Multiple Access Channel Optimal Spectrum Balancing for Upstream DSL Transmission

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***Abstract*— Upstream DSL transmission suffers from in-domain crosstalk as well as out-of-domain or alien crosstalk. Here, the use of multi-user receiver signal coordination e.g. generalized decision feedback equalization, can lead to spectacular performance gains. This paper presents a transmission scheme, referred to as MAC-OSB, which focuses on the weighted rate sum capacity by joint receiver signal coordination and transmit spectrum coordination. The proposed scheme incorporates per-user total power constraints, spectral mask constraints and discrete bit or power loading constraints. Furthermore a low-complexity scheme, referred to as MAC-ISB, is presented which performs similar to MAC-OSB. Simulations show large performance gains over existing methods especially for scenarios with significant alien crosstalk.**

***Index Terms*— Crosstalk, digital subscriber lines (DSL), dynamic spectrum management (DSM), multiple access channel (MAC), decision feedback equalization.**

I. Introduction

THE increasing demand for high speed services forces DSL systems to use higher frequencies, e.g. up to 30 Mhz for VDSL2. At these frequencies, in-domain crosstalk between different DSL lines in the same cable bundle as well as out-of-domain or alien crosstalk, becomes particularly harmfull and is a dominant source of performance degradation.

By the use of multi-user signal coordination at the transmitter side and/or the receiver side, the crosstalk can be cancelled which leads to large performance gains. In this paper, we are interested in the case where there is only coordination possible at the receiver side. This is referred to as the multiple access channel (MAC) in information theory. The driving application is upstream DSL where multiple transmitters send independent information to one central office (CO) or optical network unit (ONU), which then acts as a joint receiver. Several receiver structures have been proposed to cancel the crosstalk [1] [2]. In [3] it is shown that a simple linear zero-forcing receiver performs near-optimally in a DSL environment with only in-domain crosstalk. Unfortunately it will be demonstrated that this receiver performs very sub-optimally in scenarios with significant alien crosstalk. In [4] a procedure is presented for achieving the *unweighted* rate sum capacity in a MAC, consisting of a MMSE-GDFE receiver [2] and per-user iterative waterfilling based transmit power loading (MMSE-GDFE-IWF).

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This paper presents a transmission scheme, referred to as *MAC-OSB*, which focuses on the *weighted* rate sum capacity by joint receiver signal coordination and transmit spectrum coordination. The proposed algorithm incorporates per-user total power constraints, spectral mask constraints and discrete bit or power loading constraints. In [5] discrete bit loading was considered for the single-user case under total power and spectral mask constraints. Furthermore a low-complexity scheme, referred to as MAC-ISB, is presented which performs similar to MAC-OSB. In the simulation section the performance of these schemes is compared with existing schemes.

II. System Model

Current DSL systems use Discrete Multi-Tone (DMT) modulation, where the available frequency band is divided in a number of parallel subchannels or tones. The modems in the network are assumed to be synchronized and so each tone is capable of transmitting data independently from other tones. Transmission for a cable bundle of N users can be modelled on each tone k by

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{z}_k \qquad k = 1 \dots K. \tag{1}$$

The vector $\mathbf{x}_k = [x_k^1, x_k^2, \dots, x_k^N]^T$ contains the transmitted signals on tone k for all N users. $\mathbf{z}_k$ is the vector of additive noise on tone k which includes the alien crosstalk. To keep the formulae simple we assume that the noise is pre-whitened $E\{\mathbf{z}_k \mathbf{z}_k^H\} = I_{N\text{x}N}$. $[\mathbf{H}_k]_{n,m} = h_k^{n,m}$ is an $N \times N$ matrix containing the pre-whitened channel transfer functions from transmitter m to receiver n. The diagonal elements are the direct channels, the off-diagonal elements are the in-domain crosstalk channels. The vector $\mathbf{y}_k$ contains the received symbols. We denote the transmit power as $s_k^n \triangleq E\{|x_k^n|^2\}$. The vector containing the transmit power of user n on all tones is $\mathbf{s}^n \triangleq [s_1^n, s_2^n, \dots, s_K^n]^T$.

The upstream DSL channel is a MAC which means that on the one hand a joint decoding can be done at the receiver and on the other hand the transmit spectra can be properly set by each individual transmitter (user). Its sum capacity is given by

$$C_k = \log_2(\det(I + \mathbf{H}_k \mathbf{S}_k \mathbf{H}_k^H)), \tag{2}$$

where $\mathbf{S}_k = \text{diag}\{s_k^1, \dots, s_k^N\}$ which is diagonal because transmit signal coordination is indeed not possible.

We will consider total power constraints for every user $\sum_k s_k^n \leq P^{n,tot} \quad \forall n$ and spectral mask constraints $0 \leq s_k^n \leq s_k^{n,mask} \quad \forall n, k$.

III. Multiple Access Channel Optimal Spectrum Balancing (MAC-OSB)

For conciseness (without loss of generality) we restrict some of the formulae to the 2-user case. The data model (1) can then

be reformulated as

$$\mathbf{y}_k = \left[\begin{array}{cc} \mathbf{h}_k^1 & \mathbf{h}_k^2 \end{array}\right] \left[\begin{array}{c} x_k^1 \\ x_k^2 \end{array}\right] + \mathbf{z}_k = \mathbf{h}_k^1 x_k^1 + \mathbf{h}_k^2 x_k^2 + \mathbf{z}_k.$$

Using this reformulation the capacity formula (2) can be dissected into the following *unweighted* rate sum

$$\begin{aligned} C_k &= \log_2(\det(\mathbf{I} + \mathbf{h}_k^1 s_k^1 \mathbf{h}_k^{1,H} + \mathbf{h}_k^2 s_k^2 \mathbf{h}_k^{2,H})) \\ &= \log_2(\det(\mathbf{I} + (\mathbf{I} + \mathbf{h}_k^2 s_k^2 \mathbf{h}_k^{2,H})^{-1} \mathbf{h}_k^1 s_k^1 \mathbf{h}_k^{1,H})) \\ &\qquad + \log_2(\det(\mathbf{I} + \mathbf{h}_k^2 s_k^2 \mathbf{h}_k^{2,H})) \quad (3) \\ &= b_k^1 + b_k^2. \end{aligned}$$

The first term of (3) represents a bit loading of user 1 when user 1 is detected under crosstalk from user 2. The second term represents a bit loading of user 2 when the detection of user 2 is done after having removed the crosstalk from user 1. This exactly corresponds to the operation of the MMSE-GDFE receiver [2] and thus this receiver is indeed (unweighted rate sum) optimal for given transmit powers. Note that changing the detection order changes per-user rates, but does not change the rate sum. In [4] it is proved that if the MMSE-GDFE receiver is combined with per-user iterative waterfilling (IWF), the *unweighted* rate sum is maximized under per-user transmit power constraints (MMSE-GDFE-IWF).

Our objective will be to optimize the *weighted* rate sum

$$\tilde{C}_k = \omega_1 b_k^1 + \omega_2 b_k^2, \quad \omega_1, \omega_2 \geq 0 \qquad (4)$$

where ω_1 and ω_2 are weights given to the different users. Our overall optimization problem is

$$\begin{array}{ll} \max_{(\mathbf{s}^1,\ldots,\mathbf{s}^N,receiver)} & \sum_{k=1}^K \tilde{C}_k \\ \text{subject to} & \sum_k s_k^n \leq P^{n,tot} \quad \forall n \\ & 0 \leq s_k^n \leq s_k^{n,mask} \quad \forall n,k, \end{array}$$

where the optimization is over transmit spectra as well as receiver structures.

Theorem 3.1: For given transmit spectra, the MMSE-GDFE receiver is the optimal receiver for the weighted rate sum (4). The weights define the detection order where users with the smallest weights are detected first and users with the largest weights are detected last.

Proof: For given transmit spectra, the achievable rate region (for each tone) is known. This is a pentagon with one side making an angle of -45 degrees as shown in **Figure** 1 (2-user case). The two end points a,b of this side are achievable by a MMSE-GDFE receiver. The weighted rate sum optimum is the crossing of the pentagon and the tangent defined by the weights ω_1, ω_2. As the weights are always positive the crossing has to be one of the points a or b. As these points are achievable by the MMSE-GDFE receiver, this receiver is optimal. If $\omega_1 > \omega_2$ the optimum is point a (user 2 detected first), otherwise the optimum is point b (user 1 detected first).

The remaining question is how to find weighted rate sum optimal transmit spectra. This can be done similarly to the Optimal Spectrum Balancing (OSB) dual decomposition procedure [6] [7] which finds the optimal power loading for uncoordinated DSL transmission. The basic idea here is to use Lagrange multipliers to move the total power constraints into the cost function and so decouple the problem into independent per-tone optimization problems as follows:

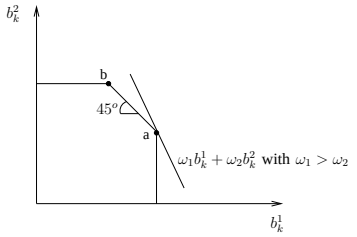


Fig. 1. Rate region on tone k

$$\max_{s_k^1,\ldots,s_k^N} \tilde{C}_k - \sum_{i=1}^N \lambda_i s_k^i \quad \text{s. t. } 0 \leq s_k^n \leq s_k^{n,mask} \quad \forall n. \quad (5)$$

By tuning the Lagrange multipliers λ_i the total power constraints can be enforced. In [7] an efficient algorithm is presented to find the optimal Lagrange multipliers. For given Lagrange multipliers the optimal power loading for the per-tone optimization problem (5) can be found by an exhaustive search over all possible power loadings. The following formulae can be used to calculate the bit loadings corresponding to certain power loadings for $\omega_1 < \omega_2$ (2-user case)

$$b_k^1 = \log_2(1 + \frac{1}{\Gamma} s_k^1 \mathbf{h}_k^{1,H} (\mathbf{I} + \mathbf{h}_k^2 s_k^2 \mathbf{h}_k^{2,H})^{-1} \mathbf{h}_k^1) \quad (6)$$

$$b_k^2 = \log_2(1 + \frac{1}{\Gamma} s_k^2 \mathbf{h}_k^{2,H} \mathbf{h}_k^2) \quad (7)$$

Note the addition (compared to (3)) of the SNR-gap to capacity Γ which takes into account QAM transmission and is a function of the desired BER, coding gain and noise margin. Note that by including the SNR-gap the theoretical optimality of the scheme is lost, as it is also the case for MMSE-GDFE-IWF [4]. However the proposed scheme provides an achievable rate region which is shown in the simulation section to outperform existing state-of-the-art schemes especially for scenarios with significant alien crosstalk.

This leads to following transmission scheme inspired by OSB to perform optimal transmit spectrum coordination (power loading) under MMSE-GDFE receive coordination, which we will refer to as *multiple access channel optimal spectrum balancing* (MAC-OSB):

Algorithm 1 MAC-OSB

pre-whiten noise
initialize weights and Lagrange multipliers
repeat
  update Lagrange multipliers
  per-tone exhaustive search over discrete power loadings
until total power constraints are satisfied

Note that, alternatively, achievable rate regions can be computed for the case of discrete (integer) bit loadings, based on exhaustive searches over all possible bit loadings, and where the corresponding power loadings s_k^1,s_k^2 are computed by inverting (6) and (7).

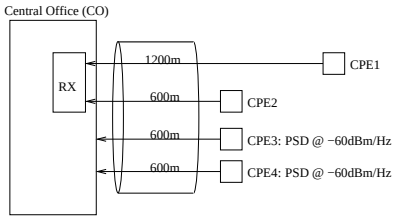


Fig. 2. VDSL upstream scenario

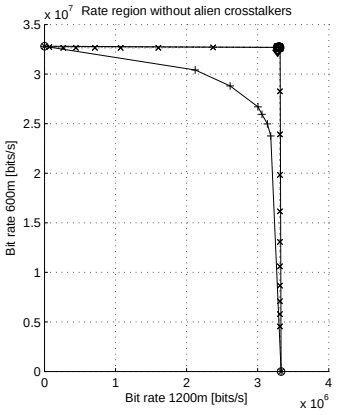


Fig. 3. No alien crosstalkers

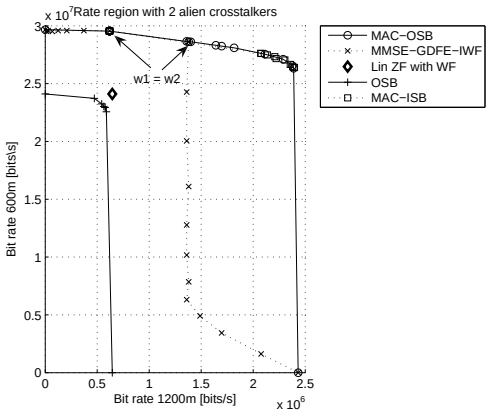


Fig. 4. Two alien crosstalkers

IV. Multiple Access Channel Iterative Spectrum Balancing (MAC-ISB)

The per-tone optimization problem of MAC-OSB is solved by an exhaustive search, with a complexity that is exponential in the number of users, i.e. B^N where B is the number of possible bit or power loadings and N is the number of users. For more than five users this becomes computationally intractable. In order to reduce this complexity one can resort to an iterative approach. This approach iteratively finds the best bit/power loading for user 1 assuming all other bit/power loadings are fixed, then similarly the loading for user 2 is optimized, etc., until user N. Then this is repeated until the loadings do not change anymore. The complexity reduces to a linear complexity LBN where L is the number of iterations which is typically very small (2 or 3). Because of the iterative approach we refer to this method as multiple access channel iterative spectrum balancing (MAC-ISB). Simulations show that this algorithm performs similar to MAC-OSB for many scenarios.

V. Simulation Results

The VDSL upstream scenario considered here is shown in **Figure** 2. At the CO the lines connected to CPE1 and CPE2 can be coordinated. DSL users CPE3 and CPE4 are two VDSL alien crosstalkers within the same cable bundle transmitting at -60 dBm/Hz. Bandplan FDD 988 is used. The line diameters are 0.4 mm. The maximum transmit power per user is 11.5 dBm for CPE1 and CPE2. The SNR gap Γ is 12.9 dB, corresponding to a coding gain of 3 dB, a noise margin of 6 dB and a target symbol error probability of 10^{-7}. The maximum bit loading is 16 bits. The goal is to maximize the upstream bit rates of CPE1 and CPE2 by choosing optimal transmit spectra with receiver coordination at the CO.

Figure 3 shows the rate regions in the absence of the two alien crosstalkers CPE3 and CPE4. In this case the noise is spatially white, and so pre-whitening is not needed, which would otherwise destroy the column-wise diagonal dominant structure (CWDD) of the channel [3]. Therefore the linear zero-forcing receiver in combination with per-user waterfilling has a near-optimal performance [3] which is very close to the performance of MAC-OSB, MAC-ISB and MMSE-GDFE-IWF. The rate region for MAC-OSB and MAC-ISB is obtained by varying the weights $\omega_i,\ i = 1, \ldots, N$. The rate region of MMSE-GDFE-IWF is created by keeping the total power of the users (except one) at the maximum and reducing one of the user's power to zero. Furthermore the rate region is plotted for OSB which shows the optimal performance for no receiver coordination and optimal transmit spectrum shaping.

Figure 4 shows the results of the scenario with the presence of the two alien crosstalkers. It can be seen that the linear zero-forcing receiver performs very sub-optimally. MAC-OSB has a much better performance than MMSE-GDFE-IWF. The MAC-ISB algorithm performs similar to MAC-OSB. Finally it is important to notice that the use of receiver coordination increases the bit rates significantly.

VI. Conclusion

A transmission scheme, referred to as MAC-OSB, has been presented which focuses on the weighted rate sum capacity in upstream VDSL scenarios by joint receiver signal coordination and transmit spectrum coordination. Furthermore a low-complexity scheme, referred to as MAC-ISB, is presented which performs similar to MAC-OSB. Simulations show that these algorithms outperform existing methods especially for scenarios with significant alien crosstalk.

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