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MMSE-based Partial Crosstalk Cancellation for Upstream VDSL[1]

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Abstract

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MMSE-based Partial Crosstalk Cancellation for Upstream VDSL

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***Abstract*—In current DSL systems, crosstalk is a major source of performance degradation. For upstream VDSL scenarios with in-domain crosstalk and AWGN, it has been shown that the crosstalk can be effectively mitigated using a linear zero-forcing canceler. Furthermore, the complexity can be reduced by only canceling the crosstalk from major crosstalkers, which is referred to as partial crosstalk cancellation. However, such approach does not work well in scenarios that also have out-of-domain (alien) crosstalk. As alien crosstalk is spatially correlated, the zero-forcing canceler performs very poorly, hence alternative structures for the canceler have to be examined. In this paper, we first demonstrate that an MMSE-based linear and nonlinear crosstalk canceler provide improved performance compared to the zero-forcing canceler in this scenario and then present efficient algorithms to perform partial crosstalk cancellation with these MMSE-based cancelers.**

***Index Terms*—DSL, crosstalk cancellation, partial crosstalk cancellation, MMSE, GDFE, alien crosstalk**

I. Introduction

To meet the demand for higher data rates, recent DSL systems use higher frequencies, e.g. up to 30 MHz [1]. At these frequencies, it has been shown that out-of-domain (alien) and in-domain crosstalk between different DSL lines in the same bundle become the dominant source of interference, limiting the performance of the DSL systems.

Crosstalk cancellation is one way to mitigate the effect of crosstalk. Several crosstalk cancellation techniques have been proposed in e.g. [2]–[6]. Different techniques may require different levels of transmitter and receiver coordination. In the case of upstream VDSL, only receiver coordination is possible, therefore a successive interference cancellation scheme performs optimally [4], [5]. However, the upstream VDSL channel matrix has a columnwise diagonally dominant (CWDD) structure, which is due to the fact that the power transmitted on a direct channel is always much larger than the power transmitted on any crosstalk channel by the same transmitter. In [6], under the assumption that the background noise is AWGN, it was shown that the CWDD structure allows a simple linear zero-forcing canceler to perform near optimally. Furthermore, due to the spatial as well as the frequency selective nature of the crosstalk, it was shown in [7] that the complexity of the cancellation can be reduced significantly using a technique called partial crosstalk cancellation, where only the interference originating from strong interferers is canceled.

However, in the case of spatially correlated background noise, e.g., when alien crosstalk is present, a prewhitening operation is needed to whiten the noise [8]. This operation destroys the CWDD structure of the channel matrix. The linear zero-forcing canceler is then found to perform very poorly. In this scenario, an alternative structure should be used that exploits the information provided by the noise correlation matrix. For this purpose, MMSE-based cancelers, both linear and nonlinear, are the natural choices. The linear MMSE canceler is a very simple structure but it significantly outperforms the linear zero-forcing canceler in the presence of spatially correlated noise. However it is still a suboptimal canceler. The MMSE-based Generalized Decision Feedback Equalizer (MMSE-GDFE), is shown to perform optimally under spatially correlated background noise [1]. It is however more complex than the linear canceler.

The complexity of the crosstalk cancellation increases with the number of lines in the DSL system. Therefore, for a large number of lines it becomes computationally infeasible. To tackle this problem partial crosstalk cancellation can again be used. One of the advantages of using a zero-forcing canceler is that there exists a very simple strategy to perform partial crosstalk cancellation. However, for MMSE-based cancelers there is no obvious strategy that can be followed to realize partial cancellation. Earlier approaches [9], [10] adopt some per-line/per canceler exhaustive search method, which is computationally complex.

In this paper, firstly, we show that the MMSE-based cancelers, i.e., the linear MMSE canceler and the MMSE-GDFE, perform significantly better than the zero-forcing canceler in the presence of spatially correlated background noise. Secondly, we present simple algorithms to perform partial crosstalk cancellation with these MMSE-based cancelers. In section II, the upstream VDSL system model is presented and the crosstalk cancellation problem in the presence of spatially

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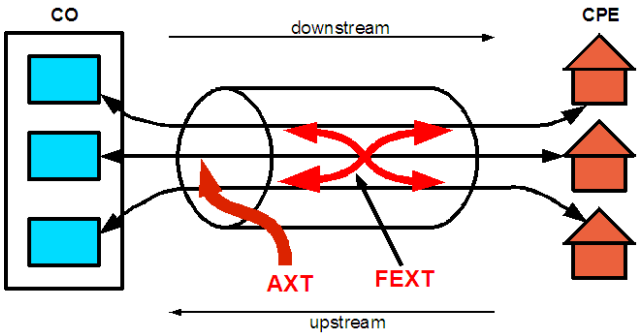


Fig. 1. A typical VDSL scenario with co-located receivers at the CO (central office) and non co-located transmitters at the CPE (customer-premises equipment) side

correlated background noise is defined. In section III the MMSE-based crosstalk cancellation techniques are reviewed and assessed. In section IV, efficient algorithms for partial crosstalk cancellation with these MMSE-based cancelers are proposed. Simulation results are presented in section V. Finally, in section VI conclusions are drawn.

II. System model

In this paper, we consider DMT-based upstream VDSL transmission, where the receivers are co-located as shown in **Fig.** 1. The system is first described by

$$\underbrace{\begin{bmatrix} y_1^k \\ \vdots \\ y_N^k \end{bmatrix}}_{\mathbf{y}^k} = \underbrace{\begin{bmatrix} h_{(1,1)}^k & \cdots & h_{(1,N)}^k \\ \vdots & \ddots & \cdots \\ h_{(N,1)}^k & \cdots & h_{(N,N)}^k \end{bmatrix}}_{\mathbf{H}^k} \underbrace{\begin{bmatrix} x_1^k \\ \vdots \\ x_N^k \end{bmatrix}}_{\mathbf{x}^k} + \underbrace{\begin{bmatrix} n_1^k \\ \vdots \\ n_N^k \end{bmatrix}}_{\mathbf{n}^k}, \quad (1)$$

where k is the tone index ($k = 1 \cdots K$), x_n^k and y_n^k are the transmitted and received symbols in line n on tone k and n_n^k is the additive spatially white Gaussian background noise (AWGN) experienced by line n on tone k, such that $E\{\mathbf{n}^k \mathbf{n}^{kH}\} = \sigma_k^2 \mathbf{I}_N$, where $\{.\}^H$ denotes the Hermitian transpose operation and $E\{.\}$ denotes the expected value operation. Finally, $h_{(n,n)}^k$ is the direct channel for line n on tone k and $h_{(n,m)}^k$ is the crosstalk channel from line m into line n. Let $E\{\mathbf{x}^k \mathbf{x}^{kH}\} = \mathbf{S}^k$. Since the transmitters are not coordinated, $\mathbf{S}^k$ is a diagonal matrix. Furthermore it is assumed that the transmitters have a fixed transmit spectrum, which can not be changed during the data transmission. Here, the channel matrix $\mathbf{H}^k$ is CWDD [6].

In the presence of alien crosstalk, the above model is generalized as equation (2) (on the top of the following page). In (2), $\mathbf{a}^k$ is the vector containing the sum of all the alien crosstalk contributions. Hence (2) can be succinctly written as

$$\begin{aligned} \mathbf{y}^k &= \mathbf{H}^k \mathbf{x}^k + \underbrace{\mathbf{a}^k + \mathbf{n}^k}_{\mathbf{z}^k} \\ &= \mathbf{H}^k \mathbf{x}^k + \mathbf{z}^k \end{aligned} \quad (3)$$

Even though the individual alien crosstalk channels are not known at the receive side, it is assumed that the total noise correlation matrix $\mathbf{R}_z^k = E\{\mathbf{z}^k \mathbf{z}^{kH}\}$ is known or can be estimated. Generally, the alien crosstalk is spatially correlated therefore the correlation matrix $\mathbf{R}_z^k$ is not a diagonal matrix.

III. MMSE-based crosstalk cancellation

The upstream VDSL scenario with receiver coordination and no transmitter coordination corresponds to the multiple access channel (MAC) in information theory. The capacity of this MAC in the AWGN case can be written as [6]

$$b = \sum_k \log_2 |\mathbf{I}_N + \sigma_k^{-2} \mathbf{H}^k \mathbf{S}^k \mathbf{H}^{kH}| \quad (4)$$

In [6] it was shown that due to the CWDD structure of the channel matrix $\mathbf{H}^k$, a simple linear zero-forcing canceler can be used to obtain near-optimal capacity approaching performance.

In the case of alien crosstalk, the background noise is spatially correlated. The capacity formula then includes a prewhitening operation and can be written as [8]

$$\tilde{b} = \sum_k \log_2 |\mathbf{I}_N + \underbrace{((\mathbf{R}_z^k)^{-1/2} \mathbf{H}^k)}_{\tilde{\mathbf{H}}^k} \mathbf{S}^k ((\mathbf{R}_z^k)^{-1/2} \mathbf{H}^k)^H|, \quad (5)$$

where $\tilde{\mathbf{H}}^k$ is the equivalent channel matrix after prewhitening. This equivalent channel matrix is no longer CWDD and hence a zero-forcing canceler no longer provides near-optimal performance. We therefore consider MMSE-based cancelers, which take the noise correlation matrix into account explicitly.

A. Linear MMSE canceler

The linear MMSE canceler output can be written as [11]

$$\hat{\mathbf{x}}^k = decision\left(\mathbf{W}^k \mathbf{y}^k\right), \quad (6)$$

where

$$\mathbf{W}^k = \mathbf{S}^k \mathbf{H}^{kH} (\mathbf{H}^k \mathbf{S}^k \mathbf{H}^{kH} + \mathbf{R}_z^k)^{-1} \quad (7)$$

is the $N \times N$ linear MMSE canceler matrix. Unlike the zero-forcing canceler, the linear MMSE canceler takes the noise correlation matrix into account. This enables the linear MMSE canceler to cancel the alien crosstalk much more efficiently than the zero-forcing canceler. In a high signal-to-noise ratio (SNR) AWGN scenario, the linear MMSE canceler approaches the zero-forcing canceler.

B. Nonlinear MMSE canceler

The linear MMSE canceler in general performs suboptimally. A non-linear MMSE canceler such as the MMSE-GDFE [1], [4], [5], [18] and the MMSE-VBLAST [16] achieves optimal performance by feeding back the already detected symbols to remove the corresponding interference.

$$\begin{bmatrix} y_1^k \\ \vdots \\ y_N^k \end{bmatrix} = \begin{bmatrix} h_{(1,1)}^k & \cdots & h_{(1,N)}^k \\ \vdots & \ddots & \cdots \\ h_{(N,1)}^k & \cdots & h_{(N,N)}^k \end{bmatrix} \begin{bmatrix} x_1^k \\ \vdots \\ x_N^k \end{bmatrix} + \underbrace{\begin{bmatrix} h_{(1,axt1)}^k & \cdots & h_{(1,axtm)}^k \\ \vdots & \ddots & \cdots \\ h_{(N,axt1)}^k & \cdots & h_{(N,axtm)}^k \end{bmatrix} \begin{bmatrix} x_{axt1}^k \\ \vdots \\ x_{axtm}^k \end{bmatrix}}_{\text{Alien Crosstalk } (\mathbf{a}^k)} + \begin{bmatrix} n_1^k \\ \vdots \\ n_N^k \end{bmatrix}. \quad (2)$$

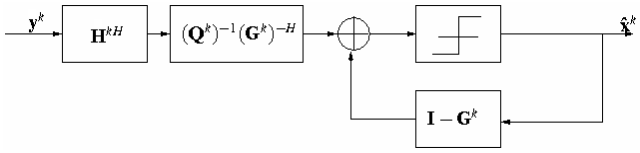


Fig. 2. MMSE-GDFE

1) MMSE-GDFE: **Fig.** 2 shows the MMSE-GDFE structure described in [1], [4], [5]. The feedforward filter $\mathbf{W}_f^k$ of the MMSE-GDFE can be written as

$$\mathbf{W}_f^k = (\mathbf{Q}^k)^{-1}(\mathbf{G}^k)^{-H}\mathbf{H}^{kH} \quad (8)$$

where $\mathbf{Q}^k$ is a diagonal matrix and $\mathbf{G}^k$ is a monic lower triangular matrix. Here $\mathbf{Q}^k$ and $\mathbf{G}^k$ can be obtained from the Cholesky factorization of

$$(\bar{\mathbf{R}}^k)^{-1} = ((\mathbf{S}^k)^{-1} + \mathbf{H}^{kH}(\mathbf{R}_z^k)^{-1}\mathbf{H}^k) \quad (9)$$

i.e.

$$(\bar{\mathbf{R}}^k)^{-1} = \mathbf{G}^k\mathbf{Q}^k\mathbf{G}^{kH}. \quad (10)$$

The lower triangular structure of $\mathbf{G}^k$ defines a detection order (line 1 is detected first, line N is detected last). When different lines have different priorities, the highest priority line should be detected last [15]. In [4], [5], it was shown that the MMSE-GDFE performs optimally in the spatially correlated noise scenario.

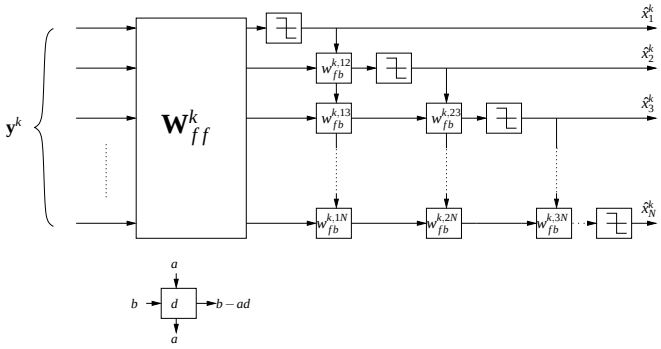


Fig. 3. MMSE-VBLAST structure

2) MMSE-VBLAST: In the MMSE-VBLAST [16], the interference from the already detected lines is subtracted from the received vector and then a MMSE linear canceler is applied to reduce the interference from the yet undetected lines. Let us define the channel $\mathbf{H}^k = [\mathbf{h}_1^k \cdots \mathbf{h}_N^k]$. The MMSE-VBLAST detection algorithm is given as,

For $n = 1 \cdots N$

$$\hat{x}_n^k := decision\left(\mathbf{w}_{ff,n}^k(\mathbf{y}^k - \sum_{l=1}^{n-1}\mathbf{h}_l^k\hat{x}_l^k)\right) \quad (11)$$

The feedforward filter is

$$\mathbf{w}_{ff,n}^k = s_n^k\mathbf{h}_n^{kH}([\mathbf{h}_n^k \cdots \mathbf{h}_N^k]\mathbf{S}_{N-n}^k[\mathbf{h}_n^k \cdots \mathbf{h}_N^k]^H + \mathbf{R}_z^k)^{-1}, \quad (12)$$

where s_n^k is the n-th diagonal element of matrix $\mathbf{S}^k$, $\mathbf{S}_{N-n}^k$ is an $(N-n)\times(N-n)$ diagonal matrix containing the power transmitted on lines n to line N and $\hat{x}_n^k$ is the estimate of x_n^k.

For $n = 1 \cdots N$

$$\hat{x}_n^k := decision\left(\mathbf{w}_{ff,n}^k\mathbf{y}^k - \sum_{l=1}^{n-1}\underbrace{\mathbf{w}_{ff,n}^k\mathbf{h}_l^k}_{\triangleq \mathbf{w}_{fb}^{k,ln}}\hat{x}_l^k\right), \quad (13)$$

where the decision feedback operation is again applied at the end as indicated in the **Fig.** 3. In [17], the MMSE-GDFE and the MMSE-VBLAST were shown to be equivalent. Due to the coupling of the feedforward and the feedback section of the MMSE-GDFE, however the partial cancellation problem becomes complicated, while the structure of the MMSE-VBLAST (**Fig.** 3) simplifies the partial cancellation problem. In the sequel, we will collectively refer to the nonlinear MMSE cancelers as "MMSE-GDFE", and use (12) and (13) as the basis for further derivation.

C. Performance

Fig. 4 plots a direct channel characteristic for a 1200m line and also shows the in-domain crosstalk from 6 other lines ranging from 600m to 1100m with increments of 100m, as simulated using the analytical channel model given in [14]. The dashed line gives an example of a typical alien crosstalk characteristic.

In the absence of alien crosstalk, it can be seen from **Fig.** 5(a) that the performance (i.e. total bit rate summed over 7 lines) of the zero-forcing canceler, the linear MMSE canceler and the MMSE-GDFE are similar. As can be seen from **Fig.** 5(b), as soon as alien crosstalk is introduced the performance decreases for the linear cancelers. The performance of the zero-forcing canceler is extremely poor whereas the linear MMSE canceler still provides relatively good performance. The best performance is given by the MMSE-GDFE. This is our motivation for using the MMSE-based cancelers. Even though the performance of the linear MMSE canceler is lower compared to the MMSE-GDFE, it has a lower complexity and

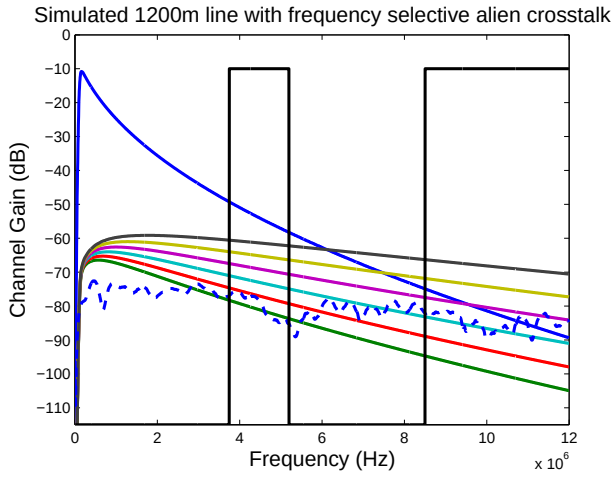
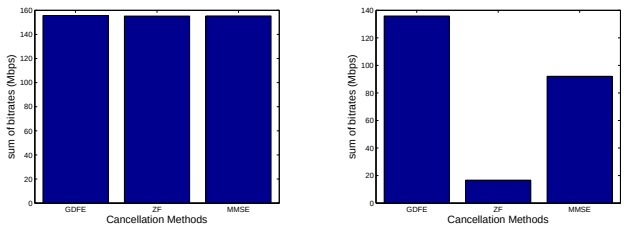


Fig. 4. Channel characteristics for a 1200m line with 6 in-domain crosstalkers ranging from 600m to 1100m and an alien-crosstalker (dashed line). The solid black rectangular line represents the transmit band for upstream VDSL.



(a) (b)

Fig. 5. Comparison of the total bit rate achieved with the zero-forcing canceler and MMSE-based cancelers when alien crosstalk is (a) not present (b) present.

hence represents an attractive intermediate solution. Even for the linear MMSE canceler, however, the complexity of full crosstalk cancellation is still $\mathcal{O}(N^2)$ [7], hence for a large number of lines the crosstalk cancellation becomes computationally heavy. This provides the motivation for developing partial crosstalk cancellation algorithms.

IV. MMSE-based Partial Crosstalk Cancellation

In [7], a simple partial crosstalk cancellation scheme for the zero-forcing canceler was presented, which exploits the spatial and frequency selectivity of the crosstalk. Only strong crosstalkers are canceled which reduces complexity significantly without any dramatic performance degradation. For the MMSE-based cancelers, however, there is no obvious strategy that can be followed to realize partial cancellation. Before delving into different schemes to select the crosstalkers to be canceled, we first establish how the partial crosstalk canceler coefficients can be computed once the lines have been selected that are used for the cancellation.

A. Computation of linear MMSE partial crosstalk canceler coefficients

Let us assume that there are N lines, therefore there are $N-1$ in-domain crosstalkers to be canceled for each individual line. Let $p_{k,n}$ be the number of observed lines, i.e., used by the crosstalk canceler, to detect the signal in line n on tone k, where $p_{k,n} = N-1$ corresponds to full crosstalk cancellation and $p_{k,n} = 0$ corresponds to no crosstalk cancellation i.e. single user detection (SUD). Let $\mathcal{M}_n^k$ be the set of indices of all the observed lines including the index of line n, i.e.,

$$\mathcal{M}_n^k \triangleq \{m_{k,n}(1) \cdots m_{k,n}(p_{k,n}+1)\}. \tag{14}$$

Similarly, let $\underline{\mathcal{M}}_n^k$ be the set of indices of the unobserved lines, i.e., lines that are not used to detect the signal in line n on tone k, i.e.,

$$\underline{\mathcal{M}}_n^k \triangleq \mathcal{N} \backslash \mathcal{M}_n^k, \tag{15}$$

where $\mathcal{N}$ is a set containing all indices $1 \cdots N$. Let $\overline{\mathbf{H}}_n^k$ be the channel matrix containing the channels corresponding to the observed lines, i.e.,

$$\overline{\mathbf{H}}_n^k \triangleq \left[[\mathbf{H}^k]_{rows \mathcal{M}_n^k, cols \mathcal{M}_n^k} \right], \tag{16}$$

and $\underline{\mathbf{H}}_n^k$ be the matrix containing all the crosstalk channels from the unobserved lines into the observed lines, i.e.,

$$\underline{\mathbf{H}}_n^k \triangleq \left[[\mathbf{H}^k]_{rows \mathcal{M}_n^k, cols \underline{\mathcal{M}}_n^k} \right]. \tag{17}$$

Similarly $\overline{\mathbf{R}}_n^k$ is defined as the noise correlation matrix of the observed lines, i.e.,

$$\overline{\mathbf{R}}_n^k \triangleq \mathbf{R}_{z,n}^k + \underline{\mathbf{H}}_n^k \underline{\mathbf{S}}_n^k \underline{\mathbf{H}}_n^{kH} \tag{18}$$

where $\underline{\mathbf{S}}_n^k$ is the diagonal matrix containing powers transmitted on the unobserved lines and $\mathbf{R}_{z,n}^k$ is the background noise correlation matrix for the observed lines. Now the linear MMSE partial crosstalk cancellation can be written as

$$\hat{x}_n^k = decision \left(\overline{\mathbf{w}}_n^k \left[\mathbf{y}^k \right]_{rows \mathcal{M}_n^k} \right), \tag{19}$$

where the partial cancellation coefficient vector $\overline{\mathbf{w}}_n^k$ can be calculated using (7) and (18) as

$$\overline{\mathbf{w}}_n^k \triangleq \overline{\mathbf{e}}_q^H \left(\mathbf{S}_n^k \overline{\mathbf{H}}_n^{kH} \left(\overline{\mathbf{H}}_n^k \mathbf{S}_n^k \overline{\mathbf{H}}_n^{kH} + \overline{\mathbf{R}}_n^k \right)^{-1} \right), \tag{20}$$

where $\overline{\mathbf{e}}_q \triangleq \left[\mathbf{I}_{p_{k,n}+1} \right]_{col\, q}$ represents the q-th column of the identity matrix of size $p_{k,n}+1$. Here, q is the position of line n in the set $\mathcal{M}_n^k$ and $\mathbf{S}_n^k$ is the diagonal matrix containing the powers transmitted on the observed lines.

B. Computation of MMSE-GDFE partial crosstalk canceler coefficients

Let $\mathcal{W}_n^k$ be the set of indices of all the observed lines by the feedforward section to detect the signal in line n on tone k, including line n, and let $\underline{\mathcal{W}}_n^k$ be the set of indices of the unobserved lines not used by the feedforward section. Similarly, let $\mathcal{B}_n^k$ be the set of indices of all the observed lines used by the feedback section to detect the signal in line n on tone k and let $\underline{\mathcal{B}}_n^k$ be the set of indices of the unobserved lines not used by the feedback section. The channel matrix corresponding to the observed lines in the feedforward section can be written as

$$\overline{\mathbf{H}}_{n,ff}^k \triangleq \left[[\mathbf{H}^k]_{rows \mathcal{W}_n^k, cols(\mathcal{W}_n^k \backslash \mathcal{B}_n^k)} \right]. \tag{21}$$

The crosstalk channel matrix corresponding to the unobserved lines can then be written as

$$\underline{\mathbf{H}}^k_{n,ff} \triangleq \left[[\mathbf{H}^k]_{rows \mathcal{W}^k_n, cols \underline{\mathcal{W}}^k_n \setminus \mathcal{B}^k_n} \right]. \quad (22)$$

Similarly $\overline{\mathbf{R}}^k_{n,ff}$ is defined as the noise correlation matrix of the observed lines, i.e.,

$$\overline{\mathbf{R}}^k_{n,ff} \triangleq \mathbf{R}^k_{z,n,ff} + \underline{\mathbf{H}}^k_{n,ff} \underline{\mathbf{S}}^k_{n,ff} \underline{\mathbf{H}}^{kH}_{n,ff} \quad (23)$$

where $\underline{\mathbf{S}}^k_{n,ff}$ is the diagonal matrix containing powers transmitted on the unobserved lines and $\mathbf{R}^k_{z,n,ff}$ is a background noise correlation matrix for the observed lines. The MMSE feedforward filter coefficient can then be computed as

$$\mathbf{w}^k_{ff,n} = \mathbf{e}^H_q \left(\mathbf{S}^k_{n,ff} \overline{\mathbf{H}}^{kH}_{n,ff} (\overline{\mathbf{H}}^k_{n,ff} \mathbf{S}^k_{n,ff} \overline{\mathbf{H}}^{kH}_{n,ff} + \overline{\mathbf{R}}^k_{n,ff})^{-1} \right), \quad (24)$$

where $\mathbf{e}_q$ is the $q-th$ column of the identity matrix whose size is defined by the cardinality of $\mathcal{W}^k_n$. here q is the position of the line n in the set $\mathcal{W}^k_n$ and $\mathbf{S}^k_{n,ff}$ is the diagonal matrix containing the power of the observed lines used by the feedforwaard section. With

$$\overline{\mathbf{h}}^k_{n,fb} \triangleq \left[[\mathbf{H}^k]_{row\, n, cols \mathcal{B}^k_n} \right]^T, \quad (25)$$

the feedback coefficient vector in line n on tone k can be written as

$$\mathbf{w}^{k,n}_{fb} = \mathbf{w}^k_{ff,n} \overline{\mathbf{h}}^k_{n,fb}. \quad (26)$$

Now the output of the MMSE-GDFE partial crosstalk canceler can be written as

$$\hat{x}^k_n = decision \left(\mathbf{w}^k_{ff,n} \mathbf{y}^k_{ff,n} - \mathbf{w}^{k,n}_{fb} \mathbf{x}^k_{fb,n} \right), \quad (27)$$

where $\mathbf{x} fb, n^k$ is a vector containing the detected symbols from all the observed lines used by the feedback section.

C. MMSE-based partial crosstalk cancellation problem

The partial crosstalk cancellation problem amounts to maximizing the total bitrate summed over all lines when using only C_{tot} cancellation taps, i.e., non-zero elements in the canceler matrices. This comes down to the following maximization problem,

$$\begin{aligned} &\max_{\mathbf{c}} \sum_{k=1}^{K} \sum_{n=1}^{N} \tilde{b}^n_k \\ &\text{s.t} \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{\substack{m=1 \\ n \neq m}}^{\tilde{N}} c^{n,m}_k \leq C_{tot}, \end{aligned} \quad (28)$$

where $\mathbf{c} = [\mathbf{c}_1, \mathbf{c}_2, ..., \mathbf{c}_K]$, $[\mathbf{c}_k]_{n,m} = c^{n,m}_k \in \{0, 1\}$.

- For the linear MMSE, $\tilde{N} = N$ therefore $\mathbf{c}_k$ is an $N \times N$ matrix containing the cancellation tap configurations $c^{n,m}_k$ on tone k. Here $c^{n,m}_k = 1$ indicates that a cancellation tap has been assigned to position (n, m), i.e., that line m is used for the detection of line n the tone k ($m \in \mathcal{M}^k_n$). Similarly, $c^{n,m}_k = 0$ indicates a cancellation tap has not been assigned to position (n, m) ($m \in \underline{\mathcal{M}}^k_n$). Finally, $\tilde{b}^n_k$ is the bit rate of line n on tone k given as

$$\tilde{b}^n_k \triangleq \log_2 \left(1 + \frac{1}{\Gamma} \frac{\left| \overline{\mathbf{h}}^k_n(q) \right|^2 s^k_n(q)}{\hat{n}} \right), \quad (29)$$

where Γ is the SNR gap, $\overline{\mathbf{h}}^k_n = \overline{\mathbf{w}}^k_n \overline{\mathbf{H}}^k_n$, $\overline{\mathbf{h}}^k_n(i)$ represents the i-th element of vector $\overline{\mathbf{h}}^k_n$, $\hat{n} = \sum_{i \neq q} \left| \overline{\mathbf{h}}^k_n(i) \right|^2 s^k_n(i) + Tr(\overline{\mathbf{w}}^k_n \overline{\mathbf{R}}^k_n \overline{\mathbf{w}}^{kH}_n)$, $Tr(\mathbf{X})$ takes the trace of the matrix $\mathbf{X}$ and $s^n_k(i)$ represents the i-th diagonal element of $\mathbf{S}^k_n$.

- For the MMSE-GDFE, the maximum feedback filter length is equal to N therefore the $\tilde{N} = 2N$. Hence $\mathbf{c}_k$ is a $N \times 2N$ matrix, where the N added columns account for the tap allocation in the feedback section. Due to the triangular structure of the feedback section the maximum number of non-zero elements in $\mathbf{c}_k$ is $N^2 + \frac{N(N-1)}{2}$. If $m \leq N$ then $c^{n,m}_k = 1$ indicates that a cancellation tap has been assigned to the feedforward section in position (n, m) ($m \in \mathcal{W}^k_n$) and when $m > N$ it indicates that a cancellation tap has been assigned to the feedback section in position $(n, m - N)$ ($m \in \mathcal{B}^k_n$). The bitrate of line n on tone k is given by

$$\tilde{b}^n_k \triangleq \log_2 \left(1 + \frac{1}{\Gamma} \frac{\left| \overline{\mathbf{h}}^k_n(q) \right|^2 s^k_n(q)}{\tilde{n}} \right), \quad (30)$$

where $\tilde{n} = \sum_{i \neq q} \left| \overline{\mathbf{h}}^k_n(i) \right|^2 s^k_n(i) + Tr(\overline{\mathbf{w}}^k_{f,n} \overline{\mathbf{R}}^k_n \overline{\mathbf{w}}^{kH}_{f,n}) - Tr(\mathbf{w}^{k,n}_{fb} \underline{\mathbf{S}}^k_{n,bb} \mathbf{w}^{k,nH}_{fb})$, $\underline{\mathbf{S}}^k_{n,bb}$ is the diagonal matrix containing powers transmitted on the observed fedback lines, $\overline{\mathbf{h}}^k_n = \overline{\mathbf{w}}^k_{f,n} \overline{\mathbf{H}}^k_n$, $\overline{\mathbf{h}}^k_n(i)$ represents the i-th element of vector $\overline{\mathbf{h}}^k_n$ and $s^n_k(i)$ represents the i-th diagonal element of $\underline{\mathbf{S}}^k_{n,ff}$.

The global optimum for (28) can be found by exhaustively searching over all possible cancellation tap configurations. However, the constraint is coupled over tones, which results in an exponential complexity in the number of tones. The overall complexity is then $\mathcal{O}(2^{KN(N-1)})$ for the linear MMSE canceler and $\mathcal{O}(2^{K\frac{3}{2}N(N-1)})$ for the MMSE-GDFE, which is generally intractable. This complexity can be made linear in K by using a dual decomposition approach. To achieve this, the constraint in the optimization problem is moved into the objective with the help of a Lagrange multiplier λ. This can then be written as

$$\begin{aligned} &\min_{\lambda} \left(\max_{\mathbf{c}} \left(\sum_{k=1}^{K} \sum_{n=1}^{N} \tilde{b}^n_k + \lambda (C^{tot} - \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{\substack{m=1 \\ m \neq n}}^{\tilde{N}} c^{n,m}_k) \right) \right) \\ &\text{subject to } \lambda \geq 0 \qquad n = 1 \cdots N \end{aligned} \quad (31)$$

In [12], for multicarrier systems with a sufficiently large number of tones, the duality gap was shown to be zero for this type of optimization problems. For a given λ, the optimization problem is decoupled over tones hence it can be solved for

each tone separately. The per-tone optimization can then be written as

$$\begin{aligned}&\text{For } k = 1 \cdots K \\ &\max_{\mathbf{c}_k} \sum_{n=1}^{N} \tilde{b}_k^n - \sum_{n=1}^{N} \sum_{\substack{m=1 \\ m \neq n}}^{\tilde{N}} \lambda c_k^{n,m}.\end{aligned} \tag{32}$$

The maximization of (32) can be performed by an exhaustive search over all possible $\mathbf{c}_k$'s. Additionally, the constraint can be enforced by tuning λ. An algorithm to efficiently tune the Lagrange multiplier is given in [13]. An exhaustive search of the partial crosstalk canceler configuration still has a complexity of $\eta K \mathcal{O}(2^{N(N-1)})$ for the linear MMSE canceler and $\eta K \mathcal{O}(2^{\frac{3}{2}N(N-1)})$ for the MMSE-GDFE, where η is the number of iterations required for the tuning of λ. However, this complexity can be further reduced based on two properties, namely line independence and line selection [13]:

1) Line independence: For each line a crosstalk cancellation configuration has to be decided on. However, from (29) and (30), it can be seen that when it is decided to observe line m for the detection of line n, this obviously only affects the performance of line n. Hence, (32) can be further decoupled over lines, leading to

$$\begin{aligned}&\text{For } k = 1 \cdots K \\ &\text{For } n = 1 \cdots N \\ &\max_{\mathbf{c}_k(n,:)} \tilde{b}_k^n - \sum_{m=1}^{\tilde{N}} \lambda c_k^{n,m}.\end{aligned} \tag{33}$$

This reduces the complexity from $\eta K \mathcal{O}(2^{N(N-1)})$ to $\eta K N \mathcal{O}(2^{(N-1)})$ for the linear MMSE canceler and from $\eta K \mathcal{O}(2^{\frac{3}{2}N(N-1)})$ to $\eta K N \mathcal{O}(2^{\frac{3}{2}(N-1)})$ for the MMSE-GDFE.

2) Line selection: For the linear MMSE canceler, each line n it has to be decided whether or not $N-1$ other lines are observed. Hence, there are 2^{N-1} possible crosstalk cancellation configurations for each line. Similarly, for the MMSE-GDFE there are $2^{(N-1+l)}$ possible crosstalk cancellation configurations for each line, where l is the number of previously decoded lines, i.e., $l = 1 \cdots (N-1)$. If $\bar{q}$ cancellation taps are available then these should be allocated such that the effectively canceled interference is maximized. For a line n if we can define a metric, $\mathcal{I}^{n,k}$ such that the $N-1$ cancellation taps for the linear MMSE canceler or $(N-1+l)$ cancellation taps for the MMSE-GDFE can be ordered, then $\bar{q}$ lines can be selected that have the largest values for this metric. In this fashion the number of possible crosstalk configurations reduces from 2^{N-1} to $N-1$ for the linear MMSE canceler and from $2^{(N-1+l)}$ to $(N-1+l)$ for the MMSE-GDFE, and so the overall complexity becomes $\eta K N(N-1)$ for the linear MMSE canceler and $\eta K \frac{3}{2} N(N-1)$ for the MMSE-GDFE. Hence the optimization problem can be written as

$$\begin{aligned}&\text{For } k = 1 \cdots K \\ &\text{For } n = 1 \cdots N \\ &\max_{\mathbf{c}_k(n,:)} \tilde{b}_k^n(\bar{q}) - \lambda \bar{q}.\end{aligned} \tag{34}$$

where $\tilde{b}_n^k(\bar{q})$ is the bit rate when $\bar{q}$ lines are observed and selected based on the selection metric $\mathcal{I}_n^k$. A simple algorithm description is provided as Alg. 1.

Algorithm 1 MMSE-based partial crosstalk canceler

```
initialize b̃_k^n(q̄), λ^0
while C ≠ C_tot do
    for tone k = 1 ··· K do
        for line n = 1 ··· N do
            compute metric I^{n,k}
            c̄_n^k = argmax_{q̄} b̃_k^n(q̄) − λ^t q̄
        end for
    end for
    compute C = Σ_k Σ_n c̄_n^k
    update λ^{t+1} = λ^t + μ(C − C_tot)
end while
```

For the ZF canceler, the metric ($\mathcal{I}_{ZF}^{n,k}$) can be easily defined based only on the interference power [7]. However, in the presence of spatially correlated background noise, the metric can not be defined so easily for the MMSE-based cancelers . In the next section, we address the issue of defining a suitable metric for the linear MMSE canceler and for the MMSE-GDFE.

D. Selection metric for linear MMSE partial crosstalk cancelers

For the linear MMSE canceler, a first option is to use the metric used in the case of the ZF canceler based only on the in-domain crosstalk power and the information about the spatially correlated background noise is ignored. So the strategy is to cancel the strongest in-domain crosstalker first. This first metric can be written as

$$\mathcal{I}_1^{n,k} = max\{|h_{(n,m)}^k|^2 s_m^k\}, \quad \text{where } m = 1 \cdots N,\ m \neq n \tag{35}$$

where $max\{x\}$ returns the index of the largest value contained in the set x. This is a good strategy when the alien crosstalk is white/nearly white or very small. However, if the background noise is spatially correlated and strong then this metric may not provide good results. In such a scenario, instead of the in-domain crosstalk information, the spatial correlation of the background noise can be used as a selection metric. This second metric can thus be defined as

$$\mathcal{I}_2^{n,k} = max\{r_{(n,m)}^k\}, \quad \text{where } m = 1 \cdots N,\ m \neq n \tag{36}$$

where $r^k_{(i,j)}$ is the entry in the i-th row and the j-th column in the noise correlation matrix $\mathbf{R}^k$ on tone k. If the alien crosstalk is much stronger than the in-domain crosstalk then this metric provides a good cancellation order. In the scenario, where both the in-domain crosstalk and the alien crosstalk are large, a metric can be defined that combines the available information about the in-domain crosstalk power and the background noise correlation. For line n, the total information provided by the background noise correlation can be written empirically as follows

$$n^{n,m,k}_{emp} \triangleq \underbrace{r^k_{(n,n)}\left(\frac{r^k_{(n,m)}}{\sum_{\substack{i\\ i\neq n}} r^k_{(n,i)}}\right)}_{I}\underbrace{\left(\frac{r^k_{(m,m)}}{|h_{(m,m)}|^2 s^k_m}\right)}_{II}. \quad (37)$$

Part I of (37) yields the amount of alien crosstalk that can be canceled if line m is selected. Part II accounts for the fact that the effective alien crosstalk that can be canceled also depends on the signal-to-interference and noise ratio (SINR) of line m.The third metric is then defined as

$$\mathcal{I}^{n,k}_3 = max\{n^{n,m,k}_{emp} + |h^k_{(n,m)}|^2 s^k_m\}, \quad (38)$$
$$\text{where} \qquad m = 1\cdots N, \; m \neq n$$

The performance achieved with $\mathcal{I}^{n,k}_3$ is found to be close to the performance achieved with the optimum tap selection scheme, i.e., obtained with an exhaustive search (see section V).

E. Selection metric for MMSE-GDFE partial crosstalk cancelers

In the MMSE-GDFE, in addition to a selection metric for the feedforward path we also need a selection metric for the feedback path. If the detection order is fixed, only already decoded lines can be used in the feedback. The feedforward path in the MMSE-GDFE is basically a linear MMSE canceler. Therefore the selection metric for the feedforward section can also be given by (38). The feedback section in the MMSE-GDFE can only cancel the in-domain interference from the already decoded lines, hence the selection metric for the feedback section can be given as (35). Combining these selection metrics, the final selection metric for the MMSE-GDFE can be written as

$$\mathcal{I}^{n,k}_4 = max\{\mathcal{I}^{n,k}_3, |h^k_{(n,g)}|^2 s^k_g\}, \quad \text{for } g = 1\cdots l, \; g \neq n, \quad (39)$$

where g represents previously detected lines. The performance achieved with $\mathcal{I}^{n,k}_4$ is found to be close to the performance achieved with the optimum tap selection scheme, i.e., obtained with an exhaustive search (see section V).

V. Simulation Results

Simulations were run for a scenario with 7 lines ranging from 600m to 1200m in 100m increments and an alien crosstalk line of 500m. The direct channels, the crosstalk channels and the alien crosstalk channel are generated using the analytical and semi-analytical model in [14]. The simulation parameters are summarized in **Table** I. The maximum number of taps that can be allocated to the linear MMSE canceler is $N \times (N-1) \times$ number of used tones. Here $N = 7$ and the number of used tones is 1147 hence the C^{max}_{tot} is 48174. With the additional lower triangular feedback section the C^{max}_{tot} in the case of MMSE-GDFE is 72261. For the linear MMSE canceler, we compare the partial crosstalk canceler selection based on metric (35), (36) and (38) with the selection obtained with an exhaustive search. The exhaustive search strategy provides a performance upper bound but has a much larger complexity. The results are shown in **Fig.** 6. It can be seen that the selection metric $\mathcal{I}^{n,k}_3$, which takes both the in-domain interference as well as the background noise information into account performs very close to the upper bound. From **Fig.** 5 (b), it can be seen that even when full crosstalk cancellation is used, the ZF-canceler achieves only 16.7 Mbps which is even well below the single user performance ($C_{tot} = 0$) of the linear MMSE.

In the case of MMSE-GDFE, the detection order is assumed to be fixed for each tone. **Fig.** 7 compares the performance of the exhaustive search based linear MMSE partial crosstalk canceler with the performance of an exhaustive search based MMSE-GDFE partial crosstalk canceler and the performance of a partial crosstalk canceler selected using the metric defined in (39). It can be seen that the performance of the partial

TABLE I
Simulation Parameters

Number of DMT tones	4096
Tone width	4.3125 kHz
symbol rate	4 kHz
Coding gain	3 dB
Noise margin	6 dB
Symbol error probability	$< 10^{-7}$
Transmit PSD	Flat (-60 dBm/Hz)
FDD band plan	998
Cable type	0.5 mm

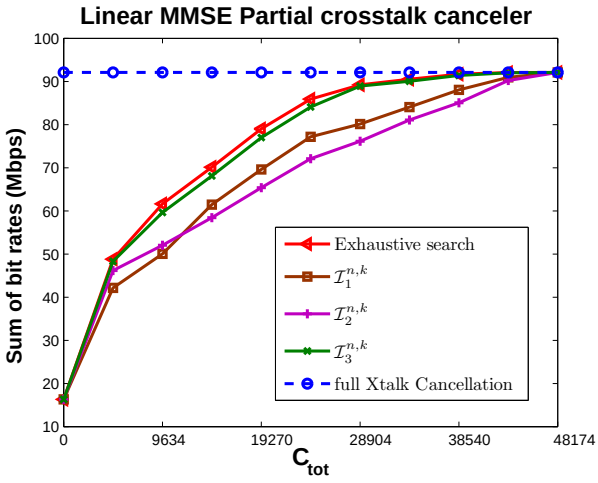


Fig. 6. Performance comparison of the different metrics for partial cancellation using a linear MMSE canceler

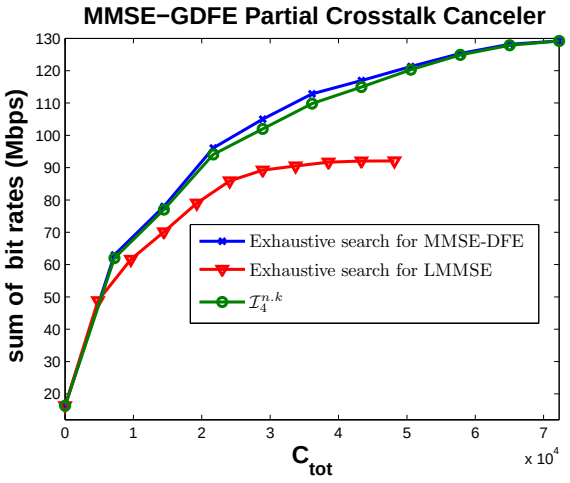


Fig. 7. Comparison between the optimum linear MMSE partial cancellation and the MMSE-GDFE partial cancellation

crosstalk canceler selected using the metric $\mathcal{I}_4^{n,k}$ performs near optimally. Furthermore, it can also be seen that the MMSE-GDFE partial crosstalk canceler consistently outperforms the linear MMSE partial crosstalk canceler for the same number of canceler taps.

VI. Conclusion

In this paper, we have considered crosstalk cancellation in upstream VDSL, where receivers are co-located, in the presence of spatially correlated background noise such as alien crosstalk . We have shown that MMSE-based crosstalk cancelers then perform significantly better than a zero-forcing crosstalk canceler. We have then developed partial crosstalk cancelers, both linear MMSE based and MMSE-GDFE based, which exploit a selection metric. It was demonstrated that a linear MMSE partial crosstalk canceler based on a tap selection metric that exploits either only in-domain crosstalk power information or only noise correlation does not perform well in the presence of spatially correlated background noise. Therefore, a simple alternative metric incorporating the information from both the in-domain crosstalk power and the noise correlation was proposed. It was also shown that a linear MMSE partial crosstalk canceler based on this metric performs near optimally. Furthermore, a simple extension of this algorithm was shown to provide a good strategy to achieve MMSE-GDFE partial crosstalk cancellation. The performance was again shown to be near optimal. Furthermore, for the same number of cancellation taps it was shown that MMSE-GDFE partial crosstalk cancellation outperforms the linear MMSE partial crosstalk cancellation.

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