Katholieke Universiteit Leuven

Departement Elektrotechniek

ESAT-SISTA/TR 08-126

JOINT TRANSMIT POWER AND FILTER TAP ALLOCATION IN DMT TRANSMITTERS WITH PER-TONE PULSE SHAPING [1]

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September 2008
Accepted for publication in Global Communications Conference (GLOBECOM), 2008

[1]This report is available by anonymous ftp from *ftp.esat.kuleuven.ac.be* in the directory *pub/sista/pkumarpa/reports/IRglobecom2008.pdf*

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Abstract

Per-tone pulse shaping has been proposed as an alternative to time domain spectral shaping for Discrete Multi-Tone (DMT) transmitters, e.g. VDSL modems. This enables the transmitter to use more tones without violating the Power Spectral Density (PSD) mask constraint for data transmission. The computational complexity of the per-tone pulse shaping and transmit power is evenly distributed over tones, however, resources (computational complexity and power) can be better exploited by using different filter lengths for different tones and by resorting to power loading. For a fixed pulse shaping filter length, the contribution of a particular tone to the stop band energy depends on the power allocated to the tone and on the distance of the tone from the band edges. The use of high order pulse shaping filters for the tones at the band edges (as well as the use of lower power) will reduce their contribution to the out of band PSD, whereas for the tones at the middle of the band these factors will have less effect on the out of band PSD. Therefore, the combination of both power loading and a variable length pulse shaping filter can be used to achieve a high data rate under resource and PSD constraints. In this paper we present an algorithm to optimally allocate the resources i.e. power and filter taps, using a dual problem formulation. This solves the problem of optimally distributing power and filter taps over tones for a given PSD mask constraint, with a relatively low complexity.

Joint transmit power and filter tap allocation in DMT transmitters with per-tone pulse shaping

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I. Introduction

Very high speed Digital Subscriber Line (VDSL) modems use Discrete Multi-Tone (DMT) modulation [1]. DMT divides the available spectrum into smaller parallel sub-bands or tones. Each tone corresponds to an orthogonal carrier. The input bit-stream is divided into several independent parallel streams which then QAM-modulate the different carriers. These QAM modulations are implemented based on an Inverse Discrete Fourier Transform (IDFT) [2]. A cyclic prefix is added to each resulting time domain symbol before transmission. If the cyclic prefix is too short namely shorter than the channel impulse response, then this results in Inter-Symbol Interference (ISI) and Inter-Carrier Interference (ICI). Highly dispersive channels such as the VDSL channel have a very long channel impulse response hence to mitigate ISI/ICI a very long cyclic prefix is needed.

This research work was carried out at the ESAT laboratory of the Katholieke Universiteit Leuven, in the frame of K.U.Leuven Research Council CoE EF/05/006 Optimization in Engineering and the Marie-Curie Fellowship EST-SIGNAL program (http://est-signal.i3s.unice.fr) under contract No. MEST-CT-2005-021175. The scientific responsibility is assumed by its authors.

The Discrete Fourier Transform (DFT) filters used to implement the DMT transmission have a poor frequency response. Firstly, the first side lobe is only 13 dB below the main lobe and secondly the rate of decay of the side lobe energy is only inversely proportional to frequency. In some applications e.g. in VDSL, the PSD of the transmitted signal is not allowed to exceed a predefined PSD mask, defining one or more pass bands and stop bands. In order to satisfy this constraint, many tones near the band edges can not be used, which then significantly reduces the available bandwidth for data transmission. One of the approaches to improve spectral characteristics is the time domain windowing of the transmit signal [3]. Recently, Phoong et al. proposed a per-tone pulse shaping method to reduce the energy in the stop band [4]. It uses a filter to shape the pulse for each tone separately. It was shown that by using this technique the number of tones that are available for data transmission can be significantly increased, compared to the case where time domain pulse shaping is used. In [4], a simple procedure for designing the pulse shaping filter is proposed, leading to a simple transmitter structure. Thus a pulse shaping filter is designed for each tone such that its stop band energy contribution is minimized. This method allows for asymmetric filters which helps in reducing the stop band energy of the tones at the band edges. This in turn helps to increase the number of tones that can be used for data transmission without violating the PSD mask constraint. The resulting transmitter structure was shown to be a dual of the per-tone equalization structure [5], where the implementation complexity of the transmitter is reduced significantly through the usage of sliding FFT operations with so called difference terms. This kind of transmit signal filtering has the same effect as the channel filtering and therefore the length of the cyclic prefix has to be increased in order to mitigate ISI [6], [7].

In DMT transmitter using per-tone pulse shaping [4], every tone is shaped using a shaping filter of fixed length and every tone is transmitted with the same amount of power. However for a fixed transmit power, tones in the middle of the pass band contribute less to the overall stop band energy and therefore a higher order pulse shaping filter does not improve the performance significantly. Therefore using a constant length pulse shaping filter for all tones may correspond to a waste of system resources. A better alternative is then to consider a

variable length pulse shaping filter for every tone. It is also pointed out in [4] that a variable length pulse shaping filter will further reduce the complexity of the transmitter. Furthermore, the number of tones available for data transmission can be increased by using a lower transmit power at the band edges which then helps increasing the achievable data rate without violating the PSD mask constraint.

It is seen that efficient methods are needed to distribute the filter taps as well as transmit power among the tones, such that a maximum data rate is achieved under resource constraints (total power and total number of filter taps) and without violating the PSD mask constraint.The Lagrange multiplier technique is used to convert a constraint optimization problem into an unconstrained problem [8], [9]. The resource allocation problem can thus be turned into a dual problem using Lagrange multipliers. This method has been used previously to optimally allocate resources in a DMT receiver with per-tone equalization [10] as well as in a DMT transmitter without the resource (i.e. transmit power and filter tap budget) constraint [11]. In this paper, an optimal resource allocation algorithm is proposed to jointly distribute the resources i.e. total transmit power and the pulse shaping filter tap budget (complexity) under a PSD constraint which is based on a dual problem formulation. In section II, a system model of the DMT transmitter employing per-tone pulse shaping is presented. In section III, the resource allocation problem is formulated, which can be used to compute the optimal transmit power and number of filter taps for each tone under a PSD mask constraint. An algorithm to solve this optimization problem will then be presented. Section IV contains some simulation results. Finally conclusion are presented in section V.

II. System Model

The following notation is adopted in the description of the DMT system. $\{.\}^T$ denotes the transpose, N is the size of the IDFT and i denotes the tone index. L is the order of the i-th pulse shaping filter, η is the cyclic prefix length. $\mathbf{x}_l$ is the output of the transmitter i.e. a vector of length $K = N+L+\eta$, corresponding to one transmitted symbol.

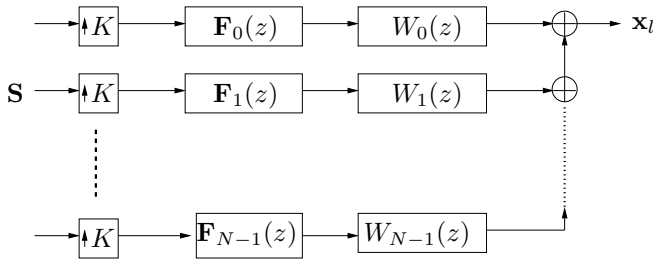

Fig. 1. DMT transmitter with per-tone shaping filter

Figure 1 shows the transmitter structure of the DMT with the per-tone shaping as derived in [4]. The filters $F_i(z)$ are the DFT filters defined as

$$F_i(z) = e^{-j2\pi i\eta/N} \sum_{l=0}^{N+\eta-1} e^{j2\pi il} z^{-l}.$$

The per-tone shaping filters, $W_i(z)$, are designed using a minimum stop band energy criterion. For simplicity we will adopt the following design criterion, presented in [4], for every filter $W_i(z)$:

$$\begin{aligned} \min_{\mathbf{w}_i} \quad & (1-\beta)\int_{\Omega 1} |F_i(e^{j\omega})W_i(e^{j\omega})|^2 d\omega \\ & +\beta \int_{\Omega 2} |F_i(e^{j\omega})W_i(e^{j\omega})|^2 d\omega \\ \text{subject to} \quad & W_i(e^{j2\pi i/N}) = 1 \end{aligned}$$

where
$\Omega 1$=stop band frequencies as defined by PSD mask
$\Omega 2$=pass band frequencies as defined by PSD mask
$W_i(e^{j\omega}) = \mathbf{e}^H \mathbf{w}_i$
$\mathbf{e}^H = \begin{bmatrix} 1 & e^{j\omega} \cdots e^{jL\omega} \end{bmatrix}$
$\mathbf{w}_i = [w_i^0 \; w_i^1 \; \cdots \; w_i^L]^T$
and where β is a small regularization constant $0 < \beta = \epsilon < 1$. This is included to prevent the stop band power from amplifying which can result in a significant increase of transmitting power [4]. The design criterion can be restated as

$$\min_{\mathbf{w}_i} \quad \mathbf{w}_i^H \mathbf{Q}_i \mathbf{w}_i \quad \text{subject to} \quad W_i(e^{j2\pi i/N}) = 1 \qquad (1)$$

where,
$\mathbf{Q}_i = (1-\beta)\int_{\Omega 1} |F_i(e^{j\omega})|^2 \mathbf{e}\mathbf{e}^H \, d\omega + \beta \int_{\Omega 2} |F_i(e^{j\omega})|^2 \mathbf{e}\mathbf{e}^H \, d\omega$

This is a linearly constrained least squares problem, which is easily solved for every i and for every filter order L.

From **Figure 1** it is seen that the transmitter output is

$$\mathbf{x}_l = \sum_i s_i \mathbf{f}_i \star \mathbf{w}_i \qquad (2)$$

where $\star$ denotes the convolution operation and s_i is the input signal at tone i, $\mathbf{f}_i$ is the DFT filter and $\mathbf{w}_i$ is the shaping filter for tone i. In [4] it has been shown that this transmitter output can be generated cheaply based on as

$$\mathbf{x}_l = \begin{pmatrix} \mathbf{0} & \mathbf{I}_\eta \\ \mathbf{I}_N & \\ \mathbf{0}_{L\times N} & \end{pmatrix} \mathbf{\Phi}^H \mathbf{D}\mathbf{s} + \sum_{i=0}^{N-1} \begin{pmatrix} -\alpha^{i(\eta+1)}\bar{\mathbf{v}}_i \\ \mathbf{0}_{(K-L)\times 1} \\ \alpha^i \bar{\mathbf{v}}_i \end{pmatrix} s_i. \qquad (3)$$

Here $\mathbf{I}_k$ is the k by k identity matrix, $\mathbf{\Phi}^H$ is the IDFT matrix, $\mathbf{s}$ is the input vector, $\alpha = e^{-j2\pi/N}$, $\mathbf{v}_i = \mathbf{U}_i \mathbf{w}_i$ and can be written as $[v_{i,0} \;\; \bar{\mathbf{v}}_i^T]^T$, $\mathbf{U}_i$ is an $(L+1)$ by $(L+1)$ upper triangular Toeplitz matrix whose first row is $[1 \; \alpha^i \cdots \alpha^{iL}]$, $\mathbf{D}$ is a diagonal matrix with diagonal elements $[v_{0,0} \; v_{1,0} \cdots v_{N-1,0}]$ consisting of the first element of $\mathbf{v}_i$ for each i.

III. Problem formulation

For a single-user system without any PSD constraint the optimal power loading can be calculated by using the water filling procedure [12]. With a PSD constraint this is no longer applicable. In [11] it was shown how to optimally allocate pulse shaping filter taps if the number of used tones and their transmit powers are known, for a system with PSD constraints. However, the number of used tones can be increased if a lower transmit power at the band edges is used. Therefore by also allocating the appropriate amount of transmit power per tone

the total achievable data rate can be maximized for a given total transmit power and total number of filter taps. This then requires a procedure to distribute the available resources (total power and total filter tap budget) optimally over the tones such that the achievable data rate is maximized and the PSD mask constraint is still met. Here we present an approach that is based on a dual problem formulation. The primal optimization problem can be written as

$$\begin{aligned} \max_{\substack{P_1 \cdots P_N \\ L_1 \cdots L_N}} \quad & \sum_i R_i \\ \text{subject to} \quad & \mathbf{T}_{mask} \geq \mathbf{T}_{spec} \\ & L_{tot} \geq \textstyle\sum_i L_i \\ & P_{tot} \geq \textstyle\sum_i P_i \\ & L_i \leq L_{max} \\ & P_i \leq P_{max} \end{aligned} \tag{4}$$

where, R_i is the achievable bit rate per tone and given as

$$R_i = log_2(1 + \frac{SNR_i}{\Gamma}),$$

SNR_i is the signal-to-noise ratio of i-th tone, Γ is the SNR gap, L_{tot} is the total filter tap budget and P_{tot} is the total power budget. L_i and P_i are the order of the shaping filter and the power allocated for the tone i. L_{max} and P_{max} is the maximum order of the pulse shaping filter and maximum transmit power allowed per tone respectively. $\mathbf{T}_{mask}$ is the vector containing M sample points of the given PSD and $\mathbf{T}_{spec}$ is the vector containing the calculated transmit PSD at each of the M sampling points.

Equation 4 can be rewritten in the dual form with Lagrange multipliers as

$$\min_{\gamma,\alpha,\lambda} \left\{ \max_{\substack{L_1 \cdots L_N \\ P_1 \cdots P_N}} (\Psi) \right\}$$

where,

$$\begin{aligned} \Psi = \quad & \textstyle\sum_i R_i + \gamma \sum_i (L_{tot} - \sum_i L_i) + \alpha \sum_i (P_{tot} - \sum_i P_i) \\ & + \textstyle\sum_j \lambda_j (T_{mask,j} - T_{spec,j}). \end{aligned} \tag{5}$$

Where α, γ and λ_j are the positive. As P_{tot}, L_{tot} and $T_{mask,j}$ are given constants hence do not influence the overall optimization problem. Therefore Ψ in the above equation can be written as

$$\begin{aligned} \Psi &= \sum_i R_i - \gamma \sum_i L_i - \alpha \sum_i P_i - \sum_j \lambda_j T_{spec,j} \\ &= \sum_i \{R_i - \gamma L_i - \alpha P_i - \sum_j \lambda_j T_{spec,j}\} \end{aligned} \tag{6}$$

By substituting,

$$T_{spec,j} = \sum_i P_i |F_i(\omega_j) \mathbf{e}^H(\omega_j) \mathbf{W}_i|^2$$

into equation (6) we obtain,

$$\Psi = \sum_i \{R_i - \gamma L_i - \alpha P_i\} - \sum_j \lambda_j \sum_i P_i |F_i(\omega_j) \mathbf{e}^H(\omega_j) \mathbf{W}_i|^2$$

i.e.

$$\Psi = \sum_i \left\{ R_i - \gamma L_i - \alpha P_i - \sum_j \lambda_j P_i |F_i(\omega_j) \mathbf{e}^H(\omega_j) \mathbf{W}_i|^2 \right\} \tag{7}$$

Hence for fixed Lagrange multipliers the problem is decoupled over tones, where the per-tone optimization function is,

$$\psi_i(\gamma, \alpha, \Lambda) = R_i - \gamma L_i - \alpha P_i - \sum_j \lambda_j P_i |F_i(\omega_j) \mathbf{e}^H(\omega_j) \mathbf{W}_i|^2 \tag{8}$$

Therefore, the per-tone optimization problem becomes,

$$\boxed{\max_{L_i, P_i} \quad \psi_i(\gamma, \alpha, \Lambda) \qquad \text{subject to } \gamma, \alpha, \Lambda \geq 0} \tag{9}$$

where Λ is a vector containing all λ_j. This per-tone optimization problem can be solved (for fixed Lagrange multiplier) by exhaustively searching over all values of P_i and L_i. The values of Lagrange multipliers can be updated until all the constraints are met. An algorithm to implement this optimization technique is given below. The complexity of the proposed algorithm is $O(M * P_{max} * L_{max})$.

Update of Lagrange multipliers (γ, α and λ) can be based on the difference between the obtained values of resources and their target values.

$$\begin{aligned} \lambda_j^{t+1} &= \lambda_j^t - \mu_j (T_{mask,j} - T_{spec,j}) \\ \gamma_i^{t+1} &= \gamma_i^t - \hat{\mu} (L_{tot} - \sum_i L_i) \\ \alpha_i^{t+1} &= \alpha_i^t - \bar{\mu} (P_{tot} - \sum_i P_i) \end{aligned}$$

μ_j, $\hat{\mu}$ and $\bar{\mu}$ are the step sizes for the updates which are always positive and these step sizes can be varied in order to drop the energy at a frequency point quickly if the output spectrum at that frequency exceeds the PSD mask or to cautiously increase it if the output spectrum is under the PSD mask. The complete algorithm is given below.

Algorithm: Joint resource allocation algorithm

Initialize λ, α, γ, μ_j, $\bar{\mu}$, $\hat{\mu}$
repeat
  For $i \in \{\text{used tones}\}$
    For $L_i = 0 \cdots L_{max}$
      compute W_i
      For $P_i = 0 \cdots P_{max}$

$$\psi_i \underset{\text{argmax}}{\Leftarrow} \{R_i - \gamma L_i - \alpha P_i - \sum_j \lambda_j P_i |F_i(\omega_j)\mathbf{e}^H(\omega_j)\mathbf{W}_i|^2\}$$

      End For
    End For
  End For
  compute $\mathbf{T}_{spec}$
  For $j = 1 \cdots M$

$$\lambda_j^{t+1} = \lambda_j^t - \mu_j(T_{mask,j} - T_{spec,j})$$

  End For

$$\gamma^{t+1} = \gamma^t - \hat{\mu}(L_{max} - \sum_i L_i)$$

$$\alpha^{t+1} = \alpha^t - \bar{\mu}(P_{max} - \sum_i P_i)$$

While (change in total filter taps and total used power)

IV. Simulation Results

In our simulation, we consider VDSL downstream transmission with a PSD mask corresponding to the FTTCab M1 deployment scenario [3]. The IDFT size is 1024, the cyclic prefix length plus the overhead due to the pulse shaping filter is 80 samples. We use $L_{max} = 16$ to be comparable to [4]. The initial values are $\lambda_j = 0$, $\alpha = 0$, $\gamma = 0$, $\mu_j = 1e8$, $\bar{\mu} = 1$ and $\hat{\mu} = 1e-7$.

For given resource constraints, we can see the working of resource allocation procedure if we compare two scenarios, one with channel knowledge and one without channel knowledge. Here, the resource allocation for a system with a channel to interference ratio as shown in the **Figure 2** is compared with a system which assumes the channel on all the tones to be 85 dB (i.e. flat channel). The channel under consideration here is a 400m twisted pair cable in a bundle of 8 with an interference channel of 400m. As the total available power is 11.5 dBm which is sufficiently high for all the tones in this case, the total power constraint in this simulation is inactive. The distribution of power over tones for given resource constraints is shown in **Figure 3**. From **Figure 3 (a)** and **(b)**, it can be seen that the resource allocation is clearly affected by the knowledge of channel state. The resource allocation algorithm tries to allocate more power to tones with higher gains when the channel is known, otherwise the transmit power is evenly distributed. This is clearly visible in **Figure 3 (c)** and **(d)**. The tap distribution over tones is given in **Figure 4**,which follows a similar pattern. **Table I** shows the number of iterations taken for the algorithm to converge for different constraint on the number of taps. It is obvious that if the resources are scarce the algorithm requires a higher number of iterations as it is then harder to distribute the resources optimally.

Furthermore from **Figure 5** it becomes evident that the data rate saturate when the number of taps is increased. The data rate approaches the data rate for the case where there are no resource constraints, which is given by the dotted line in the figure. It can also be seen that the achievable rate can be increased if the resource allocation is used than in the case where a fixed number of tones are used without any resource allocation (maximum 375 tones used in [4]).

Number of total given taps	Outer iterations to converge
1000	38
5000	12

TABLE I
NUMBER OF ITERATIONS TAKEN TO CONVERGE TO THE SOLUTION

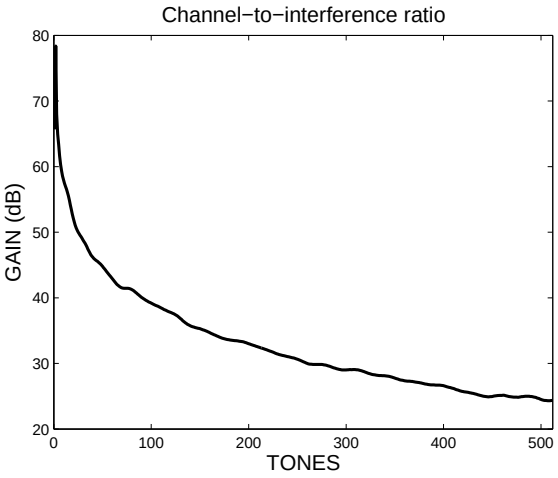

Fig. 2. Channel to interference ratio

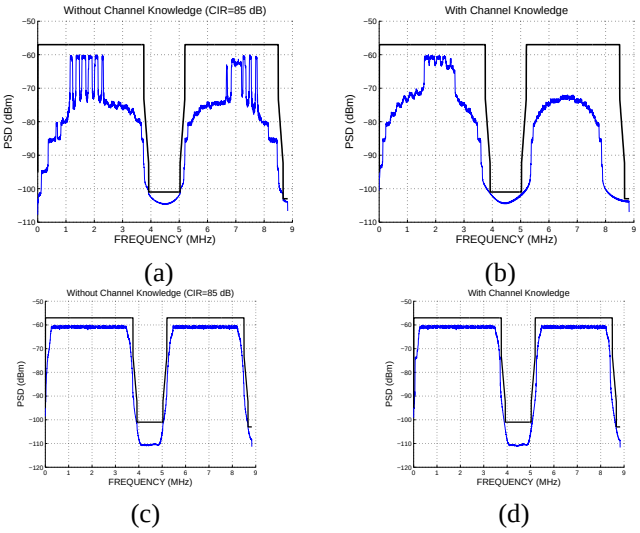

Fig. 3. Output spectrum for various resource constraints (a), (b) for 1000 taps and (c), (d) for 5000 taps

V. Conclusion

In this paper, we have proposed an efficient resource allocation algorithm to distribute the available resources (i.e. filter taps and transmit power) amongst the tones in a DMT

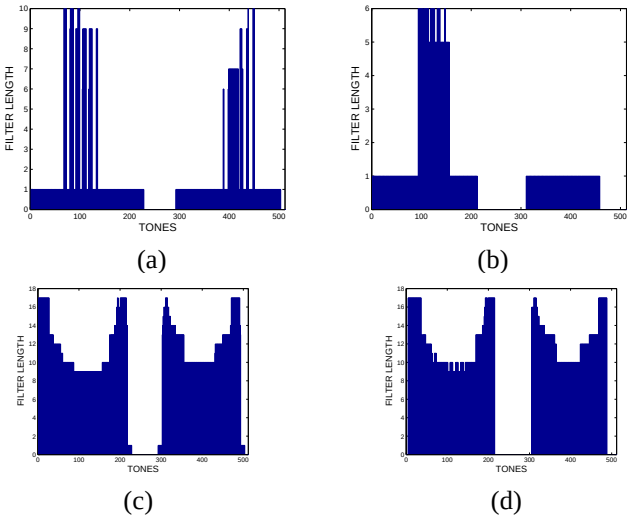

Fig. 4. Pulse shaping filter tap distribution for various resource constraints (a), (b) for 1000 taps and (c), (d) for 5000 taps

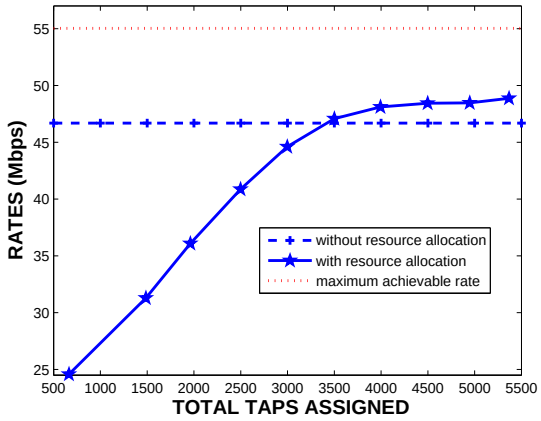

Fig. 5. Relation of data rate and the total number of available resources

transmitter with per-tone pulse shaping. We have shown that a dual problem formulation leads to an optimization that can be decoupled over tones. For given resource and PSD constraints the number of taps and transmit power can be optimally distributed by maximizing a cost function for every tone independently without violating the spectrum mask. A VDSL simulation demonstrates that the resources can be optimally distributed and the achievable rate approaches the maximum achievable bit rate.

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