low complexity, as is also demonstrated by simulations.

Resource allocation in DMT transmitters with per-tone pulse shaping

Prabin Kumar Pandey*, and Marc Moonen* and Luc Deneire†

*Department of Electrical Engineering (ESAT-SCD) Katholieke Universiteit Leuven,Heverlee-Leuven, Belgium

Email: {prabin.kumarpandey, marc.moonen}@esat.kuleuven.be

†Laboratoire I3S Algorithmes/Euclide-B

University of Nice, France

Email: deneire@i3s.unice.fr

Abstract

Per-tone pulse shaping has been proposed as an alternative to time domain spectral shaping for DMT transmitters, e.g. VDSL modems. It shapes the spectrum of individual tones such that their stop band energy contribution can be minimized. This in particular enables the transmitter to use more tones without violating the PSD mask constraint for data transmission. In per-tone pulse shaping based DMT transmitters a fixed length pulse shaping filter is typically used for every tone. Furthermore no specific power loading scheme is used. However, the contribution of a particular tone to the stop band energy depends on two factors, firstly the amount of power transmitted on the tone and secondly the distance of the tone from the band edges. Tones with sufficiently low power as well as tones sufficiently away from the band edges do not contribute much to the overall stop band energy and therefore could be assigned a lower order pulse shaping filter, which could then help to reduce the overall number of filter taps. In general the combination of power loading and variable length per-tone pulse shaping can be used to achieve a high data rate under given resource constraints. In this paper we present an algorithm to optimally allocate available resources, i.e. power and filter taps, using a dual problem formulation. This algorithm optimally allocates power and filter taps over tones for a given PSD mask constraint, with relatively low complexity, as is also demonstrated by simulations.

This research work was carried out at ESAT laboratory of Katholieke Universiteit Leuven, Leuven, Belgium, in the frame of K.U.Leuven Research Council CoE EF/05/006 Optimization in Engineering, Research Project FWO nr. G.0235.07 Design and evaluation of DSL systems with common mode signal exploitation and the Marie-Curie EST-SIGNAL program (http://est-signal.i3s.unice.fr) under contract No. MEST-CT-2005-021175. The scientific responsibility is assumed by its authors.

Part of this work has been published in ICASSP08 [1] and GLOBECOM08 [2]

Index Terms

Multicarrier communication, DMT, DSL, VDSL, resource management

I. Introduction

Very high speed Digital Subscriber Line (VDSL) modems use Discrete Multi-Tone (DMT) modulation [3]. DMT divides the available spectrum into smaller parallel sub-bands or tones. Each tone corresponds to an orthogonal carrier. The input bit-stream is divided into several independent parallel streams which then QAM-modulate the different carriers. These QAM modulations are implemented based on an Inverse Discrete Fourier Transform (IDFT) [4]. A cyclic prefix is added to each resulting time domain symbol before transmission. If the cyclic prefix is too short, i.e. shorter than the channel impulse response, then this results in inter-symbol interference (ISI) and inter-carrier interference (ICI). Highly dispersive channels such as the VDSL channel have a very long channel impulse response, hence to mitigate ISI/ICI a very long cyclic prefix is needed.

The Discrete Fourier Transform (DFT) filters used to implement the DMT have a poor frequency response. Firstly, the first side lobe is only 13 dB below the main lobe and secondly the rate of decay of the side lobe energy is only inversely proportional to frequency. In some applications e.g. in VDSL, the Power Spectral Density (PSD) of the transmitted signal is not allowed to exceed a given PSD mask defining one or more pass bands and stop bands. To satisfy this constraint, many tones near the band edges cannot be used, which then significantly reduces the available bandwidth for data transmission. One of the attempted approaches to improve the spectral characteristics of the transmit signal is to use an alternative transmit and receive basis function instead of the DFT filters [5]–[7], but this is found to make the equalization at the receiver side more complicated. Another approach is to use time domain windowing of the transmit signal [8] with and without extending the cyclic prefix and possibly an additional cyclic suffix [9]. Inter-symbol windowing, in which only the cyclic prefix and cyclic suffix is multiplied with the window function to preserve the orthogonality was presented in [10]. Similarly, intra-symbol windowing where the window function is applied to the transmit block was proposed in [11]–[13]. However, this approach destroys the orthogonality and needs a post processing at the receiver side. The idea of spectral compensation was considered in [14]–[16]. One of the simplest approaches theoretically is to filter the transmit signal [17]. Recently, Phoong et al. proposed a per-tone pulse shaping method to reduce the energy in the stop bands [18]. It uses a filter to shape the spectrum for each tone. These filters can be designed using many techniques [19]–[21]. In [18], the authors propose a simple filter design procedure, where a pulse shaping filter is designed for each tone such that its stop band energy contribution is minimized. The method allows for asymmetric filters which helps to reduce the stop band energy contribution of the tones at the band edges. This in turn helps to increase the number of tones that can be used for data transmission

without violating the PSD mask constraint. The resulting transmitter structure was derived in [18] and shown to be a dual of the per-tone equalization structure proposed in [22]. It is noted that the transmit signal filtering has the same effect as the time dispersive channel and therefore the cyclic prefix length has to be increased accordingly in order to mitigate the ISI/ICI [16], [17].

In the DMT transmitter using per-tone pulse shaping as described in [18], every tone is shaped using a shaping filter of fixed length and on every tone the same amount of power is transmitted. However for a given amount of transmit power, tones in the middle of a pass band contribute less to the overall stop band energy and therefore a high order pulse shaping filter may not improve the performance significantly, whereas for tones near the band edges using a higher order filter can indeed significantly reduce the contribution to the stop band energy. Therefore using a constant length pulse shaping filter for all tones may correspond to a waste of system resources and so a better alternative is then to consider variable length pulse shaping filters. Furthermore, the number of tones used can also be increased by using a lower transmit power at the band edges. Using a lower transmit power on a larger number of tones may help to increase the achievable data rate without violating the PSD mask constraint. In conclusion, there is a need for an efficient method to optimally allocate the filter taps as well as transmit power to tones, such that a maximum data rate is achieved with the available resources, i.e. for a given power budget and filter tap budget, and without violating the PSD mask constraint. To our knowledge such a method is not yet available.

The Lagrange multipliers method is often used to convert a constrained optimization problem into a dual unconstrained problem [23], [24]. This method has been previously used to optimally allocate resources (filter taps) in a DMT receiver with per-tone equalization [25]. In this paper, an optimal resource allocation algorithm is proposed, based on a dual problem formulation, to jointly distribute the resources (power and filter taps) in a DMT transmitter with per-tone pulse shaping combined with a specific pulse shaping filter design procedure.

In section II, a system model of the DMT transmitter employing per-tone pulse shaping is presented. In section III, the general resource allocation problem is formulated, which can be used to compute the optimal transmit power and number of filter taps for each tone under a PSD mask constraint. An algorithm to solve this optimization problem will then be presented. Section IV contains some simulation results. Finally conclusion are presented in section V.

II. System Model

A. Per tone pulse shaping

The following notation is adopted in the description of the DMT system. N is the size of the IDFT and i denotes the tone index. L is the order of the pulse shaping filter, η is the cyclic prefix length. $\mathbf{x}_l$ is the output of the transmitter i.e. a vector of length $K = N + L + \eta$, corresponding to one

transmitted symbol at time l.

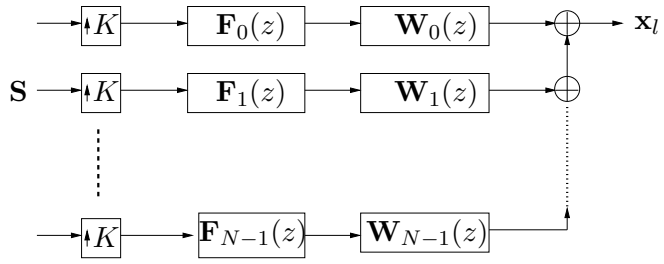

Fig. 1. DMT transmitter with per-tone shaping filter

Figure 1 shows the transmitter structure of the DMT with the per-tone shaping as derived in [18]. The filters $F_i(z)$ are the DFT filters defined as

$$F_i(z) = e^{-j2\pi i\eta/N} \sum_{l=0}^{N+\eta-1} e^{j2\pi il} z^{-l}.$$

The per-tone shaping filters, $W_i(z)$, are designed using a minimum stop band energy criterion. For simplicity we will adopt the following design criterion, presented in [18], for every filter $W_i(z)$:

$$\begin{aligned} \min_{\mathbf{w}_i} \quad & (1-\beta)\int_{\Omega_1} |F_i(e^{j\omega})W_i(e^{j\omega})|^2 d\omega \\ & +\beta\int_{\Omega_2} |F_i(e^{j\omega})W_i(e^{j\omega})|^2 d\omega \\ \text{subject to} \quad & W_i(e^{j2\pi i/N}) = 1 \end{aligned} \tag{1}$$

where

Ω_1=stop band frequencies as defined by the PSD mask

Ω_2=pass band frequencies as defined by the PSD mask

$W_i(e^{j\omega}) = \mathbf{e}^H(\omega)\mathbf{w}_i$

$\mathbf{e}^H(\omega) = \left[1\ e^{j\omega} \cdots e^{jL_i\omega}\right]$

$\mathbf{w}_i = [w_i^0\ w_i^1\ \cdots\ w_i^{L_i}]^T$

and β is a small constant $0 < \beta \ll 1$. The pass band term is included as a regularization term [18]. The design criterion can be restated as

$$\min_{W_i} \quad W_i^H \mathbf{Q}_i W_i \quad \text{subject to} \quad W_i(e^{j2\pi i/N}) = 1 \tag{2}$$

where,

$\mathbf{Q}_i = (1-\beta)\int_{\Omega_1} |F_i(e^{j\omega})|^2 \mathbf{e}\mathbf{e}^H\, d\omega + \beta\int_{\Omega_2} |F_i(e^{j\omega})|^2 \mathbf{e}\mathbf{e}^H\, d\omega$

This is a linearly constrained least squares problem, which is easily solved for every i and for every filter order L_i. The design criterion given by equation (1) thus fully specifies the pulse shaping filter coefficient $\mathbf{w}_i$, for a given tone i and filter order L_i.

From **Figure 1** we can see that the transmitter output is

$$\mathbf{x}_l = \sum_i s_i \mathbf{f}_i \star \mathbf{w}_i \tag{3}$$

where, $\star$ denotes the convolution operation and s_i is the input symbol at tone i, $\mathbf{f}_i$ is the DFT filter and $\mathbf{w}_i$ is the shaping filter for tone i. In [18] it has been shown that this transmitter output can be generated cheaply based on

$$\mathbf{x}_l = \begin{pmatrix} \mathbf{0} & \mathbf{I}_L \\ \mathbf{I}_N & \\ \mathbf{0}_{P\times N} & \end{pmatrix} \mathbf{\Phi}^H \mathbf{D}\mathbf{s} + \sum_{i=0}^{N-1} \begin{pmatrix} -\alpha^{i(L+1)}\bar{\mathbf{v}}_i \\ \mathbf{0}_{(K-P)\times 1} \\ \alpha^i \bar{\mathbf{v}}_i \end{pmatrix} s_i. \tag{4}$$

Here $\mathbf{I}_k$ is the k by k identity matrix, $\mathbf{\Phi}^H$ is the IDFT matrix, $\mathbf{s} = [s_0 \cdots s_{N-1}]^T$ is the input vector, $\alpha = e^{-j2\pi/N}$, $\mathbf{v}_i = \mathbf{U}_i \mathbf{w}_i$ and can be written as $[v_{i,0}\ \bar{\mathbf{v}}_i^T]^T$, $\mathbf{U}_i$ is an $(L+1)$ by $(L+1)$ upper triangular Toeplitz matrix whose first row is $[1\ \alpha^i \cdots \alpha^{iP}]$, $\mathbf{D}$ is a diagonal matrix with diagonal elements $[v_{0,0}\ v_{1,0} \cdots v_{N-1,0}]$ consisting of the first element of $\mathbf{v}_i$ for each i.

III. Problem formulation

A. Pulse shaping filter length optimization under fixed power loading

For a given number of used tones with equal transmit power per tone, tones in the middle of a pass band do not contribute much to the overall stop band energy, hence can be assigned a pulse shaping filter with fewer taps without significantly affecting the overall performance. This motivates the use of variable length pulse shaping filter for the different tones. This then requires a procedure to optimally distribute the filter taps over the tones, and such that the PSD mask constraint is still fulfilled. One possibility is to perform an exhaustive search over all possible combinations of pulse shaping filter lengths which satisfy the PSD mask constraint. Even though it can be implemented with a simple algorithm, this method has a prohibitively high complexity.

Here we present an alternative approach that is based on restating the constrained optimization problem as an unconstrained dual problem. Our constrained optimization problem can be written as

$$\boxed{\max_{L_1 \cdots L_N} \quad -\sum_i L_i \quad \text{subject to} \quad \mathbf{T}_{mask} \geq \mathbf{T}_{spec}} \tag{5}$$

where L_i is the pulse shaping filter order for the i-th tone, $\mathbf{T}_{mask}$ is the vector containing M sample points of the PSD mask at different frequencies and $\mathbf{T}_{spec}$ is the vector containing the similarly sampled PSD of the transmitter output.

Now we can use the method of Lagrange multipliers to integrate the constraint in the objective function. The dual problem formulation can be written as

$$\min_{\lambda_1 \cdots \lambda_M} \left\{ \max_{L_1 \cdots L_N} (\hat{\Psi}) \right\}$$

where,

$$\begin{aligned}\hat{\Psi} = \quad & -\textstyle\sum_i L_i + \sum_j \lambda_j (T_{mask,j} - T_{spec,j}) \\ \lambda_j \geq 0 \quad &\end{aligned}$$

where j is the index for the frequency samples i.e. $T_{mask,j}$ and $T_{spec,j}$ correspond to the jth frequency samples at frequency ω_j. With,

$$\begin{aligned}T_{spec,j} = \quad & \textstyle\sum_i P_i |F_i(e^{j\omega_j}) W_i(e^{j\omega_j})|^2 \\ = \quad & \textstyle\sum_i P_i |F_i(e^{j\omega_j}) \mathbf{e}^H(\omega_j) \mathbf{w}_i|^2, \end{aligned} \tag{6}$$

where $P_i = E\{|s_i|^2\}$. We can write equation (6) as,

$$\begin{aligned}\hat{\Psi} = \quad & -\textstyle\sum_i L_i - \sum_j \lambda_j \sum_i P_i |F_i(e^{j\omega_j}) \mathbf{e}^H(\omega_j) \mathbf{w}_i|^2 \\ & + \textstyle\sum_j \lambda_j T_{mask,j}\end{aligned}$$

which is equivalent to

$$\hat{\Psi} = \quad -\textstyle\sum_i \{L_i + \sum_j \lambda_j P_i |F_i(e^{j\omega_j}) \mathbf{e}^H(\omega_j) \mathbf{w}_i|^2\} \tag{7}$$

$$+ \underbrace{\sum_j \lambda_j T_{mask,j}}_{\text{constant for a given } \lambda_j} \tag{8}$$

For a given λ_j, the term $\sum_j \lambda_j T_{mask,j}$ is constant and therefore maximizing equation (7) is equivalent to maximizing

$$\hat{\Psi} = -\sum_i \{L_i + \sum_j \lambda_j \sigma_i |F_i(e^{j\omega_j}) \mathbf{e}^H(\omega_j) \mathbf{w}_i|^2\} \tag{9}$$

Hence, for fixed Lagrange multipliers λ_j the problem is decoupled over tones, and can be written for each tone as,

$$\boxed{\max_{L_i} \quad \hat{\psi_i}(\Lambda)} \tag{10}$$

$$\hat{\psi_i}(\Lambda) = -L_i - \sum_j \lambda_j P_i |F_i(e^{j\omega_j}) \mathbf{e}^H(\omega_j) \mathbf{w}_i|^2 \tag{11}$$

where Λ is a vector containing all $\lambda_j' s$. This per-tone optimization problem can be solved (for fixed Lagrange multipliers) by exhaustively searching over all values of L_i. The values of Lagrange multipliers can then be updated until all the PSD constraints are met.

An algorithm description is given below. The complexity of the proposed algorithm is $O(N * L_{max})$, where L_{max} is the maximum order of the pulse shaping filter allowed for each tone as dictated by the chosen prefix length and the channel impulse response length [18]. The Lagrange multipliers, λ_j are updated based on the difference between the PSD mask and the output PSD, i.e.

$$\lambda_j^{t+1} = \lambda_j^t - \mu_j(T_{mask,j} - T_{spec,j}) \tag{12}$$

where μ_j is the step size for the update which is always positive. The μ_j can be varied in order to drop the energy at a frequency point quickly if the output spectrum at that frequency exceeds the PSD mask and to cautiously increase it if the output spectrum is under the PSD mask. A typical initialization value for $\mathbf{\Lambda} = \{\lambda_j\}$ is $\mathbf{0}$ and then $\mu = \{\mu_j\}$ can be initialized with a value generally much larger than 1 as we need to establish a very high value for $\mathbf{\Lambda}$ in order to make the $\sum_j \lambda_j P_i |F_i(\omega_j)\mathbf{e}^H(\omega_j)\mathbf{w}_i|^2$ comparable to the value L_i. The algorithm stops when the number of taps stops updating.

Algorithm 1: Optimization of pulse shaping filter lengths

Initialize $\mathbf{\Lambda}$,μ

repeat

For $i \in \{$used tones$\}$

For $L_i = 0 \cdots L_{max}$

compute $\mathbf{w}_i$ (equation 1)

$\hat{\psi}_i \underset{\text{argmax}}{\Leftarrow} \{-L_i - \sum_j \lambda_j P_i |F_i(\omega_j)\mathbf{w}_i(\omega_j)|^2\}$

End For

End For

compute $\mathbf{T}_{spec}$

For $j = 1 \cdots M$

$\lambda_j^{t+1} = \lambda_j^t - \mu_j(T_{mask,j} - T_{spec,j})$

If $(\lambda_j^{t+1} < 0)$

$\lambda_j^{t+1} = 0$

End If

End For

While (change in total filter taps)

It is important to note that the PSD constraint in the original constrained problem is a non-convex constraint, as for a given sampling point the total power does not monotonically increase or decrease for an increase or decrease of the pulse shaping filter order of any particular tone. Furthermore there does not exist any analytical expression between the filter order and the power at a certain sampling point. Hence an analytical expression for the duality gap will be difficult to obtain. Therefore it should be noted that the final solution of the dual problem may not be the globally optimal solution of the primal problem but nevertheless practically relevant solution (see section IV).

B. Pulse shaping filter length optimization combined with power loading

For a single-user system without any PSD constraint the optimal power loading can be calculated by using the water filling procedure [9]. With a PSD constraint this is no longer applicable. In the previous section it was shown how to optimally allocate pulse shaping filter taps if the number of used tones and their transmit powers are known, for a system with PSD constraints. However, the number of used tones can be increased if a lower transmit power at the band edges is used. Therefore by also allocating the appropriate amount of transmit power for each tone the total achievable data rate can be maximized for a given total transmit power budget and total filter tap budget. This then requires a procedure to distribute the available resources (total power and total filter tap budget) optimally over the tones such that the data rate is maximized and the PSD mask constraint is still met. Here we present an approach that is again based on a dual problem formulation. The constrained optimization problem can be written as

$$\begin{aligned} \max_{\substack{P_1 \cdots P_N \\ L_1 \cdots L_N}} \quad & \sum_i R_i \\ \text{subject to} \quad & \mathbf{T}_{mask} \geq \mathbf{T}_{spec} \\ & L_{tot} \geq \textstyle\sum_i L_i \\ & P_{tot} \geq \textstyle\sum_i P_i \\ & L_i \leq L_{max} \\ & P_i \leq P_{max} \end{aligned} \tag{13}$$

where R_i is the achieved bit rate for tone i which is given as

$$R_i = log_2(1 + \frac{SNR_i}{\Gamma}),$$

SNR_i is the signal-to-noise ratio of tone i, Γ is the SNR gap, L_{tot} is the total filter tap budget and P_{tot} is the total power budget, L_i and P_i are the order of the pulse shaping filter and the power allocated to tone i, L_{max} and P_{max} are the maximum order of the pulse shaping filter and maximum transmit power allowed for each tone, $\mathbf{T}_{mask}$ is the vector containing M sample points of the given PSD and $\mathbf{T}_{spec}$ is the vector containing the calculated transmit PSD at each of these M sampling points.

Equation 13 can be rewritten in the dual form with Lagrange multipliers as

$$\min_{\gamma,\alpha,\lambda_1 \cdots \lambda_M} \left\{ \max_{\substack{L_1 \cdots L_N \\ P_1 \cdots P_N}} (\Psi) \right\}$$

where,

$$\begin{aligned}\Psi = \quad & \textstyle\sum_i R_i + \gamma \sum_i (L_{tot} - \sum_i L_i) + \alpha \sum_i (P_{tot} - \sum_i P_i) \\ & + \textstyle\sum_j \lambda_j (T_{mask,j} - T_{spec,j}). \qquad\qquad \alpha, \gamma, \lambda_j > 0 \end{aligned} \tag{14}$$

As P_{tot}, L_{tot} and $T_{mask,j}$ are given constants these do not influence the maximization for a given set of Lagrange multipliers. Therefore Ψ in the above equation can be written as

$$\begin{aligned}\Psi &= \sum_i R_i - \gamma \sum_i L_i - \alpha \sum_i P_i - \sum_j \lambda_j T_{spec,j} \\ &= \sum_i \{R_i - \gamma L_i - \alpha P_i\} - \sum_j \lambda_j T_{spec,j} \end{aligned} \tag{15}$$

By substituting $T_{spec,j}$ from equation (6) into equation (15) we obtain,

$$\Psi = \sum_i \{R_i - \gamma L_i - \alpha P_i\} - \sum_j \lambda_j \sum_i P_i |F_i(e^{j\omega_j}) \mathbf{e}^H(\omega_j) \mathbf{w}_i|^2$$

i.e.

$$\Psi = \sum_i \left\{ R_i - \gamma L_i - \alpha P_i - \sum_j \lambda_j P_i |F_i(e^{j\omega_j}) \mathbf{e}^H(\omega_j) \mathbf{w}_i|^2 \right\} \tag{16}$$

Hence for fixed Lagrange multipliers the problem is decoupled over tones, where the per-tone optimization function is,

$$\boxed{\max_{L_i, P_i} \quad \psi_i(\gamma, \alpha, \Lambda)} \tag{17}$$

$$\psi_i(\gamma, \alpha, \Lambda) = R_i - \gamma L_i - \alpha P_i - \sum_j \lambda_j P_i |F_i(e^{j\omega_j}) \mathbf{e}^H(\omega_j) \mathbf{W}_i|^2 \tag{18}$$

where Λ is a vector containing all λ_j's. This per-tone optimization problem can be solved (for fixed Lagrange multiplier) by exhaustively searching over all values of P_i (i.e. from -140dBm to P_{max}(-60dBm) in discrete steps)and L_i. The values of the Lagrange multipliers can then be updated until all the constraints are met. An algorithm description is given below. The complexity of the proposed algorithm is $O(N * N_p * L_{max})$, where N_p is the number of possible power allocation for each tone.

The Lagrange multipliers (γ, α and λ) are updated based on the difference between the obtained values for the corresponding resources and their target values, i.e.

$$\begin{aligned}\lambda_j^{t+1} &= \lambda_j^t - \mu_j (T_{mask,j} - T_{spec,j}) \\ \gamma_i^{t+1} &= \gamma_i^t - \hat{\mu} (L_{tot} - \sum_i L_i) \\ \alpha_i^{t+1} &= \alpha_i^t - \bar{\mu} (P_{tot} - \sum_i P_i) \end{aligned}$$

where μ_j, $\hat{\mu}$ and $\bar{\mu}$ are the step sizes for the updates which are always positive.

Algorithm 2: Joint resource allocation algorithm

Initialize λ, α, γ, $\mu_{\mathbf{j}}$, $\bar{\mu}$, $\hat{\mu}$

repeat

For $i \in \{\text{used tones}\}$

For $L_i = 0 \cdots L_{max}$

compute W_i (equation 1)

For $P_i = 0 \cdots P_{max}$

$$\psi_i \underset{\text{argmax}}{\Leftarrow} \{R_i - \gamma L_i - \alpha P_i - \textstyle\sum_j \lambda_j P_i |F_i(\omega_j)\mathbf{e}^H(\omega_j)\mathbf{w}_i|^2\}$$

End For

End For

End For

compute $\mathbf{T}_{spec}$

For $j = 1 \cdots M$

$\lambda_j^{t+1} = \lambda_j^t - \mu_j(T_{mask,j} - T_{spec,j})$

End For

$\gamma^{t+1} = \gamma^t - \hat{\mu}(L_{max} - \sum_i L_i)$

$\alpha^{t+1} = \alpha^t - \bar{\mu}(P_{max} - \sum_i P_i)$

While (change in filter taps and used power)

It is important here to point out that even though this is a joint resource allocation algorithm, the pulse shaping filter order for one particular tone does not have any effect on the bit rate achieved for this tone. This follows from the constrains included in design criterion (1). The bit rate for a tone is only a function of the transmit power on the tone. When all transmit powers are set, the pulse shaping filter orders are selected to ensure the overall PSD constraint is still met.

IV. Simulation Results

In our simulations, we consider VDSL downstream transmission with a PSD mask corresponding to the FTTCab M1 deployment scenario [8]. The size of the IDFT is 1024, the cyclic prefix length plus the overhead due to the pulse shaping filter is 80 samples and the SNR gap $\Gamma = 9.8dB$. We use $P_{max} = 16$ to be comparable to [18]. The initial values are $\lambda_j = 0$, $\alpha = 0$, $\gamma = 0$, $\mu_j = 1e8$, $\bar{\mu} = 1$ and $\hat{\mu} = 1e-7$.

A. Pulse shaping filter length optimization under fixed power loading

In **Figure 2** we can see that Algorithm 1 terminates after the 13-th iteration as there is no further change in the number of taps anymore. This is the minimum number of taps needed to satisfy the PSD mask with the given tones. The distribution of pulse shaping filter taps over the 375 used tones is shown in **Figure 3**. The minimum total number of taps needed to satisfy the PSD mask with the given number of tones is 3787. It is observed that the total number of taps is reduced from 6375(=17*375), when a fixed pulse shaping filter of length 17 is used for all the 375 tones as in [18], to 3787. This corresponds to a significant complexity reduction of almost 40%. It can be clearly seen that the tones at the band edges receive more filter taps than the tones in the middle of the pass bands. **Figure 4** shows the resulting output PSD and the PSD mask.

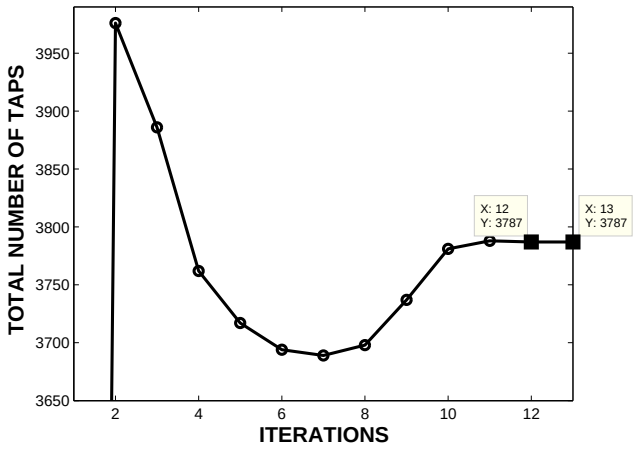

Fig. 2. Convergence of total number of pulse shaping filter taps

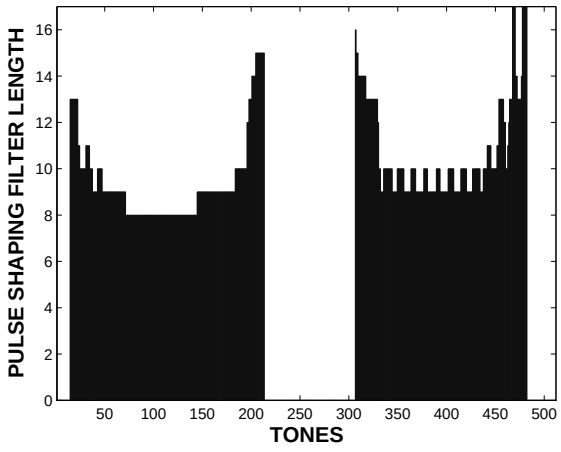

Fig. 3. Optimum distribution of number of pulse shaping filter taps over tones

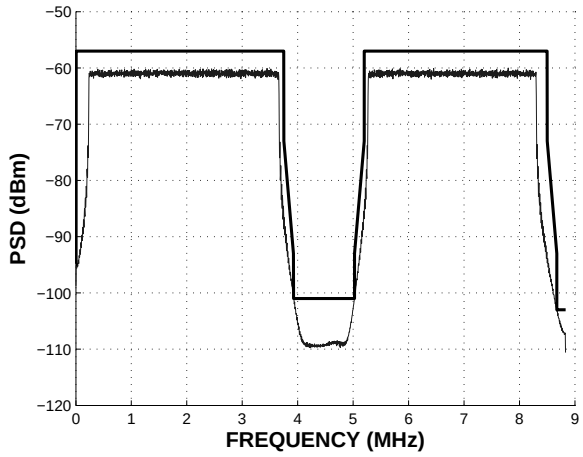

Fig. 4. Output PSD after the optimization

B. Pulse shaping filter length optimization combined with power loading

For given resource constraints, we will compare the results of the resource allocation procedure for two scenarios, one with channel knowledge and one without channel knowledge. A scenario with a channel to noise and interference ratio (CINR) as shown in the **Figure 5** is compared with a system which assumes the CINR on all the tones to be 85 dB (i.e. flat channel). The channel under consideration here is a 400m twisted pair cable in a bundle of 8 with an interference channel of 400m. As the total available power is 11.5 dBm in this case, which is larger than the total power required, the total power constraint in this simulation is inactive. The distribution of power over tones for given resource constraints is shown in **Figure 6**. It can be seen that the resource allocation is affected by the knowledge of the channel state. The resource allocation algorithm allocates more power to tones with higher CINR when the channel is known, otherwise the transmit power is more evenly distributed. The tap distribution over tones is given in **Figure 7**, which follows a similar pattern. **Table I** shows the number of iterations taken for the algorithm to converge for the different pulse shaping filter tap budgets. It is obvious that if the resources are scarce the algorithm requires a higher number of iterations, as it is then harder to distribute the resources optimally.

Furthermore from **Figure 8** it becomes evident that the achieved data rate saturates when the pulse shaping filter tap budget is increased. The data rate approaches the data rate for the case where there are no resource constraints (i.e. fixed power loading and maximum pulse shaping filter length for 375 used tones), which is given by the dotted line in the figure. It can also be seen that the achieved rate can be increased if the resource allocation is used, compared to the case where a fixed number of tones is used without any resource allocation (maximum 375 tones used in [18], number of pulse shaping filter taps=375*17=6375).

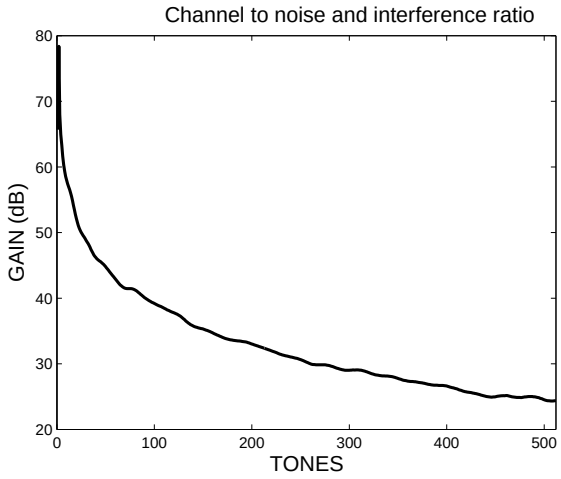

Fig. 5. Channel to noise and interference ratio

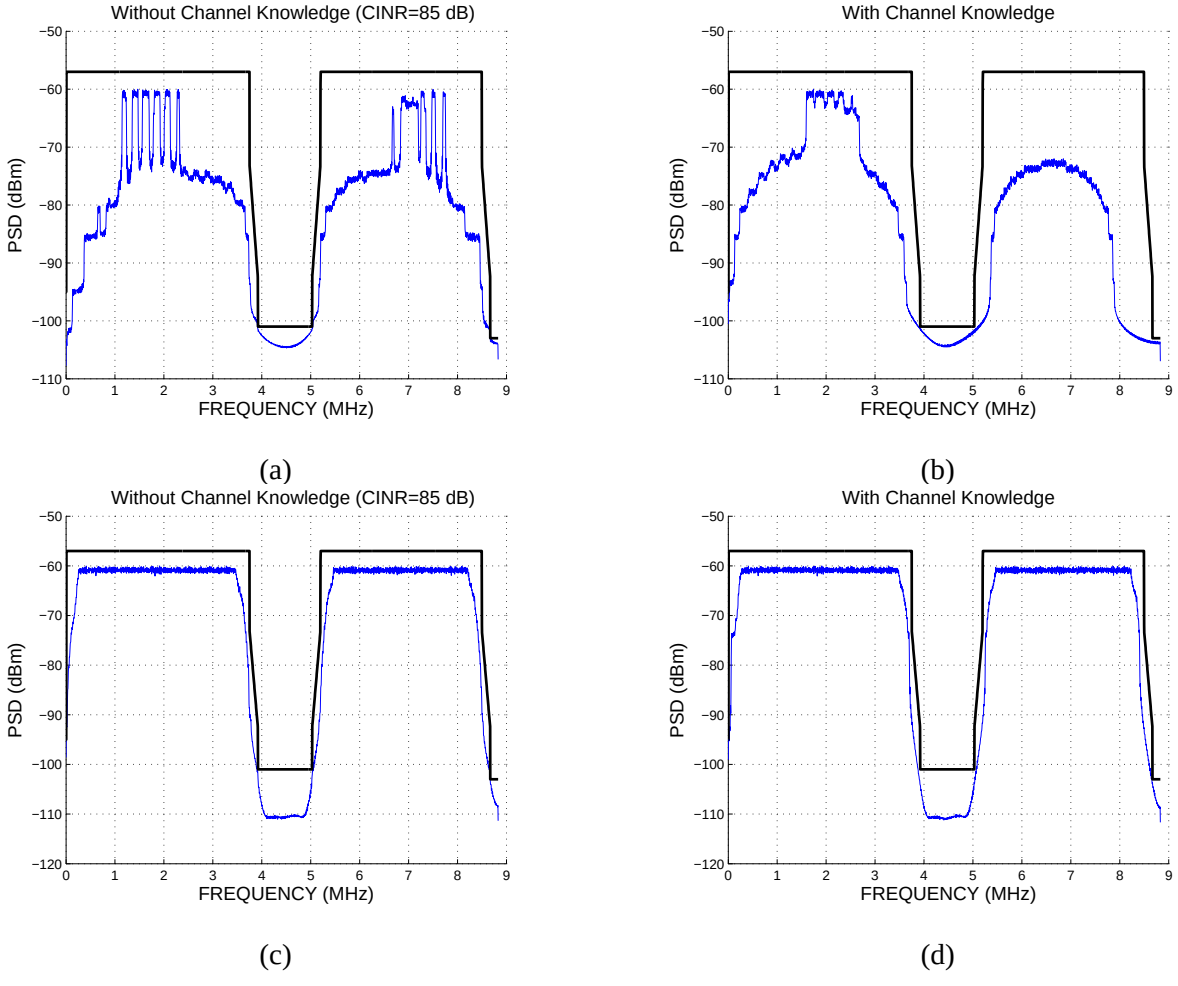

Fig. 6. Output spectrum for various resource constraints (a), (b) for 1000 taps and (c), (d) for 5000 taps

V. Conclusion

In this paper, we have proposed an efficient resource allocation algorithm to distribute the available resources (total power budget and total filter tap budget) over tones in a DMT transmitter with

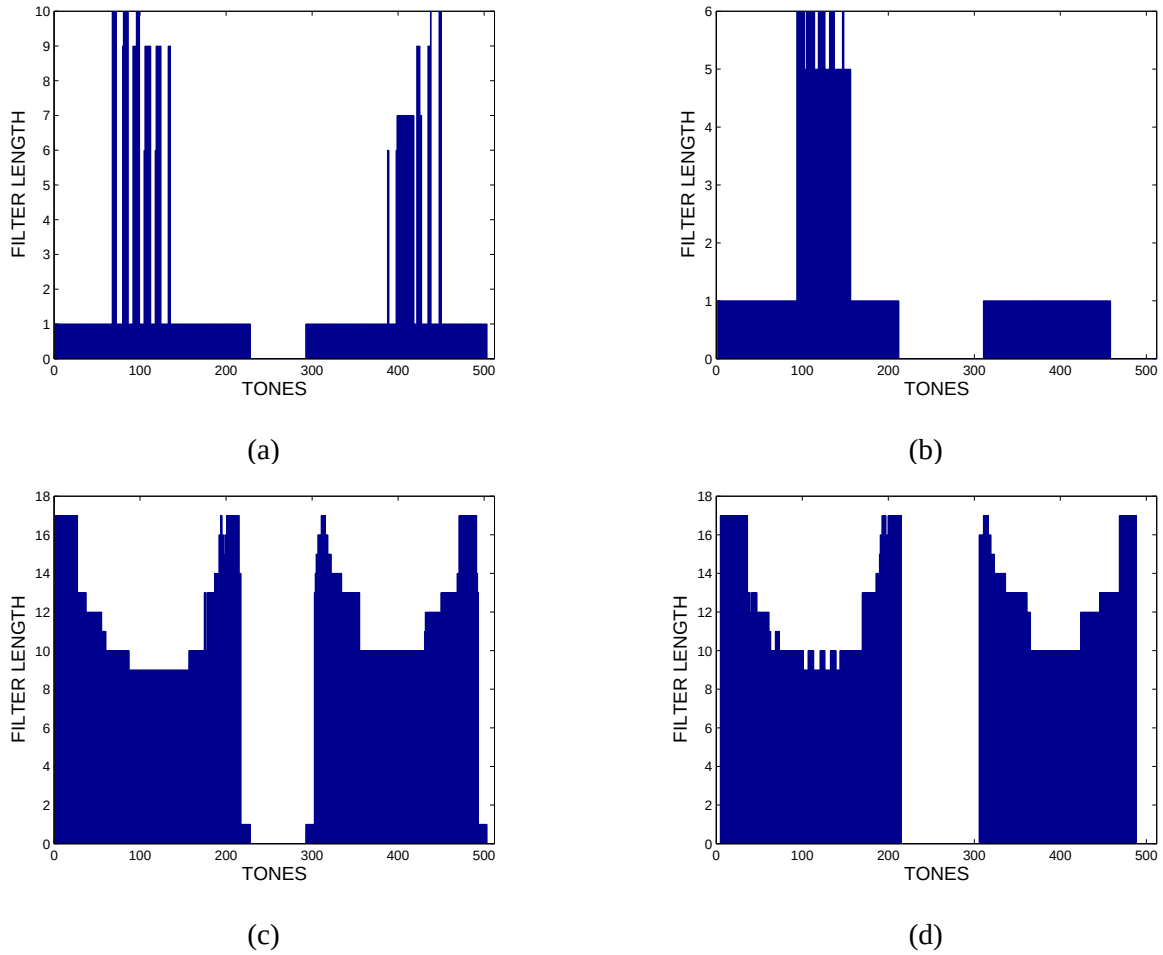

Fig. 7. Pulse shaping filter tap distribution for various resource constraints (a), (b) for 1000 taps and (c), (d) for 5000 taps

Total number of pulse shaping filter taps	Outer iterations to convergence
1000	38
5000	12

TABLE I

NUMBER OF ITERATIONS TAKEN TO CONVERGE TO THE SOLUTION

per-tone pulse shaping. We have shown that a dual problem formulation leads to an unconstrained optimization problem that is decoupled over tones and hence can be straightforwardly solved. A VDSL simulation demonstrates that under a fixed power loading, the number of pulse shaping filter taps can be reduced to approximately 40% without sacrificing performance. When combined with an optimal power loading algorithm this approach can further reduce the number of pulse shaping filter needed to achieve the same bit rate. It was also shown that the knowledge of the channel state influences the overall distribution of the resources quite significantly, especially so when the resources

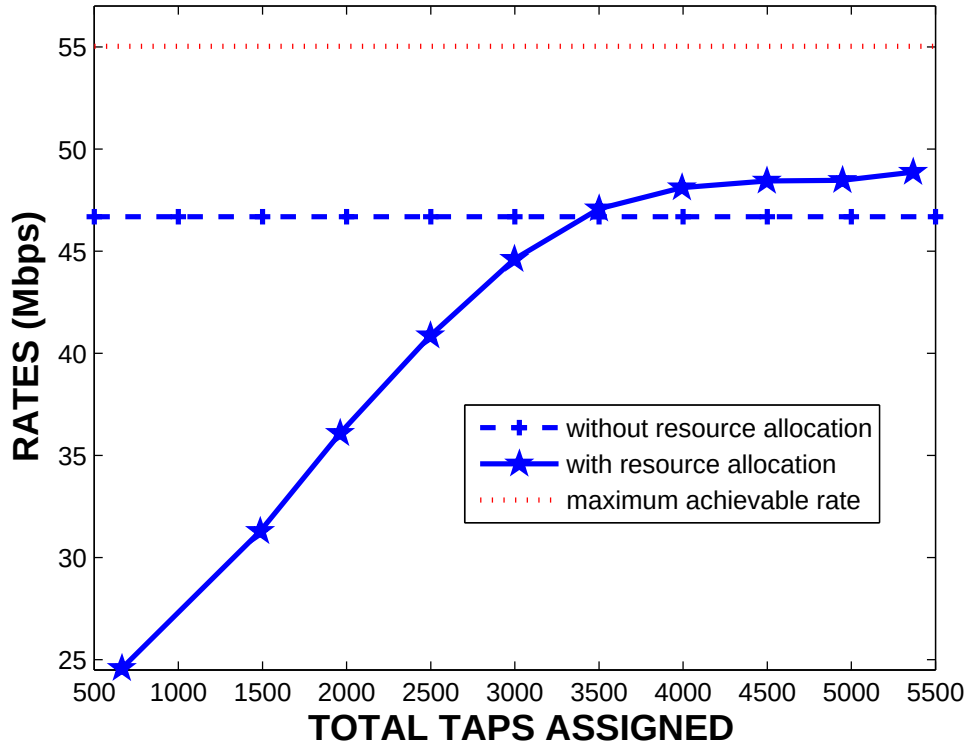

Fig. 8. Relation of data rate and the total number of available resources

are scarce.

REFERENCES

[1] P. K. Pandey, M. Moonen, and L. Deneire, "Resource allocation in DMT transmitters with per-tone pulse shaping," *accepted IEEE International Conference on Acoustics, Speech, and Signal Processing, 2008. Proceedings. (ICASSP '08).*, 2008.

[2] ——, "Joint transmit power and filter tap allocation in dmt transmitters with per-tone pulse shaping," *accepted Global Communications Conference (GLOBECOM 08).*, 2008.

[3] T. Starr, J. M. Cioffi, and P. J. Silverman, *Understanding digital subscriber line technology*. Upper Saddle River, NJ, USA: Prentice Hall PTR, 1999.

[4] J. Bingham, "Multicarrier modulation for data transmission: an idea whose time has come," *Communications Magazine, IEEE*, vol. 28, no. 5, pp. 5–14, May 1990.

[5] N. Vahlin, A.; Holte, "Optimal finite duration pulses for OFDM," *Communications, IEEE Transactions on*, vol. 44, no. 1, pp. 10–14, Jan 1996.

[6] H. Bölcskei, P. Duhamel, and R. Hleiss, "Design of pulse shaping OFDM/OQAM systems for high data-rate transmission over wireless channels," *Communications, 1999. ICC '99. 1999 IEEE International Conference on*, vol. 1, pp. 559–564 vol.1, 1999.

[7] W. Kozek and A. Molisch, "Nonorthogonal pulseshapes for multicarrier communications in doubly dispersive channels," *Selected Areas in Communications, IEEE Journal on*, vol. 16, no. 8, pp. 1579–1589, Oct 1998.

[8] "Interface between networks and customer installations: very-high speed digital subscriber lines (VDSL)," *ANSI T1E1.4/2002-031R2*, 2002.

[9] J. M. Cioffi, *Advanced digital communication, class reader, EE379C*. Stanford University, 2005.

[10] T. Magesacher, P. Ödling, and P. O. Börjesson, "Optimal intersymbol transmit windowing for multicarrier modulation," *in Proc. Nordic Signal Processing Symp. NORSIG 2006*, June 2006.

[11] Y.-P. Lin and S.-M. Phoong, "Window designs for DFT-based multicarrier systems," *IEEE Trans. on Signal Processing*, vol. 53, pp. 1015–1024, March 2005.

[12] G. Cuypers, K. Vanbleu, G. Ysebaert, and M. Moonen, "Egress reduction by intrasymbol windowing in DMT based transmitters," *Acoustics, Speech, and Signal Processing, 2003. Proceedings. (ICASSP '03). IEEE International Conference on*, vol. 3, pp. 532–535, 2003.

[13] T. Magesacher, "Optimal intra-symbol transmit windowing for multicarrier modulation," *in Proc. Intl. Symp. on Communications, Control and Signal Processing ISCCSP 2006, Marrakech, Morocco*, March 2006.

[14] J. Bingham and M. Mallory, "RFI egress supression for SDMT," *ANSI Contribution T1E1.4/96-085*, 1996.

[15] R. Baldemair, "Suppression of narrow frequency bands in multicarrier transmission systems," *in Proc. European Signal Processing Conf. EUSIPCO 2000, Tampere, Finland*, pp. 553–556, Sept. 2000.

[16] T. Magesacher, *Common-Mode Aided Wireline Communications*. PhD Thesis, Department of Information Technology, Lund University, Sweden, ISBN 91-7167-041-6, ISRN LUTEDX/TEIT-06/1037-SE, Sept. 2006.

[17] A. F. Molisch, *wideband wireless digital communications*. Upper Saddle River, NJ, USA: Prentice Hall PTR, 2000.

[18] C.-Y. Chen and S.-M. Phoong, "Per tone shaping filters for DMT transmitters," *Acoustics, Speech, and Signal Processing, 2004. Proceedings. (ICASSP '04). IEEE International Conference on*, vol. 4, pp. iv–1061–4 vol.4, 17-21 May 2004.

[19] K. W. Martin, "Small side-lobe filter design for multitone data-communication applications," *Circuits and Systems II: Analog and Digital Signal Processing, IEEE Transactions on [see also Circuits and Systems II: Express Briefs, IEEE Transactions on]*, vol. 45, no. 8, pp. 1155–1161, Aug 1998.

[20] W. Rhee, J. Chuang, and J. Cimini, L.J., "Performance comparison of OFDM and multitone with polyphase filterbank for wireless communications," *Vehicular Technology Conference, 1998. VTC 98. 48th IEEE*, vol. 2, pp. 768–772 vol.2, 18-21 May 1998.

[21] A. Tkacenko, P. Vaidyanathan, and T. Nguyen, "On the eigenfilter design method and its applications: a tutorial," *Circuits and Systems II: Analog and Digital Signal Processing, IEEE Transactions on [see also Circuits and Systems II: Express Briefs, IEEE Transactions on]*, vol. 50, no. 9, pp. 497–517, Sept. 2003.

[22] K. V. Acker, G. Leus, M. Moonen, O. van de Wiel, and T. Pollet, "Per tone equalization for DMT-based systems," *IEEE Transactions on Communications*, vol. 49, no. 1, pp. 109–119, Jan 2001.

[23] J. Nocedal and S. Write, *Numerical Optimization.* Springer verlag, 2006.

[24] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[25] P. K. Pandey and M. Moonen, "Resource allocation in ADSL variable length per-tone equalizers," *accepted IEEE Transactions on signal processing*, 2007.