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Partial crosstalk cancellation in a multi-user xDSL environment[1]

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Abstract

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Partial crosstalk cancellation in a multi-user xDSL environment

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***Abstract*— In modern DSL systems, crosstalk is a major source of performance degradation. Crosstalk cancellation schemes have been proposed to mitigate the effect of crosstalk. However, the complexity of crosstalk cancellation grows with the square of the number of lines in the binder. Fortunately, most of the crosstalk originates from a limited number of lines on a limited number of tones. As a result, a fraction of the complexity of full crosstalk cancellation suffices to cancel most of the crosstalk. The challenge is then to determine which crosstalk to cancel on which tones, given a certain complexity constraint. This paper presents an algorithm based on a dual decomposition to optimally solve this problem. The proposed algorithm naturally incorporates rate constraints and the complexity of the algorithm compares favourably to a known resource allocation algorithm, where a multi-user extension is made to incorporate the rate constraints.**

I. Introduction

Crosstalk, which is typically 10-15 dB larger than the background noise, is a major source of performance degradation in xDSL systems. One strategy for dealing with this crosstalk is crosstalk cancellation. Several crosstalk cancellation schemes have been proposed. Linear pre- and post filtering [1] requires coordination at both the transmitters and receivers. Successive interference cancellation or precompensation [2] [3] can be used if there is only coordination available at the receivers or transmitters respectively. For this level of coordination, it is shown in [4] [5] that a simple linear zero-forcing canceller or linear precompensator performs near-optimally in an xDSL environment.

Even for these simple linear cancellers, the complexity grows with the square of the number of lines. For example, in a binder of 8 VDSL lines transmitting on 4096 tones at a block rate of 4000 blocks per second, the runtime complexity of crosstalk cancellation exceeds 1 billion multiplications per second.

However, crosstalk exhibits space and tone selectivity [6]. Measurements show that most of the crosstalk originates from a limited number of lines, e.g. those in close proximity. Moreover, crosstalk coupling depends heavily on the frequency.

Because most of the crosstalk originates from a limited number of lines on a limited number of tones, a fraction of the complexity of full crosstalk cancellation suffices to cancel most of the crosstalk. This is called partial crosstalk cancellation [6] [7].

The challenge is then to determine for every user which crosstalk to cancel on which tones. In [6], an algorithm based on resource allocation is presented to solve this single-user problem. This paper presents an alternative optimal algorithm, based on a dual decomposition. The complexity of the algorithm is found to be more favourable than the complexity of the resource allocation algorithm, where a multi-user extension is made to incorporate the rate constraints.

In section II, the partial crosstalk cancellation problem is presented and then solved following a dual decomposition approach. A number of observations is made to reduce the complexity without losing the optimality of the solution. In section III the complexity of the dual decomposition algorithm is compared to the complexity of the resource allocation algorithm. Section IV concludes the paper.

II. Dual Decomposition

A. System Model

Most current DSL systems use Discrete Multi-Tone (DMT) modulation. The available frequency band is divided in a number of parallel subchannels or tones. Each tone is capable of transmitting data independently from other tones, and so the transmit power and the number of bits can be assigned individually for each tone.

Transmission for a binder of N users can be modelled on each tone k by

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{z}_k \qquad k = 1 \dots K.$$

The vector $\mathbf{x}_k = [x_k^1, x_k^2, \dots, x_k^N]^T$ contains the transmitted signals on tone k for all N users. $[\mathbf{H}_k]_{n,m} = h_k^{n,m}$ is an $N \times N$ matrix containing the channel transfer functions from transmitter m to receiver n. The diagonal elements are the direct channels, the off-diagonal elements are the crosstalk channels. $\mathbf{z}_k$ is the vector of additive noise on tone k, containing thermal noise, alien crosstalk, RFI,.... The vector $\mathbf{y}_k$ contains the received symbols.

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To take crosstalk cancellation into account, an equivalent channel $\tilde{\mathbf{H}}$ is introduced. This is the same channel as the original channel $\mathbf{H}$, but with off-diagonal elements set to zero where the crosstalk is cancelled. If user n is cancelling crosstalk originating from user m on tone k, then $\tilde{h}_k^{n,m} = 0$.

We denote the transmit power as $s_k^n \triangleq \Delta_f E\{|x_k^n|^2\}$, the noise power as $\sigma_k^n \triangleq \Delta_f E\{|z_k^n|^2\}$. The DMT symbol rate is denoted as f_s, the tone spacing as Δ_f.

It is assumed that each modem treats interference from other modems as noise. When the number of interfering modems is large, the interference is well approximated by a Gaussian distribution. Under this assumption the achievable bit loading of user n on tone k, given the transmit spectra of all modems in the system and the crosstalk cancellation configuration, is

$$b_k^n \triangleq \log_2\left(1 + \frac{1}{\Gamma}\frac{|\tilde{h}_k^{n,n}|^2 s_k^n}{\sum_{m\neq n}|\tilde{h}_k^{n,m}|^2 s_k^m + \sigma_k^n}\right), \qquad (1)$$

where Γ denotes the SNR-gap to capacity, which is function of the desired BER, the coding gain and noise margin. The data rate for user n is

$$R^n = f_s \sum_k b_k^n.$$

B. Partial Crosstalk Cancellation Problem

Because of the runtime complexity of full crosstalk cancellation, only a limited amount of crosstalk can be cancelled. The cancellation of the crosstalk from one user on some tone is done by a cancellation tap. The number of cancellation taps that can be used is constrained by the *cancellation tap constraint* C^{tot} [8]. The partial crosstalk cancellation problem amounts to finding an optimal selection of which crosstalk to cancel, thereby maximizing the capacity of the network.

Secondly, there is a *rate constraint* $R^{n,target}$ for each user. Typically, service providers offer a number of profiles to guarantee a certain Quality of Service. The rate constraint then indicates a minimum data rate required by the user.

The allocation of cancellation taps in partial crosstalk cancellation then results in the following maximization problem:

$$\begin{array}{lll} \max_{\mathbf{c}} & \sum_{n=1}^N R^n & \\ \text{s. t.} & C = \sum_k \sum_m \sum_n c_k^{n,m} \leq C^{tot} & \left\{ \begin{array}{l} n,m = 1\ldots N \\ k = 1\ldots K \end{array}\right. \\ & R^n \geq R^{n,target} & n = 1\ldots N \end{array} \qquad (2)$$

$$\text{with} \quad [\mathbf{c}_k]_{n,m} = c_k^{n,m} \quad c_k^{n,m} = \left\{ \begin{array}{lll} 0 & \Rightarrow & \tilde{h}_k^{n,m} = h_k^{n,m} \\ 1 & \Rightarrow & \tilde{h}_k^{n,m} = 0 \end{array}\right.$$

where $\mathbf{c} = [\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_K]$. $c_k^{n,m} = 1$ indicates that a cancellation tap is assigned on tone k for cancelling crosstalk on line n originating from line m. $\mathbf{c}_k$ is an N x N matrix containing the cancellation tap configurations $c_k^{n,m}$ between all users on tone k.

To find the global optimum for this optimization problem, one has to exhaustively search through all possible cancellation tap configurations $\mathbf{c}$. Because the cancellation tap constraint and the rate constraints are coupled over the tones, this results in an exponential complexity in the number of tones. By using a dual decomposition this complexity can be made linear [9] [10] [11] [12] [8]. This is done by using Lagrange multipliers to move the constraints coupled over tones into the unconstrained part of the optimization problem:

$$\begin{array}{rll} \mathbf{c}^{opt} = \text{argmax}_{\mathbf{c}} & \sum_{n=1}^N \omega_n R^n & \\ & +\lambda\big(C^{tot} - \sum_{k=1}^K \sum_{m=1}^N \sum_{n=1}^N c_k^{n,m}\big) & \\ \text{subject to} & \lambda \geq 0 & \\ & \omega_n \geq 0 \quad n = 1\ldots N & \end{array} \qquad (3)$$

where λ and ω_n are Lagrange multipliers. For a given set of λ and $\boldsymbol{\omega} = [\omega_1, \ldots, \omega_N]^T$, the optimization problem can be solved in a per-tone fashion:

$$\begin{array}{rll} \text{for } k = 1\ldots K, & & \\ \mathbf{c}_k^{opt} = \text{argmax}_{\mathbf{c}_k} & \sum_{n=1}^N \omega_n f_s b_k^n - \sum_{n=1}^N \sum_{m=1}^N \lambda c_k^{n,m} & \\ \text{subject to} & \lambda \geq 0 & \\ & \omega_n \geq 0 \quad n = 1\ldots N & \end{array} \qquad (4)$$

Maximization of (4) for given Lagrange multipliers can be performed by an exhaustive search. For each tone, all possible combinations for the canceller taps of the users should be checked. The combination giving the largest value for this expression is the optimal allocation of canceller taps.

The constraints can be enforced by choosing appropriate values for the Lagrange multipliers. The λ can be viewed as a cost for crosstalk cancellation taps. Larger values for the Lagrange multiplier result in less cancellation taps being allocated. The data rates of the users are weighted by $\boldsymbol{\omega}$, thereby giving more importance to some users. In this way, all possible trade-offs can be made to enforce the data rate constraints.

To solve (2) by (4), $\boldsymbol{\omega}$ and λ should be tuned to enforce the constraints. In [9] [10], an efficient Lagrange multiplier search procedure is presented for a similar problem. This procedure can be easily adapted for this partial cancellation problem. The basis for this procedure is relation (5), which is proven in **Appendix I**:

$$\left[\begin{array}{cc} -(\Delta\boldsymbol{\omega})^T & \Delta\lambda \end{array}\right] \left[\begin{array}{c} \Delta\mathbf{R} \\ \Delta C \end{array}\right] \leq 0 \qquad (5)$$

$\mathbf{R} = [R^1, \ldots, R^N]^T$ is a vector with the data rates and C is the number of cancellation taps corresponding to the Lagrange multipliers at hand.

Following [9] [10], relation (5) leads to update formula (6) for the Lagrange multipliers,where $(x)^+$ means $max(0, x)$ and μ is a stepsize parameter. Note that all the Lagrange multipliers are updated in parallel. This update formula is used in **Algorithm 1**, adopted from [9], to converge to the Lagrange multipliers that enforce the constraints.

The partial crosstalk cancellation problem (2) is a non-convex constrained optimization problem. Without dual decomposition, finding the global optimum requires an exhaustive search over all possible solutions. On a certain tone, a

$$\begin{bmatrix} \Delta\boldsymbol{\omega} \\ \Delta\lambda \end{bmatrix} = -\mu \begin{bmatrix} \mathbf{R} - \mathbf{R}^{target} \\ C^{tot} - C \end{bmatrix} \Rightarrow \begin{bmatrix} \boldsymbol{\omega} \\ \lambda \end{bmatrix}^{t+1} = \left(\begin{bmatrix} \boldsymbol{\omega} \\ \lambda \end{bmatrix}^{t} - \mu \begin{bmatrix} \mathbf{R} - \mathbf{R}^{target} \\ C^{tot} - C \end{bmatrix} \right)^{+} \tag{6}$$

Algorithm 1 Lagrange multiplier search algorithm

```
while distance > tolerance do
  Θ = [ω, λ]^T = best [ω, λ]^T so far
  μ = 1
  while distance ≤ previousDistance do
    previousDistance = distance
    μ = μ × 2
    ΔΘ = [Δω, Δλ]^T = update formula (6)
    [R^{Θ+ΔΘ}, C^{Θ+ΔΘ}, c] = exhaustiveSearch(Θ + ΔΘ)
    distance = ||[R^{Θ+ΔΘ} − R^{target}, C^{tot} − C^{Θ+ΔΘ}]^T||
  end while
end while
```

user has to decide which crosstalk of $N-1$ other users has to be cancelled. There are 2^{N-1} possibilities to do this. For N users and K tones, this results in a total complexity of $\mathcal{O}((2^{N-1})^{NK})$.

The dual decomposition decouples the problem over the tones, therefore reducing the exponential complexity in the number of tones K to linear complexity: $\mathcal{O}(K(2^{N-1})^N)$. This amounts to K exhaustive searches of complexity $\mathcal{O}((2^{N-1})^N)$. For an 8 user VDSL system, the complexity is reduced from $2^{7\times8\times4096}$ to $4096 \times 2^{7\times8}$. This is an enormous reduction in complexity. Moreover, as shown in the next subsection, the complexity can be even further reduced by observing that many cancellation tap configurations can be eliminated in advance.

C. Per-tone Search Complexity Reduction

To determine the optimal allocation of crosstalk cancellation taps on a certain tone, all of the $(2^{N-1})^N \approx 2^{N^2}$ possible allocations have to be evaluated. Even for a limited number of users this becomes complex. Fortunately, many of these possibilities can be eliminated based on two observations: user independence and line selection.

- *User Independence:* All users have to decide on a crosstalk cancellation configuration. This leads to an exponential complexity in the number of users N. However, from (1) it can be seen that if user n allocates a crosstalk cancellation tap to cancel crosstalk caused by user m (i.e. $\tilde{h}_k^{n,m} = 0$) this only has an influence on the capacity of user n. This corresponds to a per-user decoupling of (4), leading to

$$\begin{aligned} &\text{for } k = 1 \dots K, \\ &\text{for } n = 1 \dots N, \\ &\mathbf{c}_k^{n,opt} = \operatorname{argmax}_{\mathbf{c}_k^n} \omega_n f_s b_k^n - \textstyle\sum_{m=1}^{N} \lambda c_k^{n,m} \\ &\quad \text{subject to} \quad \lambda \geq 0 \\ &\qquad\qquad\quad\ \ \omega_n \geq 0 \quad n = 1 \dots N \end{aligned} \tag{7}$$

  As a consequence, the exponential complexity in N is reduced to linear complexity. Instead of one large search over all users, there are N independent searches for the users. This observation results in the following complexity reduction:

$$(2^{N-1})^N \rightarrow N(2^{N-1}) \tag{8}$$

- *Line Selection:* A user has to decide for $N-1$ other users whether or not to cancel the crosstalk originating from these other users. This leads to 2^{N-1} possible crosstalk cancellation configurations. However, from (1) it can be seen that to maximize the capacity, one should allocate crosstalk cancellation taps to cancel the users which are causing the largest crosstalk. Therefore, if n crosstalk cancellation taps are available, these should be used to cancel the n largest sources of crosstalk.
  As a consequence, the 2^{N-1} possibilities for crosstalk cancellation are reduced to N possibilities: cancel no crosstalker, cancel the strongest crosstalker, cancel the 2 strongest crosstalkers, ..., cancel all $N-1$ crosstalkers.

$$\begin{aligned} &\text{for } k = 1 \dots K, \\ &\text{for } n = 1 \dots N, \\ &\mathbf{c}_k^{n,opt} = \operatorname{argmax}_{\mathbf{c}_k^n} \omega_n f_s b_k^n(r) - \lambda r \\ &\quad \text{subject to} \quad \lambda \geq 0 \\ &\qquad\qquad\quad\ \ \omega_n \geq 0 \quad n = 1 \dots N \end{aligned} \tag{9}$$

  where $b(r)$ is the capacity when the r largest crosstalkers are cancelled.

When both observations are combined, N users independently have to choose one of N possible crosstalk cancellation configurations. This results in the following total complexity reduction:

$$(2^{N-1})^N \rightarrow NN \tag{10}$$

In a 8-user case, these observations reduce the number of crosstalk cancellation configurations to be evaluated from 2^{56} to 2^6. Note that despite drastic complexity reductions, the solution is still optimal.

III. Complexity comparison

In this section the complexity of the algorithm based on dual decomposition is analyzed and compared to the complexity of the optimal resource allocation algorithm of [6]. The resource allocation algorithm is a single-user algorithm. One possibility for extending this algorithm to allow multiple users is to divide the cancellation tap budget over the users in advance. By varying the cancellation tap budget allocated to each user, various trade-offs can be made in the data rates. Because of user independence and fixed individual cancellation tap

budgets, optimization of the individual users then also results in the optimization of the sum rate.

In this section however, the single-user algorithm is extended to automatically determine the correct cancellation tap budget to be allocated to the users so that rate constraints are satisfied.

A. Multi-user resource allocation algorithm

The single-user resource allocation algorithm of [6] uses the average capacity increase per allocated crosstalk cancellation tap on a certain tone:

$$v_k(r) = \frac{b_k(r) - b_k(0)}{r}, \tag{11}$$

with $b_k(r)$ the capacity on tone k when the r largest crosstalkers are cancelled (cfr. section II-C, line selection). A greedy algorithm then selects the tone k and number of crosstalkers r to cancel by searching the largest value of $v_k(r)$. The average capacity increase per allocated crosstalk cancellation tap should then be recalculated on tone k_s, based on the selected value $v_{k_s}(r_s)$, as follows:

- the average capacity increase for allocating less or equal crosstalk cancellation taps than r_s is set to zero
- the average capacity increase for allocating more crosstalk cancellation taps than r_s is recalculated as $v_k(r) = \frac{(b_k(r)-b_k(r_s))}{(r-r_s)}$, where the increase is now referenced to $b_k(r_s)$.

This is repeated until all available crosstalk cancellation taps are allocated. Note that in each iteration of the algorithm a minimum of 1 and a maximum of $N-1$ crosstalk cancellation taps is allocated. Because of this varying granularity, the crosstalk cancellation tap constraint cannot always be enforced tightly. However, the granularity is small enough to closely approach the constraint.

By introducing weights ω_n to the average capacity increase per allocated crosstalk cancellation tap, the resource allocation algorithm can be extended to automatically divide the cancellation tap budget over the users. The weights allow for some lines to be emphasized, thereby enforcing the rate constraints. To achieve a higher data rate for a user, more crosstalk cancellation taps should be allocated to that user. In order to do this, the average benefit of adding a crosstalk cancellation tap for that user is increased by a factor ω_n. A larger weight leads to more crosstalk cancellation taps allocated and thus a higher data rate.

A given set of $\omega_n{}'\mathrm{s}$ implies a cancellation tap budget for each user (which is known after the optimization is done with these $\omega_n{}'\mathrm{s}$). Because of the user independence, this again leads to an optimization of the sum rate. However, the rates are now weighted with $\omega_n{}'\mathrm{s}$, thus a weighted rate sum is optimized.

Therefore, the following relation can be derived, analogous to the derivation in appendix I:

$$\Delta\boldsymbol{\omega}\Delta\mathbf{R} \geq 0 \tag{12}$$

This is a reduced form of (5), which leads to a simplified version of update formula (6):

$$\Delta\boldsymbol{\omega} = -\mu(\mathbf{R}-\mathbf{R}^{target}) \Rightarrow \boldsymbol{\omega}^{t+1} = \left[\boldsymbol{\omega}^t - \mu(\mathbf{R}-\mathbf{R}^{target})\right]^+ \tag{13}$$

During I iterations, this update formula can then be used to steer the $\omega_n{}'\mathrm{s}$ so that the rate constraints are satisfied.

Table I presents the resulting multi-user resource allocation algorithm with its associated complexities. A $K \times N \times (N-1)$ table is initialized containing the average capacity increases per allocated crosstalk cancellation tap. For each of K tones and N users the capacity increase has to be calculated for all $N-1$ crosstalk cancellation configurations. To be able to calculate the capacity increase, the capacity without crosstalk cancellation $b_k(0)$ also has to be calculated for every tone. This results in KNN capacity calculations. Another $KN(N-1)$ multiplications and additions are required to calculate the average capacity increase per allocated crosstalk cancellation tap. The $N-1$ crosstalk cancellation configurations are based on the line selection observation of section II-C. This requires a sort over the crosstalkers for each tone. This sort can be accomplished by selecting the crosstalkers one by one and placing them in the correct position of a sorted list. Because the resulting list is sorted at all times, a binary search can be used to find the correct position to place the current crosstalker. This results in a complexity of $\sum_{i=1}^{N-1} log_2(i)$ comparisons to sort the list.

The table is then sorted to be able to efficiently find the maximum. This can be done analogous to the sorting of the crosstalkers and requires a complexity of $\sum_{i=1}^{KN(N-1)} log_2(i)$ comparisons. Note that the table of $KN(N-1)$ average capacity increases per crosstalk cancellation tap is globally searched.

Crosstalk cancellation taps can now be allocated by selecting the element with the maximum average capacity increase of the table, located at the top of the sorted list. On average, $\frac{N-1}{2}$ crosstalk cancellation taps are thereby allocated. $\frac{N-1}{2}$ elements in the table then have to be recalculated to the new reference capacity $b_k(r_s)$. This requires $\frac{N-1}{2}+1$ capacity calculations, $N-1$ multiplications and $N-1$ additions.

To keep the list sorted, $\frac{N-1}{2}$ binary searches are performed to find the new positions for the $\frac{N-1}{2}$ updated elements. This requires $\sum_{i=KN(N-1)-(\frac{N-1}{2}-1)}^{KN(N-1)} log_2(i)$ comparisons. The number of currently allocated cancellation taps is updated and compared to the cancellation tap constraint C^{tot}.

This is repeated until all available crosstalk cancellation taps are allocated. In [6] it was shown that with a runtime complexity of 30% of full crosstalk cancellation, almost all crosstalk can be cancelled. This means that approximately $\frac{KN(N-1)}{3}$ crosstalk cancellation taps have to be allocated. Taking into account that in each iteration of the algorithm $\frac{N-1}{2}$ taps are allocated, there are $\frac{KN(N-1)}{3\frac{N-1}{2}}$ iterations required on average.

Suffice it to say that the simplified joint tone-line selection algorithm presented in [6] can also be extended to the multi-user case by introducing the weights $\boldsymbol{\omega}$. However, because this

TABLE I

MULTI-USER RESOURCE ALLOCATION ALGORITHM

	capacities	multiplications	additions	comparisons
init: $v_k^n(r) = \frac{(b_k^n(r) - b_k^n(0))}{r}$ $\left\{ \begin{array}{l} k=1\ldots K \\ r=1\ldots N-1 \\ n=1\ldots N \end{array} \right.$	KNN	$KN(N-1)$	$KN(N-1)$	$KN\sum_{i=1}^{N-1} log_2(i)$
repeat				
$v_k^{\boldsymbol{\omega},n}(r) = \omega_n v_k^n(r)$	0	$KN(N-1)$	0	0
sort $v_k^{\boldsymbol{\omega},n}(r)$	0	0	0	$\sum_{i=1}^{KN(N-1)} log_2(i)$
repeat				
$(k_s, r_s, n_s) = \text{argmax}_{k,r,n}\, v_k^{\boldsymbol{\omega},n}(r)$	0	0	0	0
$v_{k_s}^{\boldsymbol{\omega},n_s}(r) = 0, \quad \forall r \leq r_s$	0	0	0	0
$v_{k_s}^{\boldsymbol{\omega},n_s}(r) = \omega_{n_s} \frac{(b_k^{n_s}(r) - b_k^{n_s}(r_s))}{(r-r_s)}, \quad \forall r > r_s$	$\frac{N-1}{2} + 1$	$N-1$	$N-1$	0
re-sort $v_k^{\boldsymbol{\omega},n}(r)$	0	0	0	$\sum_{i=KN(N-1)-(\frac{N-1}{2}-1)}^{KN(N-1)} log_2(i)$
while $\sum_{n=1}^{N}\sum_{k=1}^{K} r_k^n < C^{tot}$	0	0	1	1
update $\boldsymbol{\omega}$ based on (6)				
while rate constraints not satisfied				

algorithm is only near optimal, update formula (13) can only provide convergence to a point close to the desired data rates.

B. Multi-user dual decomposition algorithm

In the dual decomposition approach, Algorithm 1 can be used to find an appropriate distribution of the cancellation tap budget over the users, where the per-tone search is simplified based on the observations in section II-C. The resulting algorithm and complexities are shown in **Table II**. Because the updates of the Lagrange multipliers are based on the same update formula as in the resource allocation algorithm, roughly the same number of I iterations is required to enforce the constraints.

The algorithm starts by initializing a $K \times N \times N$ table of capacities for K tones, N users and N possible crosstalk cancellation configurations. To obtain the N possible crosstalk cancellation configurations, the line selection observation of section II-C is used. This requires sorting the crosstalkers which requires $KN\sum_{i=1}^{N-1} log_2(i)$ comparisons.

The algorithm then starts from some initial λ and ω's and performs K per-tone exhaustive searches. There are N possible values for λr, which can be calculated in advance. This requires N multiplications. These precalculated values are then subtracted from the corresponding elements of the $K \times N \times N$ table, after scaling them with the ω's which requires KNN multiplications. Finally, K exhaustive searches of $N \times N$ values are performed to obtain the maximum on each tone. This requires $KN(N-1)$ comparisons.

The cancellation tap constraint and the rate constraints are then checked by summing the number of taps allocated on each tone. If the constraints are not tightly satisfied, the Lagrange multipliers are updated and the per-tone search is repeated.

In **Table III** the total complexities of the multi-user resource allocation algorithm and the dual decomposition algorithm are compared.

Figure 1 shows the complexities as a function of the number of users for the resource allocation algorithm and the dual decomposition algorithm for $K = 1000$, under the assumption that $I = 50$ iterations are required to enforce the constraints. Note the significant decrease in multiplications, capacity calculations and comparisons for the dual decomposition algorithm. This complexity decrease largely overpowers the increase in the number of additions.

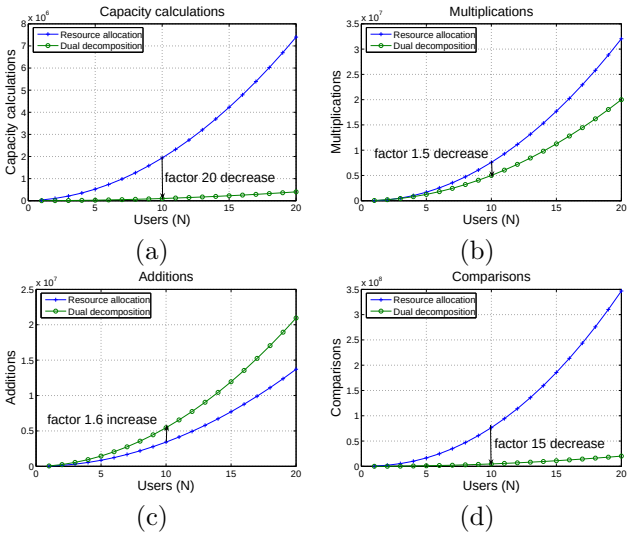


Fig. 1. Complexity comparison multi-user algorithms: (a) capacity calculations (b) multiplications (c) additions (d) comparisons

IV. CONCLUSION

In this paper we have presented an algorithm to optimally solve the partial crosstalk cancellation problem, based on a dual decomposition. The algorithm starts from a common cancellation tap budget which has to be divided over the users in such a way that rate constraints are satisfied. The dual decomposition approach naturally incorporates these rate constraints. Further, an alternative resource allocation algorithm was extended to the multi-user case to also include these rate constraints. The extension allows for the same search procedure to be used to find the optimal distribution of the cancellation tap budget over the users as used in the dual decomposition algorithm. The complexity of the dual

TABLE II

MULTI-USER DUAL DECOMPOSITION ALGORITHM

	capacities	multiplications	additions	comparisons
init: $b_k^n(r)$ $\left\{\begin{array}{l} k{=}1\ldots K \\ r{=}0\ldots N{-}1 \\ n{=}1\ldots N \end{array}\right.$	KNN	0	0	$KN\sum_{i=1}^{N-1} log_2(i)$
repeat				
for $k = 1 \ldots K$				
for $n = 1 \ldots N$				
$\mathbf{c}_k^{n,opt} = \text{argmax}_r\, \omega_n b_k^n(r) - \lambda r$	0	$N + KNN$	KNN	$KN(N-1)$
endfor				
endfor				
update $\boldsymbol{\omega}, \lambda$ based on (6)				
while $\sum_{n=1}^{N}\sum_{k=1}^{K} \mathbf{c}_k^{n,opt} \neq C^{tot}$ and rate constraints not satisfied	0	0	$(N-1)(K-1)$	1

TABLE III

COMPLEXITY COMPARISON MULTI-USER ALGORITHMS

	resource allocation	dual decomposition
capacities	$KNN + I \times \frac{KN(N-1)}{3\frac{N-1}{2}}\left(\frac{N-1}{2}+1\right)$	KNN
multiplications	$KN(N-1) + I \times \left(KN(N-1) + \frac{KN(N-1)}{3\frac{N-1}{2}}(N-1)\right)$	$I \times (N + KNN)$
additions	$KN(N-1) + I \times \frac{KN(N-1)}{3\frac{N-1}{2}} N$	$I \times \big(KNN + (N-1)(K-1)\big)$
comparisons	$KN\sum_{i=1}^{N-1} log_2(i) + I \times \sum_{i=1}^{KN(N-1)} log_2(i) +$ $\frac{KN(N-1)}{3\frac{N-1}{2}}\left(1 + \sum_{i=KN(N-1)-(\frac{N-1}{2}-1)}^{KN(N-1)} log_2(i)\right)$	$KN\sum_{i=1}^{N-1} log_2(i) + I \times \big(KN(N-1)+1\big)$

decomposition algorithm was found to compare favourably with the complexity of the resource allocation algorithm.

APPENDIX I
SEARCH ALGORITHM FOR THE LAGRANGE MULTIPLIERS

The proof presented in [9] [10] can be easily adapted for partial crosstalk cancellation. Assume a two-user scenario with signal-level control. Starting from two optimal solutions $(R^{1,\boldsymbol{\omega}_A,\lambda_A}, R^{2,\boldsymbol{\omega}_A,\lambda_A}, C^{\boldsymbol{\omega}_A,\lambda_A})$ and $(R^{1,\boldsymbol{\omega}_B,\lambda_B}, R^{2,\boldsymbol{\omega}_B,\lambda_B}, C^{\boldsymbol{\omega}_B,\lambda_B})$ corresponding to $(\boldsymbol{\omega}_A, \lambda_A)$ and $(\boldsymbol{\omega}_B, \lambda_B)$ respectively, optimality for $(\boldsymbol{\omega}_A, \lambda_A)$ implies

$$\begin{array}{rl} & \omega_{1,A} R^{1,\boldsymbol{\omega}_B,\lambda_B} + \omega_{2,A} R^{2,\boldsymbol{\omega}_B,\lambda_B} - \lambda_A C^{\boldsymbol{\omega}_B,\lambda_B} \\ \leq & \omega_{1,A} R^{1,\boldsymbol{\omega}_A,\lambda_A} + \omega_{2,A} R^{2,\boldsymbol{\omega}_A,\lambda_A} - \lambda_A C^{\boldsymbol{\omega}_A,\lambda_A} \end{array} \quad (14)$$

Optimality for $(\boldsymbol{\omega}_B, \lambda_B)$ implies

$$\begin{array}{rl} & \omega_{1,B} R^{1,\boldsymbol{\omega}_A,\lambda_A} + \omega_{2,B} R^{2,\boldsymbol{\omega}_A,\lambda_A} - \lambda_B C^{\boldsymbol{\omega}_A,\lambda_A} \\ \leq & \omega_{1,B} R^{1,\boldsymbol{\omega}_B,\lambda_B} + \omega_{2,B} R^{2,\boldsymbol{\omega}_B,\lambda_B} - \lambda_B C^{\boldsymbol{\omega}_B,\lambda_B} \end{array} \quad (15)$$

Taking the sum of (14) and (15) results in

$$\begin{array}{l} -\underbrace{\left(\omega_{1,B} - \omega_{1,A}\right)}_{\Delta\omega_1}\underbrace{\left(R^{1,\boldsymbol{\omega}_B,\lambda_B} - R^{1,\boldsymbol{\omega}_A,\lambda_A}\right)}_{\Delta R^1} \\ -\underbrace{\left(\omega_{2,B} - \omega_{2,A}\right)}_{\Delta\omega_2}\underbrace{\left(R^{2,\boldsymbol{\omega}_B,\lambda_B} - R^{2,\boldsymbol{\omega}_A,\lambda_A}\right)}_{\Delta R^2} \\ +\underbrace{\left(\lambda_B - \lambda_A\right)}_{\Delta\lambda}\underbrace{\left(C^{\boldsymbol{\omega}_B,\lambda_B} - C^{\boldsymbol{\omega}_A,\lambda_A}\right)}_{\Delta C} \leq 0 \end{array} \quad (16)$$

Relation (16) is straightforwardly extended to a multi-user scenario:

$$\left[\begin{array}{cc} -(\Delta\boldsymbol{\omega})^T & \Delta\lambda \end{array}\right]\left[\begin{array}{c} \Delta\mathbf{R} \\ \Delta C \end{array}\right] \leq 0 \quad (17)$$

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