A Robust MPC Design for Hot Rolling Mills: A Polyhedral Invariant Sets Approach

Il Seop Choi, Anthony Rossiter, Bert Pluymers, Peter Fleming and Bart De Moor

***Abstract*— The role of a hot rolling mill process is to produce strips of thickness about 0.8 $\sim$ 20 mm from heated slabs. One of the major control problems in hot rolling mills are the looper and tension control loops because these have a significant impact on the dimensional quality of a strip and process stability. Moreover, the most difficult challenge in the controller design arises from the process interaction and uncertainty affecting these loops; uncertainty comes from several sources of disturbances and model mismatch. Recently, some authors have investigated the potential benefits of MPC (Model Predictive Control). However, most authors used constrained optimisation based on nominal models and therefore, recursive feasibility and stability is not guaranteed for the uncertain case. The aim of this paper is to extend previous studies of MPC for rolling mills [9] to the robust case as well as to evaluate the efficacy of a recently proposed robust MPC (RMPC) [15] design on a multi–dimensional process.**

I. INTRODUCTION

This paper presents a case study based on one of the control problems in a hot rolling mill process, so first we give a brief introduction of a typical hot rolling mill process. The role of a hot strip mill process is to produce strips of thickness 0.8 $\sim$ 20 mm from heated slabs. Figure 1 shows a typical layout of a hot strip mill. Slabs are heated up to around $1200\,^{\circ}\mathrm{C}$ in reheating furnaces and then rolled on two reversing roughing mills which reduce the thickness of bars to around 30 mm. Then, the strip is further rolled by finishing mills composed of typically six or seven rolling stands. The rolled strip is cooled down by spraying water in a run–out table and coiled by a down coiler.

The most important control problems in hot rolling mills are concerned with the thickness, width, shape and mass flow of a strip in the finishing mills. One of these control loops is the looper and tension control [10]; this loop has significant impacts on the dimensional quality of a strip and process stability. Figure 2 shows a typical looper and tension control setup arising between two stands. Some of the control issues are that high strip tension between stands induces width shrinkage, thickness reduction and moreover can produce an edge wave on a strip while low tension makes the mass flow unstable. Therefore, strip tension should be kept to a desired value during operation to ensure proper product quality and strip threading. Typically, strip tension can be controlled by the speed of an upstream main motor.

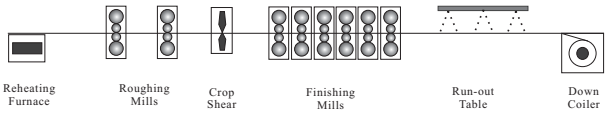


Fig. 1. Hot strip mill layout

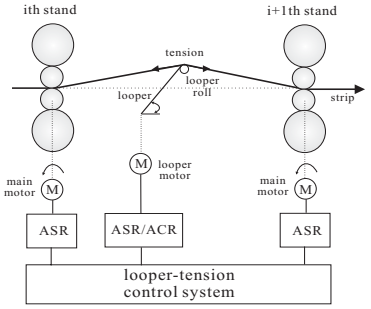


Fig. 2. Tension and looper control in finishing mills: *ASR–Automatic Speed Regulator, ACR–Automatic Current Regulator.*

A looper installed at inter–stand positions reduces tension variations by changing its angle, so it can also contribute to the quality of the products. Moreover, it can enable stable operation of the process by absorbing an excessive loop of the strip arising from a mass flow unbalance. Therefore, the looper angle should be kept to a specified value during operation in order to maintain maximum flexibility for sudden mass flow changes.

In summary, looper and tension control is achieved by simultaneous control of the strip tension and looper angle. However, this is a difficult control problem both because of significant interaction and also the need for high performance in the presence of significant uncertainty. The interaction between the looper angle and strip tension makes it difficult to design a controller and the obvious consequence is degraded control performance and stability. Uncertainty comes from several sources of disturbances and model mismatch. Up to now, many authors ([1]–[6],[10]) have proposed and applied a variety of control schemes to this control problem, but nevertheless, the increasingly strict market demand for strip quality requires further improvements in this control area.

One control design technique with obvious potential for

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these loops is MPC (Model Predictive Control), because it can give systematic handling of interaction and constraints. Unsurprisingly therefore, there have been several recent papers investigating the potential benefits of MPC. Some authors ([7],[8]) suggested an MPC controller for mass flow control design by taking account of constraints. Others [9] investigated the efficacy of an MPC scheme for the looper–tension control problem by using a MIMO model. However, most of these works have focused on constrained optimization using only nominal models and therefore, recursive feasibility and stability has not been guaranteed for the uncertain case; the nominal MPC may induce constraint violation under uncertainty resulting in severe quality defects of a strip and process instability in worst case.

This paper extends the earlier studies to the case of parameter uncertainty. In fact much of the MPC research dealing with uncertainty still requires excessive online computation or has restricted feasibility. However, a recent work [11] showed that it is possible, in some cases, to pose a robust problem in exactly the same format as a nominal problem, that is an MPC algorithm deploying just a low dimensional quadratic programming optimisation. This advance was facilitated by recent developments in the understanding and computation of robust invariant sets (e.g. [11]). Hence the aim of this paper is to evaluate the proposed robust MPC algorithm on a realistic process as well as to extend previous studies of MPC for rolling mills [9] to the robust case.

The paper is organised as follows. The process model is derived in section II, nominal MPC design and analysis for looper–tension control is discussed in section III, RMPC for LPV systems and its case study to looper–tension control are given in section IV and V respectively, and Section VI contains the conclusions.

II. PROCESS MODEL

This section introduces a detailed model of the looper and tension behaviour and shows how these can be used to construct a nominal linear model and a linear parameter varying (LPV) model (to cater for uncertainty). All the underlying equations and parameters are based on a hot strip mill process of POSCO [17].

First, define the following variables. T_l: load torque on a looper motor, T_σ: load torque component by strip tension, T_s: load torque component by strip weight, T_w: load torque component by looper weight, r_{lp}: looper roll radius, σ: strip tension, w: strip width, h: strip thickness, g: acceleration of gravity, ρ:strip density, w_{lp}: looper weight, M_l: torque of a looper motor, f:forward slip, V_i: rotating speed of a i–th work roll, T_m: rolling torque on a main motor, M_m: torque of a main motor, J_m:inertia of a main motor, l_d: torque arm, $R^{'}$:deformed roll radius, T_f: total forward tension, T_b:total backward tension, P: rolling force.

A. Tension and Looper Model

Figure 3 shows an outline of a looper. Clearly from the figure, the loop length between stands is $L^{'}(\theta(t)) = \sqrt{(x^2+y^2)} + \sqrt{(L_0-x)^2+y^2}$.

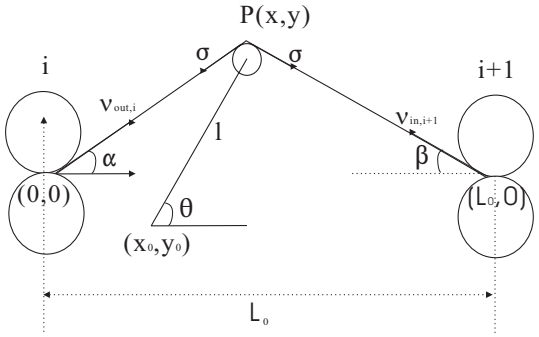


Fig. 3. Outline of a looper: $v_{out,i}$ *is the exit strip speed at the* $(i)^{th}$ *stand,* $v_{in,i+1}$ *is the entry strip speed at the* $(i+1)^{th}$ *stand.*

Define $L = L_0 - \int_0^t (v_{in,i+1} - v_{out,i})dt$, that is *the accumulated loop length which changes due to the speed difference of the strip between stands.*
From $L^{'}$ and L, inter–stand strip tension is defined as follows:

$$\sigma(t) = E[\frac{L^{'}(\theta(t)) - L(t)}{L(t)}] \tag{1}$$

where, E represents Young's modulus. The looper model is derived by applying Newton's second law with an inertia J_l, motor torque M_l and load torque T_l.

$$J_l\ddot{\theta} = M_l - T_l \tag{2}$$

Load torque T_L includes the torque by strip tension T_σ, the torque from strip weight T_s and the torque from looper weight T_w and so on.

$$T_L \approx T_\sigma + T_s + T_w \tag{3}$$

$$T_\sigma = wh\rho l[sin(\theta+\beta) - sin(\theta-\alpha)] \tag{4}$$

$$\begin{aligned} \alpha &= sin^{-1}\frac{y_0 + lsin\theta + r_{lp}}{\sqrt{(x_0+lcos\theta)^2 + (y_0+lsin\theta+r)^2}} \\ \beta &= sin^{-1}\frac{y_0 + lsin\theta + r_{lp}}{\sqrt{(L_0-x_0-lcos\theta)^2 + (y_0+lsin\theta+r)^2}} \end{aligned}$$

$$T_s = g\rho whLlcos\theta \tag{5}$$

$$T_w = gw_{lp}l_{lp}cos(\theta+\theta_{lp}) \tag{6}$$

The velocity $v_{out,i}$ of a strip which leaves i–th stand is closely related to the rotating speed of i–th main motor and forward slip.

$$v_{out,i} = (1+f)V_i \tag{7}$$

The rolling torque on a main motor is derived as follows.

$$J_m\ddot{\theta} = M_m - T_m \tag{8}$$

$$T_m = 2l_dP - R^{'}(T_f - T_b) \tag{9}$$

B. State Space Representation of Looper and Tension Model

In order to apply the linear MPC design technique we combine the equations (1)-(9) with dynamic equations of motors, and linearise them on a operating point of 13 $[N/mm^2]$ in tension and 0.35 $[rad]$ in looper angle. The resultant state space equation is constructed as follows.

$$\dot{x}(t) = A^{'}x(t) + B^{'}u(t) \tag{10}$$
$$y(t) = C^{'}x(t) + D^{'}u(t) \tag{11}$$

where, $x = [\Delta w_l \ \Delta\sigma \ \Delta\theta \ \Delta w_m \ \Delta x_l \ \Delta x_m]^T$; each state represents angular velocity of a looper motor, strip tension, looper angle, rotating speed of a main motor, integral variable of a looper controller and integral variable of a main motor controller respectively. Hence $D^{'} = 0$, $A^{'} =$

$$\begin{pmatrix} \frac{K_{l1}\phi_l}{-J_l} & \frac{K_{t\sigma}}{-J_l} & \frac{K_{t\theta}}{-J_l} & 0 & \frac{\phi_l}{J_l} & 0 \\ \frac{EK_{l\theta}}{LG_l} & \frac{EK_{v\sigma}}{-L} & 0 & \frac{E(1+f)}{-L} & 0 & 0 \\ \frac{1}{G_l} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{K_{m3}K_{g\sigma}}{-J_m} & 0 & \frac{K_{m3}\phi_m K_{m1}}{-J_m R_m} & 0 & \frac{K_{m3}\phi_m}{J_m R_m} \\ -K_{l2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -K_{m2} & 0 & 0 \end{pmatrix}$$

$$B^{'} = \begin{pmatrix} 0 & \frac{K_{l1}\phi_l}{J_l} \\ 0 & 0 \\ 0 & 0 \\ \frac{K_{m1}\phi_m K_{m3}}{R_m J_m} & 0 \\ 0 & K_{l2} \\ K_{m2} & 0 \end{pmatrix} \quad C^{'} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

where, $K_{t\theta}, K_{v\sigma}, K_{t\sigma}, K_{g\sigma}, K_{l\theta}$ represent the linearised gains on the operating point of load torque variation by angle, slip variation by tension, load torque variation by tension, rolling torque variation by tension and loop length variation by looper angle respectively. For instance, figure 4 represents the load torque variation by looper angle, and $K_{t\theta}$ is linearised by applying Taylor expansion to (3).

$$K_{t\theta} = \frac{dT}{d\theta} \approx \bar{T} + \frac{\partial T(w,h,\sigma,\theta)}{\partial\theta} \mid_{w=\bar{w},h=\bar{h},\sigma=\bar{\sigma},\theta=\bar{\theta}} \tag{12}$$

where, $\bar{T}$ is the load torque of a looper motor at operating point. This constructed linear model is used for control design and analysis of the nominal case in the next section.

III. Nominal MPC Design and Analysis for Looper–Tension Control

A. Controller Design [15]

In order to design the MPC controller continuous plant of (10,11) are converted to the following discrete equation.

$$x_{k+1} = Ax_k + Bu_k \tag{13}$$
$$y_k = Cx_k + Du_k \tag{14}$$

An unconstrained linear control law is given by

$$u = -Kx_k \tag{15}$$

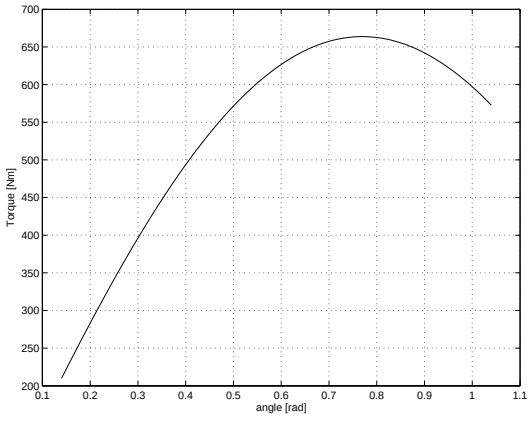


Fig. 4. Load torque on a looper motor vs. looper angle

Performance index will be assessed by the cost

$$J = \sum_{k=0}^{\infty} x_k^T Q x_k + u_k^T R u_k \tag{16}$$

Let the MPC control law([12],[13]) be:

$$\begin{aligned} u_k &= -Kx_k + c_k \quad k = 0, \cdots, n_c - 1 \\ u_k &= -Kx_k \quad\quad\ k \geq n_c \end{aligned} \tag{17}$$

where c_k are d.o.f. available for constraint handling. Substituting (13, 17) into (16), J takes the form

$$J = \sum_{j=0}^{i-1} c_j^T \hat{Q} c_j + p \ ; \quad p = x^T V x \tag{18}$$

and $\hat{Q} = B^T \Sigma B + R, \ \Sigma - \Phi^T \Sigma \Phi = Q + K^T R K$; as p is not independent on the d.o.f., it is convenient to omit it and rephrase the objective function as:

$$J = C^T W_D C \ ; \quad C = [c_0^T, \cdots, c_{n_c-1}^T]^T \tag{19}$$

where, $W_D = diag(\hat{Q}, \cdots, \hat{Q})$.

We assume that the process is subject to constraints:

$$\underline{\mathrm{u}} \leq u_k \leq \bar{u} \ ; \quad \underline{\mathrm{x}} \leq x_k \leq \bar{x}, \quad k = 1, \cdots, \infty \tag{20}$$

The MAS [14] is defined as the region within which the state and control evolutions satisfy constraints (20) with a given linear control law (15). Let the MAS be defined as

$$S_0 = \{x : M_0 x \leq d_0\} \tag{21}$$

The maximal control admissible set (MCAS) is the region within which the d.o.f. C are sufficient to ensure constraint satisfaction of the predictions[1]. The MCAS takes the form:

$$S_c = \{x : \exists C \ s.t. \ M_0 x + N_0 C \leq d_0\} \tag{22}$$

A typical paradigm for the nominal MPC minimizes the cost (18) subject to the transient and terminal constraints

[1]Use C to satisfy (20) over the first n_c steps and to ensure $x_{k+n_c} \in S_0$.

which are implicit in (22).

Algorithm 1. **Nominal MPC:** At each sampling instant, perform the optimisation:

$$\min_{C} J = C^T W_D C \quad s.t. \quad M_0 x + N_0 C \leq d_0 \tag{23}$$

Use the first block element of C in control law (17) [13].

Remark 1. This algorithm has a guarantee of recursive feasibility and convergence, for the nominal case. Moreover, for n_c large enough and K the optimal control law, the solution is the same as the optimal constrained control law.

B. Simulation Results for Nominal MPC

In this section we illustrate how the model uncertainty affects the feasibility of the constrained optimisation arising from a nominal design for looper–tension control. Through the paper we assume uncertainty comes only from linearisation; in section 2, some severe nonlinear effects were not taken into account in the nominal model due to the linearisation on an operating point (work in progress is considering the addition of unknown bounded disturbance effects).

Taking account of uncertainty, the process model (13) is represented as a LPV system

$$\begin{aligned} x_{k+1} &= A(k)x_k + B(k)u_k \\ (A(k), B(k)) &\in Co\{(A_1, B_1), \cdots, (A_m, B_m)\} \end{aligned} \tag{24}$$

For simulations algorithm 1 was implemented on the nominal system (13) with n_c=5 of d.o.f.. The data of looper and tension control are as follows: $L = 5505[mm], E = 66700[N/mm^2], J_m = 2.1[kgm^2], f = 0.0703, \phi_m = 20.0[Nm/A], J_l = 1.91[kgm^2], \phi_l = 3.326[Nm/A], G_l = 10.0$. Constraints of strip tension and looper angle are defined as $-2 \leq x_2 \leq 2\ [N/mm2], \ -0.02 \leq x_3 \leq 0.02\ [rad]$ respectively.

The simulations are performed, using a nominal control law (17), but simulated on the uncertain system (24) and nominal system (13) respectively. Figures 5, 6 show a comparison of tension and angle responses for nominal and uncertain systems; unsurprisingly uncertainty induces several constraint violations in the uncertain system simulations while constraints are respected in the nominal case. The violations arise because the nominal MCAS is inconsistent with the actual state and input evolutions and in the worst case could lead to instability or process failure. More worryingly, the optimisation itself is infeasible and hence the control law applied maybe ill-concieved without a supervisory layer. For instance figures 7, 8 show control perturbations (c_k) of a main motor for constraint satisfaction and it is clear that uncertainty has resulted in the need for much more aggressive action.

So, in summary, this simple simulation demonstrates that, as expected, for the looper-tension loop, use of a nominal model for MPC controller design does not allow a feasibility guarantee. Moreover, performance and stability may be compromised. In order to guarantee feasibility under model uncertainty a robust MPC design is required; one such algorithm is presented in the next section.

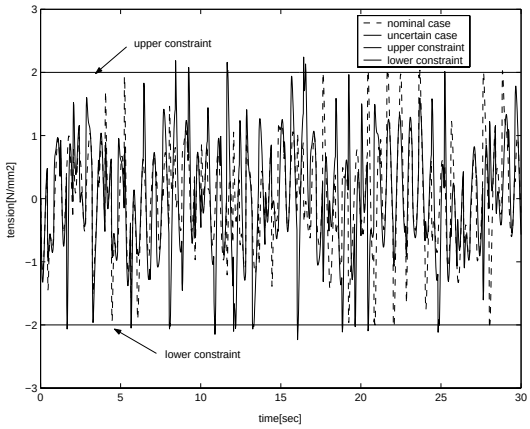


Fig. 5. A comparison of tension responses between uncertain and nominal model

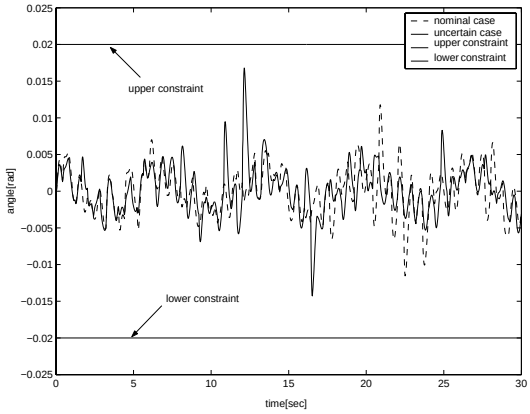


Fig. 6. A comparison of angle responses between uncertain and nominal model

IV. ROBUST MPC FOR LPV SYSTEMS

This section introduces a RMPC algorithm using polyhedral invariant sets and therefore implementable using just a quadratic programming optimisation. First we describe briefly how one can compute a robust invariant polyhedral sets for LPV systems and subsequent RMPC design based on these sets after which a case study of RMPC to looper–tension control is given in the following section.

A. Polyhedral Invariant Sets for LPV Systems

A brief summary of the key results is as follows [11]. First define the closed–loop system matrices

$$\Phi_i = A_i - B_i K, \quad i = 1, \cdots, m \tag{25}$$

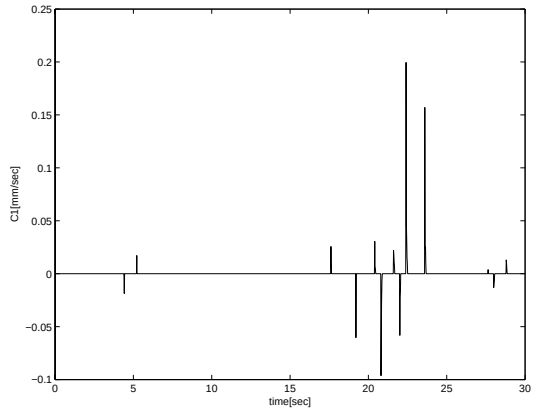


Fig. 7. control perturbation for nominal model case

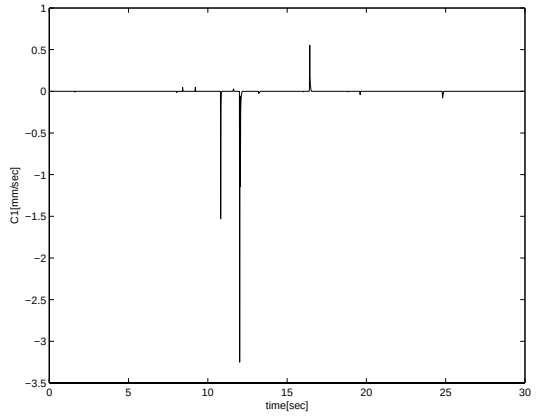


Fig. 8. control perturbation for uncertain model case

Let the MAS for the uncertain system (24), constraints (20) and control law $u = -Kx$ be given by

$$S_u = \{x : M_u x \leq d_u\} \tag{26}$$

By definition of invariance, $x \in S_u \Rightarrow \Phi_i x \in S_u, i = 1, \cdots, m$. The following algorithm enables us to compute the MAS efficiently by removing redundant constraints.

Algorithm 2. **Robust Invariant Set (MAS):**
(1) Let the constraints (20) at each sample be summarised as

$$y(k) \in \mathbf{Y} = \{y | A_y \leq b_y\}, k = 0, \cdots, \infty, \tag{27}$$
$$u(k) \in \mathbf{U} = \{u | A_u \leq b_u\}, k = 0, \cdots, \infty, \tag{28}$$

Set $M_u := [A_y^T \ A_u^T]^T$, $d_u = [b_y^T, b_u^T]^T$ and $i := 1$.

(2) Select row i from (M_u, d_u) and check $\forall j$ whether $M_{u,i}\Phi_j x \leq d_{u,i}$ is redundant with respect to the constraint defined by (M_u, d_u). Add the non–redundant constraints to (M_u, d_u) by assigning $M_u := [M_u^T \ (M_{u,i}\Phi_j)^T]^T$ and $d_u := [d_u^T \ d_{u,i}^T]^T$ for all relevant j.

(3) Set $i := i+1$. If i is strictly larger than the number of rows in $(M_u, \ d_u)$ then terminate, otherwise continue with step 2). The resulting set $S_u = \{x : M_u x \leq d_u\}$ is the MAS for the given system, constraints, and the feedback controller.

Remark 1 (MCAS): Same algorithm uses autonomous model with states $[x, \ C]$. Corresponding set takes the form of $M_r x + N_r C \leq d_r$.

B. Robust predictions and constraint handling

We give a brief introduction to robust MPC algorithm using polyhedral invariant sets for uncertainty. For more details, we refer the reader to Pluymers *et al.* [15].

Algorithm 3. **Robust MPC:**

At each sampling instant, minimise the performance index:

$$\min_C J = C^T W_D C \quad s.t. \quad M_r x + N_r C \leq d_r \tag{29}$$

$$M_r = \begin{bmatrix} M_t \\ M_v \end{bmatrix}; \quad N_r = \begin{bmatrix} N_t \\ N_v \end{bmatrix}; \quad d_r = \begin{bmatrix} d_t \\ d_v \end{bmatrix} \tag{30}$$

Use the first block element of C in control law(17).

Remark 2: Shown in [15] that this law has guarantees of recursive feasibility and stability.

V. Case Study of the RMPC application to looper–tension control

In this section we apply the efficient RMPC based on polyhedral invariant sets to looper–tension control system as a case study for multi–dimensional and MIMO case. First, we compute a robust invariant set (MCAS) for model uncertainty using algorithm 2. Then, a RMPC (algorithm 3) subjecting to satisfaction of transient and terminal constraints is implemented based on these sets after which closed–loop simulations are followed.

For uncertainty a convex set with $(A(k), B(k)) \in Co\{(A_1, B_1), \cdots, (A_m, B_m)\}$ is defined by taking account of nonlinear effects of $K_{t\sigma}$,$K_{t\theta}$, $K_{l\theta}$ and so on in (10). Constraints for inputs (rotating speed of a main motor and angular velocity of looper motor) and outputs (strip tension and looper angle) are assumed as follows: $-2.0 \leq y_1(= x_2) \leq 2.0 \ [N/mm2], \ -0.02 \leq y_2(= x_3) \leq 0.02 \ [rad], \ -2 \leq u_1 \leq 2 \ [mm/sec], \ -0.2 \leq u_2 \leq 0.2 \ [rad/sec]$. These values represent perturbations from steady state.

Figure 9 depicts target region (MAS) and feasible regions (MCAS) for $n_c = 1, 3, 5$ computed for the looper–tension control system; as expected, the feasible region is enlarged as n_c (d.o.f.) increases. The initial states inside each MCAS guarantee the satisfaction of constraints and enter the MAS by control laws with corresponding degree of freedom. Once the state enters the MAS, it can be converged to the origin by the linear control law.

Figure 10 shows a comparison of closed–loop simulations with a nominal MPC and a RMPC ($n_c = 3$) for the various initial states. The MCAS (nominal) computed from (22) has larger domain of attraction than that of the MCAS (robust) for the same d.o.f.. However, some initial states within the

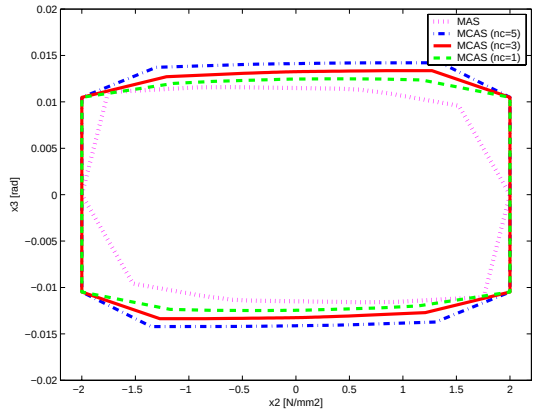


Fig. 9. Polyhedral invariant set for the looper–tension control system

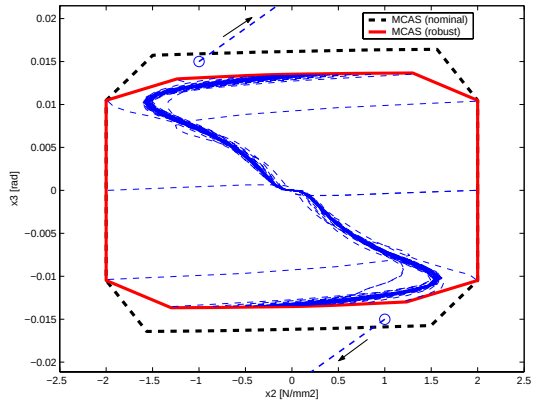


Fig. 10. The comparison of closed–loop simulations with a nominal MPC and a RMPC for various initial states

MCAS (nominal) can not guarantee the recursive feasibility because of uncertainty effects while all initial states within the MCAS (robust) can do. We perform two different closed–loop simulations in order to illustrate this;
1. Simulation from initial states inside MCAS (robust): all points converge to the origin with constraints satisfaction regardless of uncertainty dut to the guaranteed feasibility.
2. Simulation from initial states inside MCAS (nominal) but outside MCAS (robust): We choose two initial states displayed as 'o' for this and both give infeasible solutions in optimisation due to the uncertainty resulting in going out the feasible region. Therefore, as explicitly shown in the simulations the only initial points within robust invariant set can guarantee the recursive feasibility.

From this case study it is demonstrated that the RMPC based on polyhedral invariant sets can be a useful design strategy by guaranteeing the recursive feasibility for multi-dimensional systems with uncertainty. Moreover, it has obvious benefits from the use of a on–line QP optimisation.

VI. CONCLUSIONS AND FUTURE WORKS

This paper makes two main contributions. First it formulates a looper–tension model with polytopic uncertainty arising from parameter linearisation, which can be used to investigate how ignored nonlinear effects have impact on the recursive feasibility, and to compute polyhedral invariant sets for robust MPC design.

Secondly it gives a test case for the efficient polyhedral invariant set approach for LPV system which was recently published [11]. It is shown that the proposed robust MPC algorithm can be applied to looper–tension control, and gives obvious benefits.

Nevertheless this work is preliminary and future work should discuss: (i) inclusion of unknown bounded disturbance for computing polyhedral invariant sets and (ii) investigation of limitation due to the quadratic stability constraint.

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