Distributed Spectrum Management Algorithms for Multiuser DSL Networks

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***Abstract*—Modern digital subscriber line (DSL) networks suffer from crosstalk among different lines in the same cable bundle. This crosstalk can lead to a major performance degradation. By balancing the transmit power spectra, the impact of crosstalk can be minimized leading to spectacular performance gains. This is referred to as *spectrum management*. In this paper, a unifying perspective is presented on distributed spectrum management algorithms based on the Karush–Kuhn–Tucker (KKT) conditions. Furthermore, novel distributed algorithms are presented within the same KKT framework. The proposed distributed algorithms consist of local water-filling-like algorithms running in the individual modems, controlled by the spectrum management center. Extensive simulation results show that the proposed algorithms perform very well for several multi-user ADSL and VDSL scenarios.**

***Index Terms*—Digital subscriber line (DSL), distributed algorithm, interference channel, Karush–Kuhn–Tucker (KKT), multicarrier, power allocation, spectrum management.**

I. Introduction

DIGITAL subscriber line (DSL) technology remains by far the most popular broadband access technology with a 65.7% market share. In 2006, the number of DSL subscribers worldwide grew by more than 30% to reach close to 185 million [1]. The increasing demand for higher data rates forces DSL systems to use higher frequencies, e.g., up to 30 MHz for VDSL2. At these high frequencies, electromagnetic coupling becomes particularly harmful and causes interference, also called "crosstalk," among lines operating in the same cable bundle. This crosstalk is the major obstacle for performance improvement in DSL systems currently under development. *Dynamic spectrum management (DSM)* [2] refers to a set of solutions to the crosstalk problem. Basically, these solutions consist of signal level coordination and/or spectrum level coordination. In the signal level coordination solution, the crosstalk is cancelled by signal processing at the transmitter and/or the receiver. Several crosstalk canceller designs have been proposed [3], [4] [5]. However direct crosstalk cancellation is infeasible in many cases, due to complexity issues (both the amount of computation needed and the requirements of new chip sets) or as a result of unbundling (i.e., multiple operators own different lines of same cable bundle).

In this paper the focus is on spectrum level coordination, which is also referred to as spectrum management, spectrum balancing or multi-carrier power control. Here the transmit power spectrum of each modem is designed to cause minimal disturbance to other lines, while preserving a high data rate. This can significantly improve data rates beyond those of the current static spectrum management where fixed overconservative standardized transmit power spectra are used.

The problem of optimally choosing the transmit power spectra in order to maximize the data rates of the network can be formulated as an optimization problem [6], [7] which is referred to as the *spectrum management problem*. Unfortunately, this optimization problem is nonconvex and can have multiple locally optimal solutions. Three algorithms, optimal spectrum balancing (OSB) [6], branch and bound optimal spectrum balancing (BB-OSB) [7], and a prismatic branch and bound algorithm (PBB) [8], respectively, were proposed to compute the globally optimal solution, i.e., the globally optimal transmit spectra for the network of interfering DSL lines. Unfortunately, their complexity grows exponentially with the number of modems. A cheaper iterative approach, called *iterative spectrum balancing* (ISB), was simultaneously proposed in [9] and [10]. Here, the complexity is reduced through a series of line searches. OSB, BB-OSB, PBB, and ISB are centralized algorithms which means that they rely on a spectrum management center (SMC) to optimize the transmit power spectra for all modems simultaneously. Note that it is assumed that the SMC has access to estimates of the noise and the crosstalk channels obtained through training procedures or with the aid of crosstalk models and knowledge of the loop topology. *Distributed spectrum management* applies a different paradigm where each modem executes a spectrum management algorithm locally to compute its transmit power spectrum. One of the first and most famous distributed algorithms is iterative water-filling (IW) [11]. In this algorithm, each modem maximizes its own bit rate, where the crosstalk from the other modems is simply treated as noise. Although this leads to a "selfish" solution, the advantage is that the modems do not need

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any information from the other modems and so the iterative water-filling algorithm is a completely autonomous algorithm.

Another autonomous algorithm is the autonomous spectrum balancing (ASB) algorithm [12], that utilizes the concept of a reference line. Each modem optimizes its own bit rate taking into account the damage caused on the reference line avoiding a "selfish" behavior leading to a better network performance.

Recently another distributed spectrum management paradigm was introduced in [13] and is referred to as SCALE (Successive Convex Approximation for Low-complExity). The algorithm involves iterative convex approximations. Here, each modem executes a spectrum management algorithm locally. In addition, limited message-passing is allowed between the modems and the SMC. The resulting method can be seen as a distributed computation across the DSL network.

In this paper, the focus is on distributed spectrum management algorithms. First, it is shown how current distributed spectrum management algorithms can be viewed from a similar framework. This framework consists of choosing an (iterative) approximation of the nonconvex spectrum management problem and deriving a distributed solution based on the KKT conditions of the chosen approximation. Furthermore, novel distributed spectrum management algorithms are proposed within the same KKT framework. The first algorithm involves an iterative convex approximation approach. This is similar to SCALE; however, it chooses a different convex approximation based on a linearization of the nonconcave part of the objective function. This choice leads to an interesting water-filling-like structure. The second algorithm is based on the KKT conditions of the spectrum management problem. The third algorithm is a multiple starting point extension of the second algorithm which leads to a better performance for multiple user scenarios with multiple locally optimal solutions. All the proposed algorithms consist of local water-filling-like algorithms running in the modems, steered by a SMC.

The paper is organized as follows. Section II describes the system model for a network of interfering DSL modems. In Section III, the nonconvex spectrum management problem is briefly formulated. Section IV presents current state-of-the-art distributed spectrum management algorithms from a KKT perspective. In Section V, three novel spectrum management algorithms are proposed and discussed. Finally, in Section VI extensive simulation results compare the proposed algorithms with state-of-the-art distributed and centralized spectrum management algorithms for several multiuser ADSL and VDSL scenarios.

II. System Model

Most current DSL systems use discrete multi-tone (DMT) modulation. The basic idea of DMT is to split the available bandwidth into a large number of subcarriers also called tones. Under certain conditions (sufficiently long cyclic prefix and perfect tone synchronization), each tone is capable of transmitting data independently from other tones, and so the transmit power and the number of bits can be assigned individually for each tone. This gives a large flexibility in optimally shaping the transmit spectrum. The transmission for a binder of N modems can be modeled on each tone k by

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{z}_k, \quad k = 1 \dots K.$$

where K is the number of tones. The vector $\mathbf{x}_k = \left[x_k^1, x_k^2, \dots, x_k^N\right]^T$ contains the transmitted symbols on tone k for all N modems. $[\mathbf{H}_k]_{n,m} = h_k^{n,m}$ is an $N \times N$ matrix containing the channel transfer functions from transmitter m to receiver n on tone k. The diagonal elements are the direct channels, the off-diagonal elements are the crosstalk channels. $\mathbf{z}_k$ is the vector of additive noise on tone k, containing thermal noise, alien crosstalk, RFI, etc. The vector $\mathbf{y}_k$ contains the received symbols.

The transmit power is denoted as $s_k^n \triangleq \Delta_f E\left\{|x_k^n|^2\right\}$, the noise power as $\sigma_k^n \triangleq \Delta_f E\left\{|z_k^n|^2\right\}$. The vector containing the transmit power of modem n on all tones is $\mathbf{s}^n \triangleq [s_1^n, s_2^n, \dots, s_K^n]^T$. The DMT symbol rate is denoted as f_s, the tone spacing as Δ_f.

In this model, no signal coordination is assumed between modems and the signals from other modems are treated as noise: such a multiuser channel is referred to as an "interference channel." When the number of interfering modems is large, the interference is well approximated by a Gaussian distribution. Under this assumption the achievable bit loading for modem n on tone k, given the transmit spectra $\mathbf{s}_k \triangleq \left[s_k^1, s_k^2, \dots, s_k^N\right]^T$ of all modems in the system, is

$$b_k^n \triangleq \log_2\left(1 + \frac{1}{\Gamma}\frac{|h_k^{n,n}|^2 s_k^n}{\sum\limits_{m \neq n} |h_k^{n,m}|^2 s_k^m + \sigma_k^n}\right) \tag{1}$$

where Γ denotes the signal-to-noise ratio (SNR) gap to capacity, which is a function of the desired bit error ratio (BER), the coding gain and noise margin [14].

The data rate for modem n and the total power used by modem n are, respectively

$$R^n = f_s \sum_k b_k^n \text{ and } P^n = \sum_k s_k^n.$$

III. Spectrum Management Problem

The problem of optimally balancing the transmit power spectra in order to maximize the data rates of the DSL network is referred to as the rate-adaptive spectrum management problem. The objective is to find the optimal transmit spectra for a bundle of interfering DSL lines, maximizing a weighted sum of data rates subject to per-modem total power constraints and spectral mask constraints. This can be formulated as the following nonconvex optimization problem:

$$\begin{aligned} &\max_{\mathbf{s}^1,\dots,\mathbf{s}^N} \sum_{n=1}^{N} w_n R^n \\ &\text{s.t.} \sum_k s_k^n \leq P^{n,\text{tot}} \quad \forall\, n, \\ &\text{s.t. } 0 \leq s_k^n \leq s_k^{n,\text{mask}} \quad \forall\, n,\ \forall\, k \end{aligned} \tag{2}$$

where $P^{n,\mathrm{tot}}$ denotes the total power budget for modem n and $s_k^{n,\mathrm{mask}}$ denotes the spectral mask for modem n on tone k. The weights w_n are used to put more emphasis on some modems. In [7] it is explained how these weights can be adjusted when extra data rate constraints have to be satisfied. All the algorithms explained in this paper can be easily extended with data rate constraints.

IV. A KKT Perspective on State-of-the-Art Distributed Spectrum Management Algorithms

In this section it will be shown how state-of-the-art distributed spectrum management algorithms can be viewed from a similar framework. This framework consists of two steps. The first step is approximating the nonconvex spectrum management problem (2). This can be done by a (non)convex approximation or by iterative (non)convex approximations. The second step is deriving a distributed solution based on the KKT conditions of these approximations. This unifying perspective simplifies the comparison of the different algorithms to better understand their strengths and limitations. Furthermore, it shows that a key design parameter is choosing a good (iterative) approximation. Possible design criteria that can be considered in the choice of approximations are complexity, (sub)optimality, convergence properties, communication overhead, etc. It will be shown how state-of-the-art distributed algorithms fit in this framework by explaining their approximation and how they are derived based on the KKT conditions of their approximation. However there are more possible (iterative) approximations to tackle the spectrum management problem each of them having different properties. In Section V it will be shown how other approximations lead to algorithms with interesting properties.

A. KKT Perspective on Iterative Water-Filling

The iterative water-filling algorithm [11] is an extension of the single-user water-filling algorithm [15] to multiple users. Each modem iteratively applies the single-user water-filling algorithm while the crosstalk of the other modems is fixed. In this way each modem maximizes its own bit rate without taking into account the damage caused to the other modems. This approach generally does not lead to the solution of the spectrum management problem (2). In fact, it corresponds to solving the following optimization problem for each modem n:

$$\begin{aligned}&\max_{s_1^n,\ldots,s_K^n} w_n f_s \sum_k \log_2\left(1+\frac{1}{\Gamma}\frac{|h_k^{n,n}|^2 s_k^n}{\sum_{m\neq n}|h_k^{n,m}|^2 s_k^m+\sigma_k^n}\right)\\ &\text{s.t. } \sum_k s_k^n \le P^{n,\mathrm{tot}}\\ &\text{s.t. } 0\le s_k^n \le s_k^{n,\mathrm{mask}} \quad \forall\, k. \end{aligned} \tag{3}$$

The objective function is a concave function as the crosstalk of the other modems $\sum_{m\neq n}|h_k^{n,m}|^2 s_k^m$ is fixed. Problem (3) can be viewed as a convex approximation of (2). If the crosstalk is negligible, approximation (3) will lead to a solution similar to the solution of (2). In the case of large crosstalk, solving approximation (3) leads to a different and suboptimal solution. The advantage of the approximation is its convexity. The globally optimal (and distributed) solution of this convex problem (for the nth modem, with fixed crosstalk from the other modems) can be found by solving the system obtained by its KKT conditions. The KKT conditions for modem n are as follows:

$$\frac{w_n f_s |h_k^{n,n}|^2}{\log(2)\left(|h_k^{n,n}|^2 s_k^n+\sum_{m\neq n}\Gamma|h_k^{n,m}|^2 s_k^m+\Gamma\sigma_k^n\right)} -\lambda_n+\mu_k^n-\nu_k^n=0 \quad \forall\, k \tag{4}$$

$$\sum_{k=1}^K s_k^n - P^{n,\mathrm{tot}} \le 0 \tag{5}$$

$$-s_k^n \le 0 \quad \forall\, k \tag{6}$$

$$s_k^n \le s_k^{n,\mathrm{mask}} \quad \forall\, k \tag{7}$$

$$\lambda_n \ge 0 \tag{8}$$

$$\mu_k^n \ge 0 \quad \forall\, k \tag{9}$$

$$\nu_k^n \ge 0 \quad \forall\, k \tag{10}$$

$$\lambda_n\left(\sum_{k=1}^K s_k^n - P^{n,\mathrm{tot}}\right)=0 \tag{11}$$

$$-\mu_k^n s_k^n = 0 \quad \forall\, k \tag{12}$$

$$\nu_k^n\left(s_k^n - s_k^{n,\mathrm{mask}}\right)=0 \quad \forall\, k. \tag{13}$$

Using (4) the optimal transmit powers s_k^n for fixed dual variables $\lambda_n, \nu_k^n, \mu_k^n$ are given by the following closed-form formula:

$$s_k^n = \frac{w_n f_s}{\log(2)\left(\lambda_n+\nu_k^n-\mu_k^n\right)} - \frac{\sum_{m\neq n}\Gamma|h_k^{n,m}|^2 s_k^m+\Gamma\sigma_k^n}{|h_k^{n,n}|^2} \tag{14}$$

where λ_n corresponds to constraint $\sum_k s_k^n \le P^{n,\mathrm{tot}}$, ν_k^n corresponds to constraint $s_k^n \le s_k^{n,\mathrm{mask}}$ and μ_k^n corresponds to constraint $0 \le s_k^n$. The KKT conditions are then equivalent to the conditions in (15), shown at the top of the next page, which can be summarized as follows:

$$s_k^n = \left[\frac{w_n f_s}{\log(2)\lambda_n} - \frac{\Gamma\left(\sum_{m\neq n}|h_k^{n,m}|^2 s_k^m+\sigma_k^n\right)}{|h_k^{n,n}|^2}\right]_0^{s_k^{n,\mathrm{mask}}} \tag{16}$$

$$\lambda_n\left(\sum_{k=1}^K s_k^n - P^{n,\mathrm{tot}}\right)=0 \tag{17}$$

where $[x]^a$ means $\min(x,a)$ and $[x]_b$ means $\max(x,b)$. Note that the spectral mask constraints are easily taken into account. Furthermore λ_n has to be chosen so that the complementary slackness condition (17) is satisfied. Note that the total power constraints will always be active. This follows from the particular choice of convex approximation (3). A modem always benefits from allocating more power and therefore will use its total available power. The λ_n can be found using a distributed approach [7], [13] [16], [17] for updating λ_n:

$$\lambda_n = \left[\lambda_n + \epsilon\left(\sum s_k^n - P^{n,\mathrm{tot}}\right)\right]_0. \tag{18}$$

$$
\begin{aligned}
&1)\quad \text{If } \frac{w_n f_s}{\log(2)\lambda_n} - \frac{\sum\limits_{m\neq n} \Gamma |h_k^{n,m}|^2 s_k^m + \Gamma\sigma_k^n}{|h_k^{n,n}|^2} \leq 0,\\
&\qquad \text{then } s_k^n = 0,\ \nu_k^n = 0,\ \mu_k^n = \lambda_n - \frac{w_n f_s |h_k^{n,n}|^2}{\log(2)\left(\sum\limits_{m\neq n} \Gamma |h_k^{n,m}|^2 s_k^m + \Gamma\sigma_k^n\right)}.\\
&2)\quad \text{If } 0 \leq \frac{w_n f_s}{\log(2)\lambda_n} - \frac{\sum\limits_{m\neq n} \Gamma |h_k^{n,m}|^2 s_k^m + \Gamma\sigma_k^n}{|h_k^{n,n}|^2} \leq s_k^{n,\max},\\
&\qquad \text{then } s_k^n = \frac{w_n f_s}{\log(2)\lambda_n} - \frac{\sum\limits_{m\neq n} \Gamma |h_k^{n,m}|^2 s_k^m + \Gamma\sigma_k^n}{|h_k^{n,n}|^2},\ \nu_k^n = 0,\ \mu_k^n = 0.\\
&3)\quad \text{If } \frac{w_n f_s}{\log(2)\lambda_n} - \frac{\sum\limits_{m\neq n} \Gamma |h_k^{n,m}|^2 s_k^m + \Gamma\sigma_k^n}{|h_k^{n,n}|^2} \geq s_k^{n,\max},\\
&\qquad \text{then } s_k^n = s_k^{n,\max},\ \mu_k^n = 0,\ \nu_k^n = \frac{w_n f_s}{\log(2)\left(s_k^{n,\max} + \frac{\sum\limits_{m\neq n} \Gamma |h_k^{n,m}|^2 s_k^m + \Gamma\sigma_k^n}{|h_k^{n,n}|^2}\right)} - \lambda_n.\\
&4)\quad \lambda_n \left(\sum_{k=1}^{K} s_k^n - P^{n,\text{tot}}\right) = 0
\end{aligned}
\tag{15}
$$

where ϵ is a step size parameter. Alternatively, one can use a simple bisection search on the λ_n for every modem n.

This leads to a distributed algorithm where every modem n iteratively applies formula (16) and bisects on λ_n to converge to the optimal solution of approximation (3). The IW algorithm is shown in Algorithm 1, where δ is a small value indicating the required accuracy in total power, γ is a small value for stopping the bisection search on λ_n in the case of an inactive total power constraint and $\Lambda^{\max}$ is a maximum value for the λ_n.

To summarize, it is shown that the iterative water-filling algorithm can be seen as a convex approximation of the spectrum management problem (2). The choice of this approximation leads to near-optimal performance for small crosstalk scenarios. The KKT conditions are then used to derive the distributed (and fully autonomous) iterative water-filling procedure.

B. KKT Perspective on SCALE

The SCALE algorithm, proposed in [13], is based on iterative convex approximations of the spectrum management problem (2). The following lower bound on $\log(1+z)$ is used:

$$
\begin{aligned}
\alpha_k^n \log z + \beta_k^n &\leq \log(1+z)\\
\text{where } \alpha_k^n &= \frac{z_0}{1+z_0}\\
\text{and } \beta_k^n &= \log(1+z_0) - \frac{z_0}{1+z_0}\log z_0.
\end{aligned}
\tag{19}
$$

This bound is tight with equality at a chosen value $z_0 > 0$ when the constants $\{\alpha_k^n, \beta_k^n\}$ are chosen as specified above. This leads to the following approximation of problem (2):

$$
\begin{aligned}
&\max_{\mathbf{s}^1,\ldots,\mathbf{s}^N} \sum_n w_n f_s\\
&\qquad \times \sum_k \alpha_k^n \log_2\left(\frac{1}{\Gamma}\frac{|h_k^{n,n}|^2 s_k^n}{\sum\limits_{m\neq n} |h_k^{n,m}|^2 s_k^m + \sigma_k^n}\right) + \beta_k^n\\
&\text{s.t. } \sum_k s_k^n \leq P^{n,\text{tot}} \quad \forall\, n\\
&\text{s.t. } 0 \leq s_k^n \leq s_k^{n,\max} \quad \forall\, n\ \forall\, k.
\end{aligned}
\tag{20}
$$

Using the transformation $\tilde{s}_k^n = \log(s_k^n)$, the approximation (20) is equivalent to the following convex optimization problem

$$
\begin{aligned}
&\max_{\tilde{\mathbf{s}}^1,\ldots,\tilde{\mathbf{s}}^N} \sum_n w_n f_s\\
&\qquad \times \sum_k \alpha_k^n \log_2\left(\frac{1}{\Gamma}\frac{|h_k^{n,n}|^2 e^{\tilde{s}_k^n}}{\sum\limits_{m\neq n} |h_k^{n,m}|^2 e^{\tilde{s}_k^m} + \sigma_k^n}\right) + \beta_k^n\\
&\text{s.t. } \sum_k e^{\tilde{s}_k^n} \leq P^{n,\text{tot}}, \quad \forall\, n,\\
&\text{s.t. } e^{\tilde{s}_k^n} \leq s_k^{n,\max}, \quad \forall\, n\ \forall\, k.
\end{aligned}
\tag{21}
$$

where the constraint $0 \leq s_k^n$ is always satisfied if $\tilde{s}_k^n$ is transformed back to s_k^n.

Algorithm 1 IW

1: Modem $\mathbf{n}$ algorithm:
2: Initialize $s_k^n = 0, \forall k$
3: loop {Execute at regular intervals}
4: $\lambda_n^{\min} = 0, \lambda_n^{\max} = \Lambda^{\max}$
5: $\lambda_n = (\lambda_n^{\max} + \lambda_n^{\min})/2$
6: while $|\sum_k s_k^n - P^{n,\text{tot}}| > \delta$ and $\lambda_n > \gamma$ do
7: $\lambda_n = (\lambda_n^{\max} + \lambda_n^{\min})/2$
8: Update s_k^n using (16), $\forall k$
9: if $\sum_k s_k^n > P^{n,\text{tot}}$ then
10: $\lambda_n^{\min} = \lambda_n$
11: else
12: $\lambda_n^{\max} = \lambda_n$
13: end if
14: end while
15: end loop

Algorithm 2 SCALE

1: Modem $\mathbf{n}$ algorithm:
2: Initialize $s_k^n = 0, \alpha_k^n = 1, \forall k$
3: loop {Execute at regular intervals}
4: $\lambda_n^{\min} = 0, \lambda_n^{\max} = \Lambda^{\max}$
5: $\lambda_n = (\lambda_n^{\max} + \lambda_n^{\min})/2$
6: Receive messages $P_k^{\text{SCALE},n}$ from SMC, $\forall k$
7: while $|\sum_k s_k^n - P^{n,\text{tot}}| > \delta$ and $\lambda_n > \gamma$ do
8: $\lambda_n = (\lambda_n^{\max} + \lambda_n^{\min})/2$
9: Update s_k^n using (22), $\forall k$
10: if $\sum_k s_k^n > P^{n,\text{tot}}$ then
11: $\lambda_n^{\min} = \lambda_n$
12: else
13: $\lambda_n^{\max} = \lambda_n$
14: end if
15: end while
16: Measure $\text{int}_k^n = \sum_{p \neq n} |h_k^{n,p}|^2 s_k^p + \sigma_k^p$, $\forall k$
17: Update α_k^n with (19), $\forall k$
18: Transmit $\frac{\alpha_k^n}{\text{int}_k^n}$ to SMC, $\forall k$
19: end loop
20: SMC algorithm:
21: loop
22: Receive messages $\frac{\alpha_k^n}{\text{int}_k^n}$ from modems n, $\forall k$
23: Compute messages $P_k^{\text{SCALE},n}$
and send to each modem n, $\forall k$.
24: end loop

Algorithm 3 ASB-2

1: Modem $\mathbf{n}$ algorithm:
2: Initialize $s_k^n = s_k^{n,\text{mask}}/2, \forall k$
3: Initialize all reference line parameters
4: loop {Execute at regular intervals}
5: $\lambda_n^{\min} = 0, \lambda_n^{\max} = \Lambda^{\max}$
6: $\lambda_n = (\lambda_n^{\max} + \lambda_n^{\min})/2$
7: while $|\sum_k s_k^n - P^{n,\text{tot}}| > \delta$ and $\lambda_n > \gamma$ do
8: $\lambda_n = (\lambda_n^{\max} + \lambda_n^{\min})/2$
9: Update s_k^n using (26), $\forall k$
10: if $\sum_k s_k^n > P^{n,\text{tot}}$ then
11: $\lambda_n^{\min} = \lambda_n$
12: else
13: $\lambda_n^{\max} = \lambda_n$
14: end if
15: end while
16: end loop

Now a similar approach [see (4)–(15)] can be followed as in the previous section to derive a distributed algorithm based on the KKT conditions. In particular, based on the KKT stationarity condition and the complementarity conditions of approximation (21), the following fixed point equation can be derived:

$$s_k^n = \left[\frac{w_n f_s \alpha_k^n}{\log(2)\lambda_n + \underbrace{\sum_{m \neq n} \frac{w_m f_s \alpha_k^m \Gamma |h_k^{m,n}|^2}{\sum_{p \neq m} \Gamma |h_k^{m,p}|^2 s_k^p + \Gamma \sigma_k^m}}_{P_k^{\text{SCALE},n}}} \right]_0^{s_k^{n,\text{mask}}}. \tag{22}$$

Note that the transformation $s_k^n = e^{\tilde{s}_k^n}$ is used to transform back to s_k^n as explained in [13] (where the constraint $0 \leq s_k^n$ is added as $\tilde{s}_k^n$ is transformed back to s_k^n).

Every modem iteratively applies formula (22) and bisects on λ_n to converge to the solution of the approximated problem (21). By regularly updating the approximation parameters α_k^n with formula (19) the network converges to a locally optimal solution of problem (2). Note that the algorithm cannot be executed in a fully distributed manner as the term $P_k^{\text{SCALE},n}$(22) depends on information which is not available to each modem. However this information can be acquired with some limited message-passing as explained in [13]. This leads to Algorithm 2 adopted from [13].

In [18], a slightly different approximation is proposed based on a high SNR approach as follows:

$$\log z \approx \log(1+z). \tag{23}$$

This leads to an approximation which is similar to the approximation of SCALE where the parameters are fixed to $\alpha_k^n = 1$ and $\beta_k^n = 0$. Unlike the SCALE algorithm, this high SNR approach does not necessarily converge to a local optimum of problem (2). If the high SNR assumption holds, this approach leads to a near-optimal performance. Note that for this approximation, no approximation parameters need to be updated; however, this reduction in complexity is negligible.

C. KKT Perspective on ASB

The ASB algorithm, proposed in [12], utilizes the concept of a reference line. This reference line should be chosen so as to represent a statistical average of all crosstalk victim lines within a typical network. In [12], heuristics are provided for choosing the reference line. Once the reference line is fixed each modem optimizes its own bit rate taking into account the damage caused on the reference line.

This can be viewed as the following approximation (24) of problem (2), where $h_k^{n,\mathrm{ref}}$, h_k^{ref}, s_k^{ref}, σ_k^{ref}, and w_n^{ref} are, respectively, the crosstalk channel from the nth modem into the reference line, the channel attenuation, the power, the noise, and the weight of the reference line. These values need to be chosen in advance and are constants. Every modem n has to solve the following optimization problem:

$$\begin{aligned}
&\max_{s_1^n,\ldots,s_K^n} w_n f_s \sum_k \log_2\left(1+\frac{1}{\Gamma}\frac{|h_k^{n,n}|^2 s_k^n}{\sum_{m\neq n}|h_k^{n,m}|^2 s_k^m+\sigma_k^n}\right)\\
&\quad \underbrace{+w_n^{\mathrm{ref}} f_s \sum_k \log_2\left(1+\frac{1}{\Gamma}\frac{\left|h_k^{\mathrm{ref}}\right|^2 s_k^{\mathrm{ref}}}{\left|h_k^{n,\mathrm{ref}}\right|^2 s_k^n+\sigma_k^{\mathrm{ref}}}\right)}_{T_n}\\
&\text{s.t.} \sum_k s_k^n \le P^{n,\mathrm{tot}}\\
&\text{s.t.}\ 0\le s_k^n \le s_k^{n,\mathrm{mask}} \quad \forall\, k. \qquad (24)
\end{aligned}$$

Although approximation (24) is not convex, the main observation of ASB is that it can be solved in an easy way. The KKT stationarity condition of approximation (24) leads to (25), shown at the bottom of the page.

In [12], the authors propose to write this equation as a cubic equation and to solve it by checking the three roots as well as the boundary solutions $s_k^n=0$ and $s_k^n=s_k^{n,\mathrm{mask}}$. In [12], an extension is proposed to include multiple reference lines. However, for M reference lines, the ASB algorithm needs to solve a polynomial equation of degree $2M+1$ for each modem n on each tone k.

To reduce this complexity, we propose an alternative solution similarly to the approach of the previous sections. This will be referred to as ASB-2. Taking into account the KKT complementarity conditions of (24), (25) can be reformulated as a fixed point equation such that s_k^n, indicated by the letter Q in formula (25), is isolated as [see (26), shown at the bottom of the page].

$P_k^{\mathrm{ASB-2},n}$ is here expressed for one reference line, but it is easy to extend to M reference lines. Note that this fixed point equation does not necessarily converge to the same solution as the solution proposed in [12]. Every modem iteratively applies formula (26) and bisects on λ_n. If the reference lines are fixed for every modem, the algorithm can be executed in an autonomous manner which is a great asset for practical implementation. This leads to Algorithm 3. If line 9 of Algorithm 3 is changed by checking the three roots of cubic (25) and the boundary solutions $s_k^n=0$ and $s_k^n=s_k^{n,\mathrm{mask}}$, the ASB algorithm from [12] is obtained.

$$\begin{aligned}
\forall\, n,k:\ &\frac{w_n f_s |h_k^{n,n}|^2}{\log(2)\left(|h_k^{n,n}|^2 \underbrace{s_k^n}_{Q}+\sum_{m\neq n}\Gamma|h_k^{n,m}|^2 s_k^m+\Gamma\sigma_k^n\right)}\\
&-\frac{w_n^{\mathrm{ref}} f_s \left|h_k^{\mathrm{ref}}\right|^2 s_k^{\mathrm{ref}}\Gamma\left|h_k^{n,\mathrm{ref}}\right|^2}{\log(2)\left(\Gamma\left|h_k^{n,\mathrm{ref}}\right|^2 s_k^n+\Gamma\sigma_k^{\mathrm{ref}}\right)\left(\left|h_k^{\mathrm{ref}}\right|^2 s_k^{\mathrm{ref}}+\Gamma\left|h_k^{n,\mathrm{ref}}\right|^2 s_k^n+\Gamma\sigma_k^{\mathrm{ref}}\right)}\\
&-\lambda_n-\nu_k^n+\mu_k^n=0. \qquad (25)
\end{aligned}$$

$$s_k^n=\left[\frac{\frac{w_n f_s}{\log(2)}}{\lambda_n+P_k^{\mathrm{ASB-2},n}}-\frac{\sum_{m\neq n}\Gamma|h_k^{n,m}|^2 s_k^m+\Gamma\sigma_k^n}{|h_k^{n,n}|^2}\right]_0^{s_k^{n,\mathrm{mask}}} \qquad (26)$$

with

$$P_k^{\mathrm{ASB-2},n}=\frac{w_n^{\mathrm{ref}} f_s \left|h_k^{\mathrm{ref}}\right|^2 s_k^{\mathrm{ref}}\Gamma\left|h_k^{n,\mathrm{ref}}\right|^2}{\log(2)\left(\Gamma\left|h_k^{n,\mathrm{ref}}\right|^2 s_k^n+\Gamma\sigma_k^{\mathrm{ref}}\right)\left(\left|h_k^{\mathrm{ref}}\right|^2 s_k^{\mathrm{ref}}+\Gamma\left|h_k^{n,\mathrm{ref}}\right|^2 s_k^n+\Gamma\sigma_k^{\mathrm{ref}}\right)}.$$

V. Novel Distributed Spectrum Management Algorithms

In this section, three novel distributed spectrum management algorithms are proposed within the same KKT framework introduced in the previous section. The focus is on a distributed approach where the modems execute a local spectrum management algorithm and where in addition, some limited message passing is allowed between the modems and the SMC. Such an approach is interesting from a practical point of view. It combines a fast local algorithm adapting fast to channel and noise changes with the operation of the SMC to infrequently steer the local algorithms so that the network converges to a better performance. A similar approach is also used in [19], [20], and [13].

As explained in the beginning of Section IV, the choice of the type of (iterative) approximations has an influence on many factors like complexity, (sub)optimality, convergence properties, communication overhead, convex rate region, practical implementation, etc. One constraint for our spectrum management algorithms is that we want the local algorithms to be water-filling type procedures based on the following formulas:

$$s_k^n = \frac{w_n f_s}{\log(2)\lambda_n \delta_k^n} - \frac{\Gamma\left(\sum_{m\neq n} |h_k^{n,m}|^2 s_k^m + \sigma_k^n\right)}{|h_k^{n,n}|^2} \tag{27}$$

$$s_k^n = \frac{w_n f_s}{\log(2)\lambda_n + \epsilon_k^n} - \frac{\Gamma\left(\sum_{m\neq n} |h_k^{n,m}|^2 s_k^m + \sigma_k^n\right)}{|h_k^{n,n}|^2} \tag{28}$$

where δ_k^n and ϵ_k^n are constants and where (28) can be obtained from (27) through a simple transformation as follows:

$$\delta_k^n = 1 + \frac{\epsilon_k^n}{\log(2)\lambda_n}. \tag{29}$$

Water-filling formulas are well-established and understood in the communications field. Many discrete bit loading algorithms are based on water-filling type of procedures. In Section V-D it will be shown how the MS-DSB algorithm proposed in Section V-C can be used together with existing (water-filling inspired) fast bit loading algorithms [19].

Finally the convergence and complexity of the proposed algorithms is discussed.

A. Convex Approximation Distributed Spectrum Balancing (CA-DSB)

The first spectrum management algorithm is based on an iterative convex approximation approach similar to SCALE. However it chooses a different convex approximation so that a water-filling type of formula is obtained. This approach is also described in [16]. Here, it will be presented from a KKT perspective.

Note that the spectrum management problem (2) can be reformulated as follows:

$$\begin{aligned}
&\max_{\mathbf{s}^1,\ldots,\mathbf{s}^N} \sum_n w_n f_s \sum_k \log_2\left(\sum_{m=1}^N \left|\tilde{h}_k^{n,m}\right|^2 s_k^m + \Gamma\sigma_k^n\right) \\
&\qquad \underbrace{-\sum_n w_n f_s \sum_k \log_2\left(\Gamma\left(\sum_{m\neq n} |h_k^{n,m}|^2 s_k^m + \sigma_k^n\right)\right)}_{A=\text{nonconcave part}} \\
&\text{s.t.} \sum_k s_k^n \leq P^{n,\text{tot}} \quad \forall\, n \\
&\text{s.t. } 0 \leq s_k^n \leq s_k^{n,\text{mask}} \quad \forall\, n\ \forall\, k \\
&\qquad \text{with } |\tilde{h}_k^{n,m}|^2 \begin{cases} = \Gamma\,|h_k^{n,m}|^2, & n \neq m \\ = |h_k^{n,n}|^2, & n = m. \end{cases}
\end{aligned} \tag{30}$$

The basic idea is to approximate the nonconcave (convex) part A of the objective function of (30) in a point $\mathbf{s}_{k,\text{ap}}$ by a lower bound hyperplane as follows:

$$\begin{aligned}
&-\left(\sum_{m\neq n} a_k^{m,n} s_k^m + c_k^n\right) \\
&\qquad \leq \underbrace{-\log_2\left(\Gamma\left(\sum_{m\neq n} |h_k^{n,m}|^2 s_k^m + \sigma_k^n\right)\right)}_{A_k^n}
\end{aligned} \tag{31}$$

with equality in point $\mathbf{s}_{k,\text{ap}}$.

The approximation parameters c_k^n, $a_k^{m,n}$ $(\forall\ m)$ are readily obtained based on solving a linear system of N equations in N unknowns or based on the gradient and function value of A_k^n (31) at $\mathbf{s}_{k,\text{ap}}$. This lower bound approximation leads to the following convex optimization problem:

$$\begin{aligned}
&\max_{\mathbf{s}^1,\ldots,\mathbf{s}^N} \sum_n w_n f_s \sum_k \log_2\left(\sum_{m=1}^N \left|\tilde{h}_k^{n,m}\right|^2 s_k^m + \Gamma\sigma_k^n\right) \\
&\qquad -\sum_n w_n f_s \sum_k \left(\sum_{m\neq n} a_k^{m,n} s_k^m + c_k^n\right) \\
&\text{s.t.} \sum_k s_k^n \leq P^{n,\text{tot}} \quad \forall\, n \\
&\text{s.t. } 0 \leq s_k^n \leq s_k^{n,\text{mask}} \quad \forall\, n\ \forall\, k.
\end{aligned} \tag{32}$$

By iteratively solving problem (32), obtaining $\mathbf{s}_k^* \ \forall\, k$, and updating the approximation parameters $a_k^{m,n}$, c_k^n for $\mathbf{s}_{k,\text{ap}} = \mathbf{s}_k^*$ using (31), the sequence of convex subproblems produces a monotonically increasing objective value which converges to a locally optimal solution of the spectrum management problem (2).

The KKT conditions of (32), in particular the KKT stationarity condition and complementarity conditions, can be used to

derive the following fixed point equation to find the spectra that solve the convex approximation (32):

$$s_k^n = \left[\left(\frac{\frac{w_n f_s}{\log(2)}}{\underbrace{\lambda_n + \sum_{m \neq n} \omega_m f_s a_k^{n,m} - \sum_{m \neq n} \frac{\frac{w_m f_s \Gamma \left| h_k^{m,n} \right|^2}{\log(2)}}{\sum_p \left| \tilde{h}_k^{m,p} \right|^2 s_k^p + \Gamma \sigma_k^m}}_{P_k^{\text{CA-DSB},n}}} \right) - \frac{\sum_{m \neq n} \Gamma \left| h_k^{n,m} \right|^2 s_k^m + \Gamma \sigma_k^n}{\left| h_k^{n,n} \right|^2} \right]_0^{s_k^{n,\max}}. \quad (33)$$

Note that formula (33) is a water-filling type of equation, more specifically it corresponds to formula (28), where $\epsilon_k^n = P_k^{\text{CA-DSB},n}$. Although the term $P_k^{\text{CA-DSB},n}$ depends on s_k^n $\forall n$, it is viewed as a constant which is updated only infrequently by the SMC. Every modem iteratively applies formula (33), where $P_k^{\text{CA-DSB},n}$ is a constant and bisects on λ_n, as shown in Algorithm 4. Note that only one fixed point iteration is performed for every λ_n. In addition, the approximation parameters $a_k^{m,n}$, c_k^n and the term $P_k^{\text{CA-DSB},n}$ are updated infrequently (31) by the SMC to converge to a locally optimal solution of the spectrum management problem. The algorithm will be referred to as CA-DSB, i.e., convex approximation distributed spectrum balancing.

B. Distributed Spectrum Balancing (DSB)

The second distributed spectrum management algorithm starts from the nonconvex spectrum management problem (2) directly without involving any (iterative) approximation. Based on the KKT conditions of (2) a distributed solution is derived. It is known that the KKT conditions are necessary for a solution in nonlinear programming to be locally optimal. Here, this will be used the other way around. There will be looked for a solution satisfying the KKT conditions which may correspond to a local or a global optimum.

The starting point is the KKT stationarity condition of the reformulated spectrum management problem (30):

$$\begin{aligned} \forall n,k: & \frac{w_n f_s \left| h_k^{n,n} \right|^2}{\log(2) \left(\left| h_k^{n,n} \right|^2 \underbrace{s_k^n}_{Q} + \sum_{m \neq n} \Gamma \left| h_k^{n,m} \right|^2 s_k^m + \Gamma \sigma_k^n \right)} \\ & + \sum_{m \neq n} \frac{\frac{w_m f_s \Gamma \left| h_k^{m,n} \right|^2}{\log(2)}}{\sum_p \left| \tilde{h}_k^{m,p} \right|^2 s_k^p + \Gamma \sigma_k^m} \\ & - \sum_{m \neq n} \frac{\frac{w_m f_s \Gamma \left| h_k^{m,n} \right|^2}{\log(2)}}{\sum_{p \neq m} \Gamma \left| h_k^{m,p} \right|^2 s_k^p + \Gamma \sigma_k^m} \\ & - \lambda_n - \nu_k^n + \mu_k^n = 0. \end{aligned} \quad (34)$$

By using the KKT complementarity conditions, (34) can be reformulated as a fixed point equation such that s_k^n, indicated by the letter Q in formula (34), is isolated as follows:

$$s_k^n = \left[\frac{\frac{w_n f_s}{\log(2)}}{\lambda_n + P_k^{\text{DSB}_1,n} - P_k^{\text{DSB}_2,n}} - \frac{\sum_{m \neq n} \Gamma \left| h_k^{n,m} \right|^2 s_k^m + \Gamma \sigma_k^n}{\left| h_k^{n,n} \right|^2} \right]_0^{s_k^{n,\max}} \quad (35)$$

with $P_k^{\text{DSB}_1,n} = \sum_{m \neq n} \frac{w_m f_s \Gamma \left| h_k^{m,n} \right|^2 / \log(2)}{\sum_{p \neq m} \Gamma \left| h_k^{m,p} \right|^2 s_k^p + \Gamma \sigma_k^m}$

and

$$P_k^{\text{DSB}_2,n} = \sum_{m \neq n} \frac{w_m f_s \Gamma \left| h_k^{m,n} \right|^2 / \log(2)}{\sum_p \left| \tilde{h}_k^{m,p} \right|^2 s_k^p + \Gamma \sigma_k^m}.$$

Note that formula (35) is a water-filling type of equation, more specifically it corresponds to formula (28) where $\epsilon_k^n = P_k^{\text{DSB}_1,n} - P_k^{\text{DSB}_2,n}$. Although the term $P_k^{\text{DSB}_1,n} - P_k^{\text{DSB}_2,n}$ depends on s_k^n $\forall n$, it is viewed as a constant which is updated only infrequently by the SMC. Every modem iteratively applies formula (35) and bisects on λ_n.

Note that formula (35) can be reformulated as (36), shown at the bottom of the page.

$$\begin{aligned} s_k^n &= \left[\frac{\frac{w_n f_s}{\log(2)}}{\lambda_n + \sum_{m \neq n} \frac{w_m f_s \Gamma \left| h_k^{m,n} \right|^2}{\log(2)} \left(\frac{1}{\text{int}_k^m} - \frac{1}{\text{rec}_k^m} \right)} - \frac{\text{int}_k^n}{\left| h_k^{n,n} \right|^2} \right]_0^{s_k^{n,\max}} \\ \text{where } \text{int}_k^n &= \sum_{m \neq n} \Gamma \left| h_k^{n,m} \right|^2 s_k^m + \Gamma \sigma_k^n \\ \text{rec}_k^n &= \sum_m \left| \tilde{h}_k^{n,m} \right|^2 s_k^m + \Gamma \sigma_k^n. \end{aligned} \quad (36)$$

Algorithm 4 CA-DSB

1: Modem **n** algorithm:
2: Initialize $s_k^n = s_k^{n,\text{mask}}/2$, $\forall k$
3: loop {Execute at regular intervals}
4: $\lambda_n^{\min} = 0, \lambda_n^{\max} = \Lambda^{\max}$
5: $\lambda_n = (\lambda_n^{\max} + \lambda_n^{\min})/2$
6: Receive messages $P_k^{\text{CA}-\text{DSB},n}$ from SMC, $\forall k$
7: while $|\sum_k s_k^n - P^{n,\text{tot}}| > \delta$ and $\lambda_n > \gamma$ do
8: $\lambda_n = (\lambda_n^{\max} + \lambda_n^{\min})/2$
9: Update s_k^n using (33), $\forall k$
10: if $\sum_k s_k^n > P^{n,\text{tot}}$ then
11: $\lambda_n^{\min} = \lambda_n$
12: else
13: $\lambda_n^{\max} = \lambda_n$
14: end if
15: end while
16: Measure $\text{int}_k^n = \sum_{p \neq n} |h_k^{n,p}|^2 s_k^p + \sigma_k^p$, $\forall k$
17: Transmit int_k^n, s_k^n to SMC, $\forall k$
18: end loop
19: SMC algorithm:
20: loop
21: Receive messages int_k^n, s_k^n from modems n, $\forall k$
22: Update $a_k^{n,m}$ by solving linear system (31) using int_k^n, s_k^n, $\forall k, n$
23: Compute messages $P_k^{\text{CA}-\text{DSB},n}$ and send to each modem n, $\forall k$.
24: end loop

Algorithm 5 DSB

1: Modem **n** algorithm:
2: Initialize $s_k^n = s_k^{n,\text{mask}}/2$, $\forall k$
3: loop {Execute at regular intervals}
4: $\lambda_n^{\min} = 0, \lambda_n^{\max} = \Lambda^{\max}$
5: $\lambda_n = (\lambda_n^{\max} + \lambda_n^{\min})/2$
6: Receive messages $(P_k^{\text{DSB}_1,n} - P_k^{\text{DSB}_2,n})$ from SMC, $\forall k$
7: while $|\sum_k s_k^n - P^{n,\text{tot}}| > \delta$ and $\lambda_n > \gamma$ do
8: $\lambda_n = (\lambda_n^{\max} + \lambda_n^{\min})/2$
9: Update s_k^n using (35), $\forall k$
10: if $\sum_k s_k^n > P^{n,\text{tot}}$ then
11: $\lambda_n^{\min} = \lambda_n$
12: else
13: $\lambda_n^{\max} = \lambda_n$
14: end if
15: end while
16: Measure $\text{int}_k^n = \sum_{p \neq n} |h_k^{n,p}|^2 s_k^p + \sigma_k^p$, $\forall k$
17: Compute messages $\left(\frac{1}{\text{int}_k^n} - \frac{1}{\text{rec}_k^n}\right)$, $\forall k$ and send to SMC
18: end loop
19: SMC algorithm:
20: loop
21: Receive messages $\left(\frac{1}{\text{int}_k^n} - \frac{1}{\text{rec}_k^n}\right)$ from modems n, $\forall k$
22: Compute messages $(P_k^{\text{DSB}_1,n} - P_k^{\text{DSB}_2,n})$ and send to each modem n, $\forall k$.
23: end loop

The quantity int_k^n is the received interference and noise of modem n on tone k and the quantity rec_k^n is the received signal of modem n on tone k. Both quantities are already measured by current state-of-the-art modems. The algorithm is shown in Algorithm 5 and it will be referred to as DSB, i.e., distributed spectrum balancing. A very similar algorithm, called Modified Iterative Water-filling (MIW), is recently proposed in [21]. MIW and DSB have the same transmit power update formulas and so use the same concept of water-filling in addition with a tone dependent penalty, called taxation term in [21]. Furthermore, MIW extends the focus to the asynchronous DSL transmission case [22], where it is shown that MIW, and so also DSB, can simply be extended to incorporate intercarrier interference (ICI). DSB incorporates spectral mask constraints and so MIW can similarly be extended to incorporate these extra constraints.

Note that there are a number of correspondences between SCALE, CA-DSB and DSB. Term $P_k^{\text{DSB}_1,n}$ of (35) is very similar to term $P_k^{\text{SCALE},n}$ of the SCALE formula (22). Furthermore, term $P_k^{\text{DSB}_2,n}$ of (35) is the same as the second term of $P_k^{\text{CA}-\text{DSB},n}$ of CA-DSB (33). In fact if we use $a_k^{n,m} = (\Gamma |h_k^{m,n}|^2 / \log(2))/(\sum_{p \neq m} \Gamma |h_k^{m,p}|^2 s_k^p + \Gamma \sigma_k^m)$ in (33) then term $P_k^{\text{DSB}_1,n} - P_k^{\text{DSB}_2,n}$ of (35) equals term $P_k^{\text{CA}-\text{DSB},n}$ of (33). Finally, note that if in ASB the T_n of (24) is chosen as $\sum_{m \neq n} w_m R^m$, the DSB algorithm is obtained. This fact can be used to develop good heuristics for choosing the reference line. One should tune the parameters of the reference line for ASB so that it optimally follows $\sum_{m \neq n} w_m R^m$ for varying $s_k^n \ \forall \ k$.

C. Multiple Starting Point Distributed Spectrum Balancing (MS-DSB)

The third spectrum management algorithm is an extension of DSB with multiple starting points. The distributed algorithms SCALE, CA-DSB and DSB are algorithms that converge to a locally optimal solution of problem (2). The spectrum management problem (2) is a nonconvex optimization problem that can have multiple locally optimal solutions. These solutions can differ significantly in obtained weighted data rate sum. This means that locally optimal algorithms like SCALE, CA-DSB, and DSB can sometimes perform quite sub-optimally compared to the global optimum, depending on the DSL scenario.

TABLE I
$N+2$ INITIAL TRANSMIT POWERS FOR EACH TONE k USED BY MS-DSB

1	$\mathbf{0}$
2	$\mathbf{s}_k^{\max}/2$
$3 \ldots (N+2)$	$\mathbf{e_n}.s_k^{n,\max}$, for $n = 1 \ldots N$

An important factor that determines the obtained locally optimal solution is the initial transmit power spectrum. For SCALE as presented in [13], it was suggested to start from initial transmit power spectra equal to zero $s_k^n = 0 \; \forall \; n, k$. For CA-DSB and DSB, it is suggested to start from initial transmit power spectra $s_k^n = s_k^{n,\max}/2 \; \forall \; n, k$. The heuristic for this initial choice is that if there is a uniform distribution for the possible power loadings, the maximum difference between the initial power loading and the obtained power loading is minimized on average. Note that if there are no spectral mask constraints, one can put $s_k^{n,\max} = P^{n,\text{tot}}$.

In simulation Section VI-A, it will be observed that CA-DSB and DSB, with the same initial transmit power spectra, generally converge to the same solution. In some cases the SCALE algorithm gives a different solution, meaning it converges to a different locally optimal solution. If the initial power spectra are chosen equally for the three algorithms, they generally converge to the same solutions. This is confirmed by many simulations. This means that SCALE, CA-DSB, and DSB converge to locally optimal solutions which depend on the initial power spectra.

Based on the above observations, we propose to combine DSB with a multiple starting point approach. For each tone, multiple initial transmit powers are chosen and for each choice the transmit powers are updated iteratively so as to converge to a local optimum. The best of the converged results is then chosen for each tone independently. By using multiple initial transmit powers it is more likely that the global optimum for that tone will be found. In order to keep the complexity low, we propose to use $N+2$ initial transmit power sets. These initial transmit power sets are given in Table I, where $\mathbf{0}$ is the zero vector of dimension N, $\mathbf{s}_k^{\max}$ is a vector of dimension N with the corresponding spectral masks on tone k for the N modems and $\mathbf{e_n}.s_k^{n,\max}$ is the unit vector in the nth dimension multiplied by the corresponding spectral mask. The first transmit power set corresponds to the initial transmit power set of the SCALE algorithm. The second transmit power set corresponds to the initial transmit power set of the CA-DSB and DSB algorithms. This choice ensures that the obtained local optimum on tone k is at least as good as those obtained by the SCALE, CA-DSB, and DSB algorithms. The remaining N transmit power sets are chosen to correspond to initial transmit power sets where only one modem is active in each tone k. The reasoning behind this choice is that we observe that for large crosstalk the locally optimal solutions can get isolated along the axes. A fully random approach would then fail to find these isolated locally optimal points unless many (i.e., many more than $N+2$) initial points are chosen randomly. A recent work [23] actually proves that in the case of large crosstalk, the optimal solution converges to an FDMA solution where only one user is active in each tone k.

TABLE II
WATER-FILLING PENALTIES FOR DISTRIBUTED ALGORITHMS

	per-user penalty	per-tone penalty term
IW	λ_n	0
CA-DSB	λ_n	$P_k^{\text{CA}-\text{DSB},n}$
DSB	λ_n	$P_k^{\text{DSB}_1,n} - P_k^{\text{DSB}_2,n}$
ASB-2	λ_n	$P_k^{\text{ASB}-2,n}$

This leads to Algorithm 6. Each modem executes the modem part of the DSB algorithm locally so that it can react fast to channel and noise changes. The SMC executes the multiple starting point extension. The SMC regularly receives messages $(1/\text{int}_k^n - 1/\text{rec}_k^n)$ from the N modems and computes the quantities $P_k^{\text{DSB}_1,n} - P_k^{\text{DSB}_2,n}$. The loop at line 5 will search for the Lagrange multipliers so that the active power constraints are satisfied, using a subgradient approach at line 6 as in formula (18) (see also [7] and [17]). Note that all the Lagrange multipliers are updated in parallel. Then for each tone and for each initial transmit power set, the modems iterate using the DSB formula (35). The best of the converged results is used. When the loop is converged the quantities $P_k^{\text{DSB}_1,n} - P_k^{\text{DSB}_2,n}$ are communicated to the modems so that they can steer their transmit spectra to improve the network performance. This algorithm will be referred to as MS-DSB, i.e., multiple starting point distributed spectrum balancing.

D. Per-Tone Penalty Water-Filling Algorithms

The water-filling formula (16) of Section IV-A is a well-established formula in the communications field. In this formula, the Lagrange multiplier λ_n can be seen as a penalty for modem n: the larger λ_n, the smaller the power allocated to the nth modem. The interesting part is that this penalty λ_n is equal for all the tones k of modem n. Formulas (27) and (28) can be seen as water-filling formulas with the addition of a *per-tone penalty*. For each modem n, each tone k is penalized differently. In formula (27) this is done by a penalizing scaling factor whereas in formula (28) this is done by a penalizing term. The CA-DSB, DSB, and ASB-2 formulas, (33), (35), and (26), respectively, can be viewed as water-filling formulas with the addition of a *per-tone penalty term*. These penalties are summarized in Table II.

In [19], the scaled iterative water-filling (SIW) algorithm is proposed which is based on per-tone scaling factors. Furthermore a fast bit loading implementation is proposed. The MS-DSB algorithm can be used in combination with this fast bit loading implementation as follows. Based on the per-tone penalty terms the equivalent per-tone scaling factors can be obtained by following closed-form formula:

$$\delta_k^n = 1 + \frac{\left(P_k^{\text{DSB}_1,n} - P_k^{\text{DSB}_2,n}\right)}{\lambda_n} \quad \forall \, k, n. \tag{37}$$

Note that this per-tone scaling factor depends on the Lagrange multiplier λ_n. This information is available at the SMC and so the corresponding scaling factors can be computed. These scaling factors are regularly updated and communicated to the

Algorithm 6 MS-DSB	Algorithm 7 MS-DSB (bit loading version)
1: SMC algorithm:	1: SMC algorithm:
2: loop {Execute at regular intervals}	2: loop {Execute at regular intervals}
3: Receive messages $\left(\frac{1}{\text{int}_k^n} - \frac{1}{\text{rec}_k^n}\right)$ from modems n, $\forall k$	3: Receive messages $\left(\frac{1}{\text{int}_k^n} - \frac{1}{\text{rec}_k^n}\right)$ from modems n, $\forall k$
4: Compute messages $(P_k^{\text{DSB}_1,n} - P_k^{\text{DSB}_2,n})$ $\forall n, k$.	4: Compute messages $(P_k^{\text{DSB}_1,n} - P_k^{\text{DSB}_2,n})$ $\forall n, k$.
5: while complementary slackness conditions (11) not satisfied do	5: while complementary slackness conditions (11) not satisfied do
6: update Lagrange multipliers using (18)	6: update Lagrange multipliers using (18)
7: for all tones k do	7: for all tones k do
8: for each initial transmit power set do	8: for each initial transmit power set do
9: for iterations do	9: for iterations do
10: for each modem do	10: for each modem do
11: Apply formula (35)	11: Apply formula (35)
12: end for	12: end for
13: end for	13: end for
14: end for	14: end for
15: Take best of all converged solutions	15: Take best of all converged solutions
16: end for	16: end for
17: end while	17: end while
18: Compute per-tone dependent penalties $P_k^{\text{DSB}_1,n} - P_k^{\text{DSB}_2,n}$, $\forall n, k$	18: Compute DSB scaling factors δ_k^n (37)
19: Send $P_k^{\text{DSB}_1,n} - P_k^{\text{DSB}_2,n}$ to all modems n	19: Send δ_k^n to all modems $n, \forall k$
20: end loop	20: end loop
21: Modem n algorithm:	21: Modem n algorithm:
22: loop	22: loop
23: Apply modem part of DSB algorithm	23: Apply fast scaled bit loading algorithm [19]
24: end loop	24: At regular intervals:
	25: Receive scaling factors $\delta_k^n, \forall k$
	26: Transmit $\left(\frac{1}{\text{int}_k^n} - \frac{1}{\text{rec}_k^n}\right) \forall k$ to SMC
	27: end loop

modems where the fast bit loading algorithms [19] are implemented. This fits well in the band-preference framework proposed in [19] and [20]. In Algorithm 7 this bit loading version of the MS-DSB algorithm is shown.

To reduce the number of communication messages between the SMC and the modems a single scaling factor can be computed for a group of adjacent tones which are likely to have similar properties in a DSL channel. This was also proposed in [19]. The total number of tones K can be grouped into M bands. For each band, only one scaling factor is computed which represents all the K/M tones in this band. This approximation can be improved if a curve of higher degree is fitted for the tones within one band, leading to a slightly increased message passing.

E. Complexity Analysis

The complexity of the proposed algorithms can be separated into the message-passing communication overhead and the complexity of the power updates by the modems.

The complexity of the power updates refers to the complexity of the modems to converge to their transmit powers satisfying the total power constraints for fixed per-tone penalties, multiplied with the total number of updates of the per-tone penalties. For fixed per-tone penalties the CA-DSB, DSB and ASB-2 algorithms consist of two levels of cycles. An outer cycle iterates over the modems and an inner cycle bisects on λ_n for each modem until total power constraints are satisfied. The number of outer cycles ν is typically only 3 for the transmit powers to converge to within 1% of the previous cycle. The number of inner iterations (bisections on Lagrange multipliers λ_n) is denoted as I_b. In order to achieve an accurate precision, I_b can be fixed at 34 similarly to the approach in [12]. Furthermore, K transmit powers have to be computed for a given λ_n. In order to converge to the powers satisfying the total power constraints for fixed per-tone penalties, this leads to a complexity of $\nu I_b N K$ times updating powers using (33), (35), or (26) for CA-DSB, DSB, or ASB-2, respectively.

For the CA-DSB and DSB algorithms, this complexity has to be multiplied with the total number of updates of the per-tone penalties which will be referred to as L. We assume that the per-tone penalties of all the modems are updated at the same

TABLE III
ALGORITHM COMPLEXITY AND COMMUNICATION OVERHEAD FOR DISTRIBUTED SPECTRUM MANAGEMENT ALGORITHMS WHERE ν:NUMBER OF OUTER CYCLES OVER MODEMS; I_b:NUMBER OF BISECTIONS ON LAGRANGE MULTIPLIER λ_n; K:NUMBER OF TONES; N:NUMBER OF MODEMS; L:NUMBER OF UPDATES OF PER-TONE PENALTIES; μ:NUMBER OF ITERATIONS OVER MODEMS WITHIN ONE TONE; I:NUMBER OF CENTRALIZED LAGRANGE MULTIPLIER UPDATES. THE UNITY OF THE COMMUNICATION OVERHEAD IS THE NUMBER OF MESSAGES. THE UNITY OF THE ALGORITHM COMPLEXITY IS THE NUMBER OF EVALUATIONS OF THE CORRESPONDING POWER UPDATE FORMULA (x) IN ORDER TO CONVERGE

Algorithm	Complexity [no. of power updates using (x)]	Communication Overhead [no. of messages]
IW	$\nu I_b KN$ times (16)	0
SCALE	$\nu I_b LKN$ times (22)	$2LKN$
ASB-2	$\nu I_b KN$ times (26)	0
CA-DSB	$\nu I_b LKN$ times (33)	$3LKN$
DSB	$\nu I_b LKN$ times (35)	$2LKN$
MS-DSB	$\mu ILKN(N+2)$ times (35)	$2LKN$

time. L depends on the scenario. For small crosstalk scenarios, L is typically only 2. Furthermore this number does not increase with the number of modems if small crosstalk modems are added. For large crosstalk scenarios, L is typically 20–40, and there is a small increase in L if strong crosstalk modems are added to the cable bundle. Note that the updating of the per-tone penalties corresponds to the act of iteratively linearizing the nonconcave part of (30). From the optimization perspective, such strategy is known to have sublinear convergence. However for the considered DSL scenarios the convergence is quite satisfying. This leads to a total complexity of $\nu I_b LNK$ times updating powers using (33) or (35) for CA-DSB or DSB, respectively.

For the MS-DSB algorithm the main computation happens in the SMC. A difference is that the Lagrange multipliers $\lambda_n \; \forall \, n$ are updated in parallel which typically requires less than 50 updates [7]. This leads to a complexity of $\mu ILKN(N+2)$ times (35) where $(N+2)$ refers to the number of initial transmit power sets, I refers to the number of Lagrange multiplier updates which is typically less than 50 and μ is the number of iterations over the users within one tone as shown in Algorithm 6 which is typically only 3.

Second, there is a message-passing overhead for CA-DSB, DSB, and MS-DSB. In a system where all the modems update their per-tone penalties at the same time the total communication complexity for DSB and MS-DSB will be $2LNK$ messages. This can be explained as follows. Between each modem and the SMC, K messages have to be communicated in both directions for updating the K per-tone penalties, leading to $2K$ messages. For N users, this leads to $2NK$ messages. Furthermore, there are typically L updates of the per-tone penalties as explained in the previous paragraphs, leading to $2LNK$ messages. For CA-DSB there are two messages to the SMC instead of one (line 17 of Algorithm 4) leading to a total of $3LNK$ messages. This is summarized in Table III. Note that for IW and SCALE, the implementations of Algorithm 1 and 2 are used, respectively.

F. Convergence Analysis

First, the convergence of the CA-DSB algorithm is discussed. This algorithm basically consists of two parts. The first part is a message-passing protocol that updates the approximation parameters leading to iterative convex approximations. The second part is an iterative update of each users' transmit powers for fixed per-tone penalties which corresponds to lines 7–15 of Algorithm 4. In the following theorems the convergence is discussed of these two parts.

Theorem 5.1: By iteratively solving convex subproblem (32) and updating the approximation parameters $a_k^{m,n}$ using (31), the sequence of convex subproblems has a monotonically increasing objective value which converges to a locally optimal solution of the spectrum management problem (2).

Proof of Theorem 5.1: Reformulation (30) of the spectrum management problem can be shortly formulated as follows:

$$\max_{\mathbf{s}\in\Omega} h_1(\mathbf{s}) - h_2(\mathbf{s}) \equiv \min_{\mathbf{s}\in\Omega} g_1(\mathbf{s}) - g_2(\mathbf{s}) \equiv \min_{\mathbf{s}\in\Omega} f(\mathbf{s}) \quad (38)$$

where $h_1(\mathbf{s})$ and $h_2(\mathbf{s})$ are concave and respectively correspond to the first and second part of the objective function of (30). Furthermore $g_1(\mathbf{s})$ and $g_2(\mathbf{s})$ are convex, where $g_i(\mathbf{s}) = -h_i(\mathbf{s})$ and $f(\mathbf{s})$ is nonconvex, where $f(\mathbf{s}) = g_1(\mathbf{s}) - g_2(\mathbf{s})$. The constraint set is a convex set $\Omega = \{\mathbf{s} \in \mathbf{R}^{NK} | \sum_{k=1}^{K} s_k^n \leq P^{n,\mathrm{tot}} \; \forall n; \; 0 \leq s_k^n \leq s_k^{n,\mathrm{mask}} \; \forall n,k\}$.

The proposed lower bound convex approximation by CA-DSB (32) can be formulated as

$$\min_{\mathbf{s}\in\Omega} g_1(\mathbf{s}) - (g_2(\mathbf{s}_{\mathrm{ap}}) + \nabla g_2(\mathbf{s}_{\mathrm{ap}})^T(\mathbf{s}-\mathbf{s}_{\mathrm{ap}}))$$

where $\mathbf{s}_{\mathrm{ap}}$ is the approximation point.

We make the technical assumption that $g_1(\mathbf{s})$ has an upper bound on its curvature, i.e., $\|\nabla^2 g_1(\mathbf{s})\|_2 \leq C$ for all $\mathbf{s} \in \Omega$. This implies that $g_1(\mathbf{s}+\mathbf{p}) \leq g_1(\mathbf{s}) + \nabla g_1(\mathbf{s})^T\mathbf{p} + C/2\|\mathbf{p}\|_2^2$. Due to convexity of $g_2(\mathbf{s})$, we also have

$$f(\mathbf{s}+\mathbf{p}) \leq f(\mathbf{s}) + \nabla f(\mathbf{s})^T\mathbf{p} + \frac{C}{2}\|\mathbf{p}\|_2^2. \quad (39)$$

The key goal is to prove that the sequence $\mathbf{s}(t+1)$ for $t = 1, 2, \ldots$ produces a monotonically decreasing objective value

which converges to a local optimum of (38), where $\mathbf{s}(t+1)$ is defined as follows:

$$\mathbf{s}(t+1) = \arg\min_{\mathbf{s}\in\Omega} g_1(\mathbf{s}) - \Big(g_2\big(\mathbf{s}(t)\big) + \nabla g_2\big(\mathbf{s}(t)\big)^T\big(\mathbf{s}-\mathbf{s}(t)\big)\Big)$$

and where $\mathbf{s}(t)$ is the value of the transmit powers at iteration t.

To this end, we prove that a) the sequence is monotonically decreasing $f(\mathbf{s}(t+1)) \le f(\mathbf{s}(t))$ and that b) it converges to a local optimum of (38).

a)

$$\begin{aligned} f(\mathbf{s}(t)) &= g_1\big(\mathbf{s}(t)\big) - g_2\big(\mathbf{s}(t)\big) \\ &= g_1\big(\mathbf{s}(t)\big) - \Big(g_2\big(\mathbf{s}(t)\big) + \frac{\partial g_2\big(\mathbf{s}(t)\big)}{\partial \mathbf{s}}\big(\mathbf{s}(t)-\mathbf{s}(t)\big)\Big) \\ &\ge \min_{\mathbf{s}\in\Omega} g_1(\mathbf{s}) - \Big(g_2(\mathbf{s}(t)) + \frac{\partial g_2\big(\mathbf{s}(t)\big)}{\partial \mathbf{s}}\big(\mathbf{s}-\mathbf{s}(t)\big)\Big) \\ &= g_1\big(\mathbf{s}(t+1)\big) - \Big(g_2\big(\mathbf{s}(t)\big) + \frac{\partial g_2\big(\mathbf{s}(t)\big)}{\partial \mathbf{s}}\big(\mathbf{s}(t+1)-\mathbf{s}(t)\big)\Big) \\ &\ge g_1\big(\mathbf{s}(t+1)\big) - g_2\big(\mathbf{s}(t+1)\big) = f(\mathbf{s}(t+1)) \end{aligned} \tag{40}$$

where (40) uses the lower bound property (31). If the sequence is bounded below, we have $f(\mathbf{s}(t)) \to f^*$. Due to continuity of f and compactness of Ω, the sequence $(\mathbf{s}(t))_{t\in\mathbb{N}}$ has at least one accumulation point. Let us call this $\mathbf{s}^*$.

b) We now show that $\mathbf{s}^*$ is a local optimum for (38). Due to convexity of Ω, the necessary conditions for a point $\mathbf{s}^*$ to be a local optimum for (38) are

$$\forall \mathbf{y} \in \Omega : \nabla f(\mathbf{s}^*)^T(\mathbf{y}-\mathbf{s}^*) \ge 0. \tag{41}$$

We now show that (41) holds. For this aim let us assume that (41) is *not* satisfied in the limit, i.e.,

$$\exists \mathbf{y} \in \Omega : \quad \nabla f(\mathbf{s}^*)^T(\mathbf{y}-\mathbf{s}^*) < 0. \tag{42}$$

For this fixed $\mathbf{y} \in \Omega$, due to continuity of $\nabla f(\mathbf{s})^T(\mathbf{y}-\mathbf{s})$ w.r.t. $\mathbf{s}$, there is a neighborhood $\mathcal{N}$ of $\mathbf{s}^*$ and an $\epsilon > 0$ and $M > \epsilon/C$ so that for all $s \in \mathcal{N}$ holds that $\nabla f(\mathbf{s})^T(\mathbf{y}-\mathbf{s}) < -\epsilon$ and $\|\mathbf{y}-\mathbf{s}\|_2^2 < M$. Let us now regard the convex subproblem at any iterate $\mathbf{s}(t)$ in this neighborhood. It holds that

$$\begin{aligned} f(\mathbf{s}(t+1)) &\le \min_{\mathbf{s}\in\Omega} g_1(\mathbf{s}) - (g_2(\mathbf{s}(t)) + \nabla g_2(\mathbf{s}(t))^T(\mathbf{s}-\mathbf{s}(t))) \\ &\le \min_{\mathbf{s}\in\Omega} f(\mathbf{s}(t)) + \nabla f(\mathbf{s}(t))^T(\mathbf{s}-\mathbf{s}(t))) + \frac{C}{2}\|\mathbf{s}-\mathbf{s}(t)\|_2^2 \\ &\le f(\mathbf{s}(t)) + \nabla f(\mathbf{s}(t))^T(\mathbf{z}-\mathbf{s}(t)) + \frac{C}{2}\|\mathbf{z}-\mathbf{s}(t)\|_2^2. \end{aligned}$$

where $\mathbf{z}$ can be an arbitrary point in Ω.

By taking a point $\mathbf{z} = \mathbf{s}(t) + \mu(\mathbf{y}-\mathbf{s}(t))$ for the $\mathbf{y}$ above, we obtain

$$\begin{aligned} f(\mathbf{s}(t+1)) &\le f(\mathbf{s}(t)) + \mu\nabla f(\mathbf{s}(t))^T(\mathbf{y}-\mathbf{s}(t)) \\ &\quad + \frac{C}{2}\mu^2\|\mathbf{y}-\mathbf{s}(t)\|_2^2 \\ &\le f(\mathbf{s}(t)) - \mu\epsilon + \frac{CM}{2}\mu^2 \end{aligned}$$

and we can minimize w.r.t. to μ, on the line, to yield

$$f(\mathbf{s}(t+1)) \le \min_{\mu} f(\mathbf{s}(t)) - \mu\epsilon + \frac{CM}{2}\mu^2 = f(\mathbf{s}(t)) - \frac{\epsilon}{2CM}$$

where the minimum is attained at $\mu = \epsilon/CM \le 1$. This constant decrease at any iterate $\mathbf{s}(t)$ in the neighborhood $\mathcal{N}$ (which contains infinitely iterates $\mathbf{s}(t)$ because $\mathbf{s}^*$ is an accumulation point) is a contradiction to $f(\mathbf{s}(t)) \to f^*$. This means (41) holds and thus the iterative approach converges to a stationary point of (38). ■

Theorem 5.2: For fixed per-tone penalties $P_k^{\mathrm{CA-DSB},n}$, if $\max_{m\neq n,k} \Gamma\,|h_k^{n,m}|^2 / |h_k^{n,n}|^2 < 1/(N-1)$, then the sequential or parallel update of each users' transmit powers using (33) converges to the unique fixed point in an N-user system.

Proof of Theorem 5.2: The main observation here is that the update formula (33) of CA-DSB for fixed per-tone penalties has exactly the same structure as the update formula of the ASB-S2 algorithm of [12] for some arbitrary reference line. In [12] convergence conditions for the ASB-S2 are derived based on contraction mapping in the transmit power updates. For an N-user system it is proven that sequential or parallel updates of each users' transmit powers converge to the unique fixed point when $\max_{m\neq n,k} \Gamma\,|h_k^{n,m}|^2 / |h_k^{n,n}|^2 < 1/(N-1)$. These conditions do not depend on the values of the per-tone penalties. So ASB-S2 and CA-DSB have a similar convergence behavior and Theorem 5.2 is proven.

In Section V-B, it was mentioned that for $a_k^{n,m} = (\Gamma\,|h_k^{m,n}|^2 / \log(2))/(\sum_{p\neq m} \Gamma\,|h_k^{m,p}|^2 s_k^p + \Gamma\sigma_k^m)$, the per-tone penalties of DSB and CA-DSB are exactly the same. In fact this is the closed form solution of the system of N equations that the SMC part of CA-DSB has to solve. So the DSB algorithm is similar to the CA-DSB algorithm except that part of the calculation of the approximation parameters is done in the modems. CA-DSB and DSB lead to the same per-tone penalties and so have similar convergence properties. MS-DSB is a multiple starting point extension of DSB and so has the same convergence properties. ■

As explained in [12], the convergence conditions are not satisfied for every possible DSL scenario. However, it should be mentioned that convergence problems have never been encountered during extensive simulations of multi-user ADSL and VDSL scenarios (Section VI).

VI. Simulation Results

In this section, simulation results are shown for the distributed spectrum management algorithms. The following parameter settings are assumed for the DSL scenarios. The twisted pair lines have a diameter of 0.5 mm (24 AWG). The maximum transmit power is 20.4 dBm [24] for the ADSL scenarios and 11.5 dBm

TABLE IV
COMPARISON DSM ALGORITHMS FOR ASYMMETRIC N-USER ADSL DOWNSTREAM SCENARIO FIG. 1: COLUMNS INDICATE DSM ALGORITHMS, ROWS INDICATE NUMBER OF MODEMS, THREE CELLS PER ROW AND COLUMN INDICATING THE SIMULATION TIME, PERCENTAL PERFORMANCE IN WEIGHTED DATA RATE SUM AND NUMBER OF LAGRANGE MULTIPLIER UPDATES, RESPECTIVELY. FOR ISB, TWO NUMBERS IN EACH CELL ARE GIVEN FOR TWO DIFFERENT ITERATION ORDERS AS EXPLAINED IN SECTION VI-A

(a) Without spectral mask constraints

Modems	OSB	BB-OSB	ISB	IW	SCALE	ASB-2	CA-DSB	DSB	MS-DSB
2	3min	30s	14.00s / 14.03s	0.03s	0.56s	0.04s	0.08s	0.05s	1.18s
	100.0%	99.0%	100.0% / 100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	20	25	23 / 23	-	-	-	-	-	23
3	3h	3min	25.97s / 26.10s	0.08s	1.09s	0.08s	0.17s	0.1s	1.44s
	100.0%	99.8%	99.9% / 99.9%	99.8%	100.0%	99.9%	100.0%	100.0%	100.0%
	20	17	23 / 23	-	-	-	-	-	21
4	5d	20min	48.23s / 81.05s	0.13s	1.69s	0.10s	0.33s	0.19s	2.19s
	100.0%	99.6%	99.8% / 99.9%	99.5%	100.0%	99.6%	100.0%	100.0%	100.0%
	21	21	25 / 43	-	-	-	-	-	24
5	-	3h	76.56s / 493.06s	0.24s	2.75s	0.66s	3.13s	1.66s	5.20s
	-	100%	91.2% / 99.4%	71.8%	98.3%	86.9%	92.8%	92.8%	100.0%
	-	35	25 / 63	-	-	-	-	-	35
6	-	1d	102.49s / 167.00s	0.28s	3.48s	0.80s	5.21s	2.61s	7.30s
	-	99.4%	94.1% / 96.6%	86.9%	89.5%	98.3%	99.4%	99.4%	100.0%
	-	20	24 / 39	-	-	-	-	-	23
7	-	7d	137.79s / 321.06s	0.36s	7.15s	0.94s	5.89s	3.24s	6.21s
	-	99.6%	90.6% / 98.4%	92.7%	83.6%	99.8%	100.0%	100.0%	100.0%
	-	17	23 / 53	-	-	-	-	-	15

(b) With spectral mask constraints

Modems	OSB	BB-OSB	ISB	IW	SCALE	ASB-2	CA-DSB	DSB	MS-DSB
2	9s	0.76s	0.63s / 0.63s	0.04s	0.05s	0.05s	0.04s	0.02s	0.10s
	100.0%	79.9%	100.0% / 100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	1	1	1	-	-	-	-	-	1
3	9min	2.19s	1.17s / 1.19s	0.05s	0.06s	0.08s	0.10s	0.08s	0.11s
	100.0%	89.3%	100.0% / 100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	1	1	1	-	-	-	-	-	1
4	6h	10.81s	1.93s / 1.97s	0.09s	4.69s	0.14s	0.33s	0.29s	0.13s
	100.0%	92.2%	100.0% / 100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	1	1	1	-	-	-	-	-	1
5	-	70.69s	2.96s / 3.02s	0.22s	5.32s	0.60s	2.83s	1.69s	0.18s
	-	93.0%	100.0% / 100.0%	68.5%	100.0%	81.3%	89.8%	89.8%	100.0%
	-	1	1	-	-	-	-	-	1
6	-	10.96min	4.33s / 4.47s	0.38s	5.30s	0.75s	6.39s	4.56s	0.36s
	-	94.6%	92.6% / 98.0%	88.5%	93.8%	97.7%	99.1%	99.2%	100.0%
	-	1	1	-	-	-	-	-	1
7	-	1.4h	6.08s / 6.30s	0.47s	8.17s	0.87	9.34s	5.99s	0.46s
	-	95.6%	90.3% / 97.9%	95.5%	86.4%	99.9%	100.0%	100.0%	100.0%
	-	1	1	-	-	-	-	-	1

for the VDSL scenarios. The SNR gap Γ is 12.9 dB, corresponding to a coding gain of 3 dB, a noise margin of 6 dB, and a target symbol error probability of 10^{-7}. The tone spacing Δ_f is 4.3125 kHz. The DMT symbol rate f_s is 4 kHz. For all scenarios, the weights w_n are chosen equal for all users $(n = 1 \ldots N)$, namely $w_n = 1/N$. The simulations are performed in Matlab.

A. ADSL Performances

The asymmetric and symmetric ADSL Downstream (DS) scenarios considered here are shown in Figs. 1 and 2 respectively. The simulations are performed for a two-user case $(N = 2)$ up to a seven-user case $(N = 7)$. The four-user scenario, for example, consists of active modems 1, 2, 3, and 4, where modems 5, 6, and 7 are inactive.

In Table IV(a), simulation results are shown for the asymmetric downstream scenarios for two up to seven modems without spectral mask constraints. The "OSB" column refers to the Optimal Spectrum Balancing algorithm proposed in [6] where the power loading is discretized in 100 equal dBm steps. This fine discretization leads to a good approximation of the continuous solution. A finer discretization would increase the complexity too much here. The "BB-OSB" column refers to the branch and bound OSB algorithm proposed in [7] where discrete bit loading is considered ranging from 0 up to 16 bits in steps of 1 bit. Note that the discrete nature of BB-OSB leads to a weighted sum rate loss with respect to the OSB algorithm.

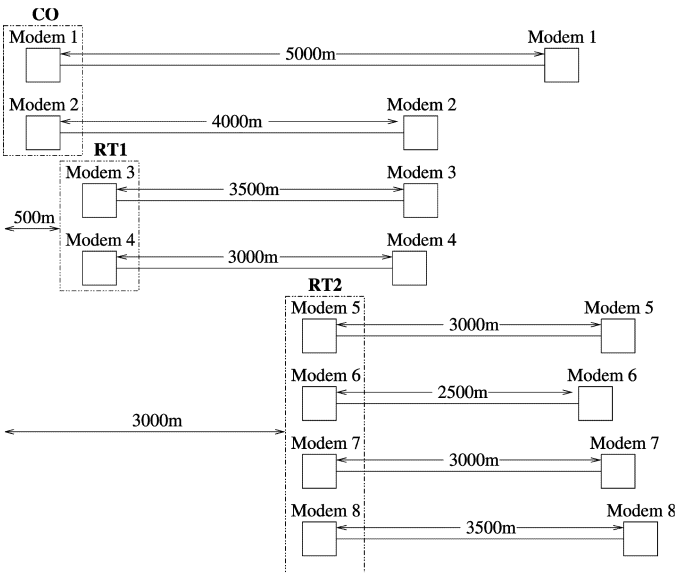


Fig. 1. Asymmetric N-user ADSL scenario with different loop lengths and different CO-RT distances.

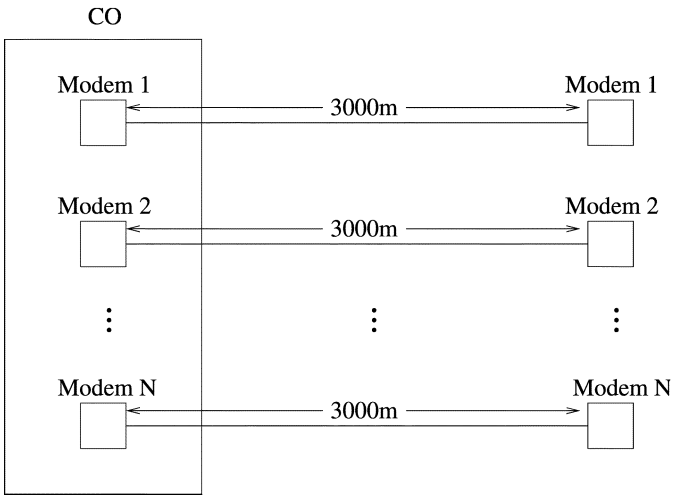


Fig. 2. Symmetric N-user ADSL scenario with equal loop lengths of 3000 m.

The "ISB" column refers to the ISB algorithm proposed in [9] and [10] with power loading. For the Lagrange multiplier search of OSB and ISB, the algorithms described in [7] and [25] are used.

The "IW", "SCALE", "ASB-2", "CA-DSB", "DSB", and "MS-DSB" columns refer to the methods described in Sections IV-A, IV-B, IV-C, V-A, V-B, and V-C, respectively. Note that the parameters of the reference line for the ASB-2 algorithm are chosen based on the guidelines provided in [12].

For each number of modems and for each spectrum management algorithm, there are three cells representing the simulation time, the performance in weighted data rate sum and the number of Lagrange multiplier updates to converge to the total power constraints, respectively. Note that the ISB column has two numbers for each cell. The reason is that the performance of the ISB algorithm depends on the iteration order. The number of possible iteration orders is $N!$ for an N-modem scenario. However, in order to reduce the total number of simulated iteration orders, just N iteration orders are simulated for every scenario. For instance, for the four-user case, ISB is performed for the iteration orders 1-2-3-4, 2-3-4-1, 3-4-1-2, and 4-1-2-3. The left number in the cell then represents the smallest value in simulation time, percental weighted data rate sum and number of Lagrange multiplier updates of the four simulated iteration orders. The right number represents the largest value in simulation time, percental weighted data rate sum, and number of Lagrange multiplier updates of the four simulated iteration orders. Note that the difference between these two values increases even more if more iteration orders are simulated, but we have restricted ourselves to simulating N iteration orders just to show the dependence of ISB on the iteration order.

The OSB algorithm takes too long to simulate for more than 4 users and so only simulation results are shown up to four users. Therefore, the performance in weighted data rate sum is measured with respect to the MS-DSB algorithm. This is the reason why the performance of the MS-DSB algorithm is always 100%. Note that this does not mean that the MS-DSB algorithm performs globally optimal for every DSL scenario.

From Table IV(a), it can be seen that OSB and BB-OSB have an exponential complexity in the number of modems. This large computational burden excludes practical implementation. However the results of OSB and BB-OSB can be used to evaluate the performance of the other algorithms. The ISB algorithm is much faster but shows suboptimality for scenarios with more than four users. The performance also strongly depends on the iteration order. Note that the number of Lagrange multiplier updates to converge to the total power constraints are less than 65 for all the scenarios.

The distributed algorithms of Table IV(a) are significantly faster than the centralized algorithms. Note that these algorithms (except for MS-DSB) update their Lagrange multipliers individually, so there is no total number of joint Lagrange multipliers updates.

IW performs suboptimally for the scenarios with more than four users. The reference line approach of ASB-2 performs clearly better than IW. CA-DSB and DSB perform similarly because their initial transmit power spectra are equal, as explained in Section V-C. SCALE performs better for the five-user scenario but worse for the six- and seven-user scenarios. In the seven-user case it performs worse than IW. This is due to the fact that it converges to a bad locally optimal solution. Note that it is verified by simulations that SCALE leads to the same solutions as CA-DSB and DSB when it uses the same initial transmit power spectra.

It can be seen that the MS-DSB algorithm performs at least as good as the SCALE, CA-DSB and DSB algorithms. Note that MS-DSB is not a globally optimal algorithm but its probability to converge to the globally optimal solution is larger than for SCALE, CA-DSB and DSB. Furthermore, its increase in simulation time with respect to the other distributed algorithms is small. Note that MS-DSB updates its Lagrange multipliers in parallel whereas, for SCALE, CA-DSB, and DSB, the Lagrange multipliers are updated individually by the modems.

Table IV(b) shows the simulation results for the downstream asymmetric scenario with spectral mask constraints. Note that there is no Lagrange multiplier search needed in this case. The

TABLE V
COMPARISON DSM ALGORITHMS FOR SYMMETRIC N-USER ADSL DOWNSTREAM SCENARIO FIG. 2: COLUMNS INDICATE DSM ALGORITHMS, ROWS INDICATE NO. OF MODEMS, THREE CELLS PER ROW AND COLUMN INDICATING THE SIMULATION TIME, PERCENTAL PERFORMANCE IN WEIGHTED DATA RATE SUM AND NO. OF LAGRANGE MULTIPLIER UPDATES, RESPECTIVELY. FOR ISB, TWO NUMBERS IN EACH CELL ARE GIVEN FOR TWO DIFFERENT ITERATION ORDERS AS EXPLAINED IN SECTION VI-A

(a) Without spectral mask constraints

Modems	OSB	BB-OSB	ISB	IW	SCALE	ASB-2	CA-DSB	DSB	MS-DSB
2	3min	30s	17.70s / 18.09s	0.05s	1.11s	0.05s	0.07s	0.03s	1.91s
	100.0%	99.3%	100.0% / 100.0%	99.6%	100.0%	100.0%	100.0%	100.0%	100.0%
	35	35	29 / 29	-	-	-	-	-	33
3	3h	3min	19.77s / 19.95s	0.10s	1.60s	0.07s	0.12s	0.05s	1.17s
	100.0%	99.3%	99.9% / 100.0%	99.9%	100.0%	100.0%	100.0%	100.0%	100.0%
	22	22	17 / 17	-	-	-	-	-	17
4	5d	20min	33.17s / 33.63s	0.13s	1.76s	0.10s	0.25s	0.15s	1.77s
	100.0%	99.2%	100.0% / 100.0%	99.7%	100.0%	100.0%	100.0%	100.0%	100.0%
	28	28	17 / 17	-	-	-	-	-	19
5	-	3h	25.03s / 42.80s	0.13s	2.59s	0.13s	0.47s	0.24s	1.85s
	-	99.3%	99.9% / 100.0%	99.6%	100.0%	99.8%	100.0%	100.0%	100.0%
	-	28	8 / 14	-	-	-	-	-	13
6	-	1d	71.28s / 72.10s	0.25s	3.09s	0.25s	0.63s	0.32s	5.04s
	-	99.1%	99.7% / 99.7%	99.4%	100.0%	99.8%	100.0%	100.0%	100.0%
	-	39	16 / 16	-	-	-	-	-	16
7	-	7d	131.31s / 132.47s	0.30s	3.95s	0.30s	0.84s	0.47s	7.47s
	-	98.8%	100.0% / 100.0%	99.4%	100.0%	99.8%	100.0%	100.0%	100.0%
	-	36	21 / 21	-	-	-	-	-	18

(b) With spectral mask constraints

Modems	OSB	BB-OSB	ISB	IW	SCALE	ASB-2	CA-DSB	DSB	MS-DSB
2	9s	8.01s	0.63s / 0.65s	0.02s	0.09s	0.07s	0.11s	0.06s	0.05s
	100.0%	89.1%	100.0% / 100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	1	1	1	-	-	-	-	-	1
3	9min	2.49s	1.18s / 1.25s	0.04s	0.10s	0.09s	0.09s	0.08s	0.07s
	100.0%	88.9%	100.0% / 100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	1	1	1	-	-	-	-	-	1
4	6h	12.17s	1.94s / 1.97s	0.05s	0.08s	0.14s	0.12s	0.11s	0.09s
	100.0%	88.9%	100.0% / 100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	1	1	1	-	-	-	-	-	1
5	-	77.05s	3.01s / 3.07s	0.07s	0.14s	0.19s	0.19s	0.12s	0.15s
	-	88.6%	100.0% / 100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	-	1	1	-	-	-	-	-	1
6	-	9.14min	4.40s / 4.50s	0.07s	0.12s	0.23s	0.24s	0.16s	0.32s
	-	88.6%	100.0% / 100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	-	1	1	-	-	-	-	-	1
7	-	1.13h	6.18s / 6.28s	0.18s	0.16s	0.28s	0.48s	0.21s	0.45s
	-	88.6%	100.0% / 100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	-	1	1	-	-	-	-	-	1

first choice of the Lagrange multipliers equal to zero leads to a solution already satisfying the total power constraints. The total power constraints are thus not active. Furthermore the same conclusions can be drawn as for Table IV(a). Table V(a) and (b) shows the simulation results for the downstream symmetric scenario without and with spectral mask constraints, respectively. It can be seen that all the algorithms perform similarly. So symmetric DSL scenarios are not really a challenge and one should opt for one of the low-complexity algorithms like IW, SCALE, CA-DSB, or DSB.

Summarizing, it can be stated that SCALE, CA-DSB, and DSB generally lead to good locally optimal solutions. However their performance depends on the initial transmit power spectra. For some DSL scenarios, the nonconvex spectrum management problem can have several locally optimal solutions. In this case, SCALE, CA-DSB, and DSB can perform quite suboptimally. MS-DSB tackles this issue by carefully choosing multiple initial transmit spectra. Although this approach does not guarantee a globally optimal solution, it leads to an improved performance with a small increase in complexity.

B. VDSL Performances

In Fig. 3, a four-user VDSL upstream scenario is shown. This scenario consists of one long line and three short lines causing a

TABLE VI
DATA RATE PERFORMANCE COMPARISON OBTAINED BY IW AND DSB ALGORITHMS FOR UPSTREAM VDSL SCENARIO FIG. 3

	IW	DSB
weighted sum rate [Mb/s]	14.5	14.8
Modem 1 [Mb/s]	0.2	1.1
Modem 2 [Mb/s]	19.2	19.2
Modem 3 [Mb/s]	19.1	19.4
Modem 4 [Mb/s]	19.3	19.3

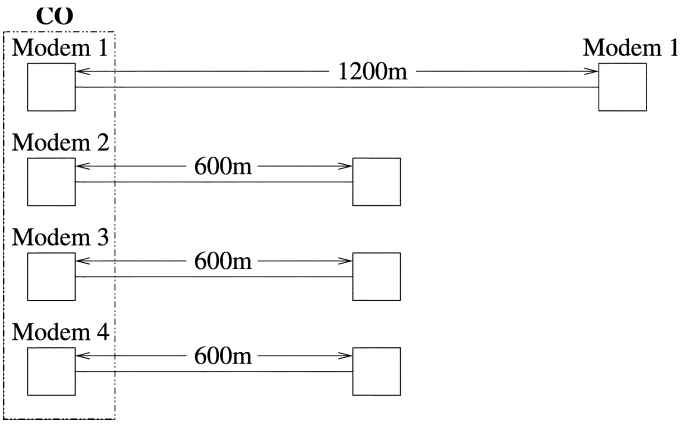


Fig. 3. Four-user upstream VDSL simulation scenario 1 with three near users (600 m) and one far user (1200 m).

TABLE VII
DATA RATE PERFORMANCE COMPARISON OBTAINED BY IW AND DSB ALGORITHMS FOR UPSTREAM VDSL SCENARIO FIG. 4

	IW	DSB
weighted sum rate [Mb/s]	14.8	18.4
Modem 1 [Mb/s]	0.026	1.10
Modem 2 [Mb/s]	0.026	1.10
Modem 3 [Mb/s]	29.3	28.6
Modem 4 [Mb/s]	29.6	42.4

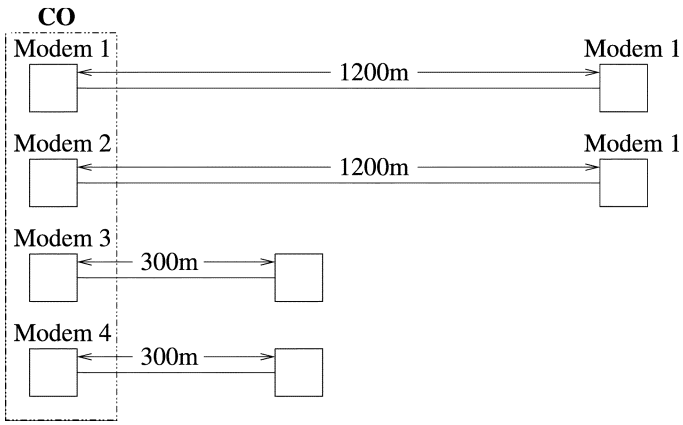


Fig. 4. Four-user upstream VDSL simulation scenario 2 with two near users (300 m) and two far users (1200 m).

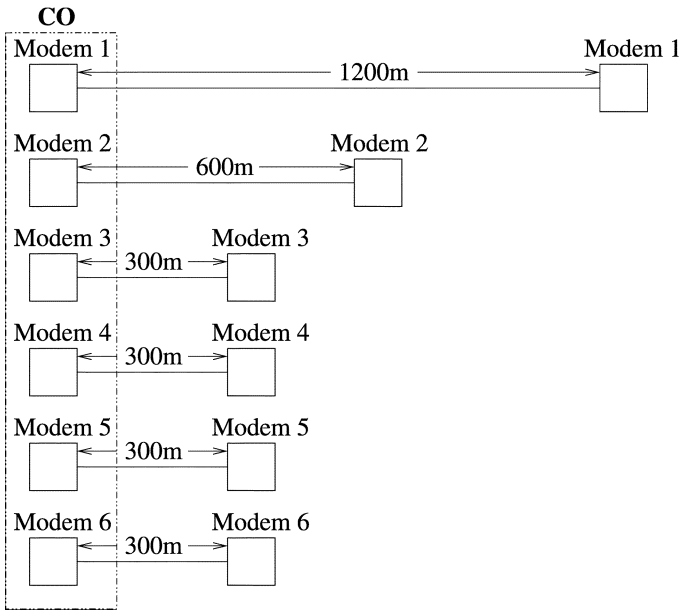


Fig. 5. Six-user upstream VDSL simulation scenario with four users at 300 m, one user at 600 m, and one user at 1200 m from the central office (CO).

lot of crosstalk into the long line. If the spectrum is not managed properly the performance of the long line will be very poor. In Table VI, the results are shown for IW and DSB. It can be seen that the weighted data rate sum is slightly larger for DSB. However, in terms of bit rates of the individual modems, DSB succeeds in increasing the bit rate of the long line up to 550% with respect to IW (that achieves a "selfish optimum") while keeping the same bit rates on the short lines.

In Fig. 4, an even more asymmetric VDSL upstream scenario is shown. The comparison between the performance for IW and DSB is shown in Table VII. The bit rates of the long lines are increased by a factor 42. The bit rate of the second short line is increased by a factor 1.4. The bit rate of the first short line is decreased by only 2.7%. For these heavily unbalanced VDSL scenarios appropriate spectrum management can thus indeed lead to significant performance improvements.

Note that all users for the IW algorithm use their full power budget for the scenarios of Figs. 3 and 4. For given equal weights, we have verified that for these scenarios this solution leads to the maximum weighted sum rate obtained by IW and to that end the comparison is done in a fair way.

Finally, in Fig. 5, a six-user upstream VDSL scenario is shown. The transmit power spectra obtained by IW and DSB are shown in Figs. 6 and 7, respectively, where the numbers indicate the modems. It can be seen that the IW transmit spectra are very selfish with full power transmission in frequency bands that are also used by other modems. The transmit power spectra of DSB are very different. The line of 1200 m uses tones 1–166 while the other modems decrease their transmit spectra in this frequency band. This decrease in transmit spectrum is called a back-off. The 600m line transmits at full power on tones 167–337 while it backs off on tones 1–166. Modem 3 uses tones 338–732 and backs off on the other tones. Modem 6 uses tones 733–1147 and backs off on the other tones. Modems 4 and 5 do not dominate any tones but back off on tones 1–732. These modems do not use their full power budget. When the crosstalk interference increases further, the optimal power loading will

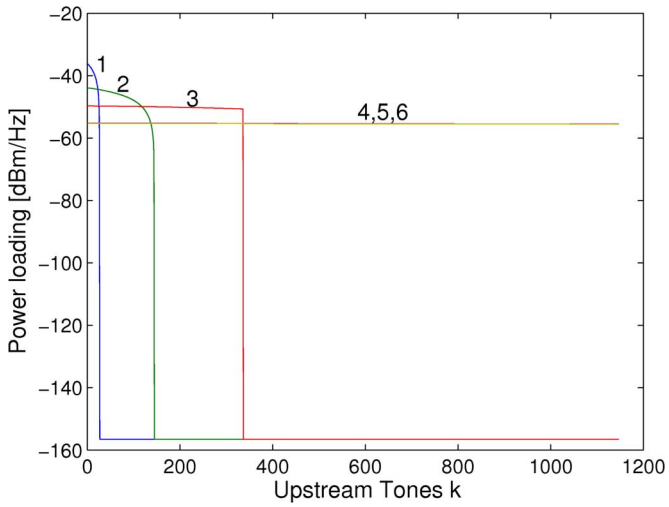


Fig. 6. Transmit power loading obtained by the IW algorithm for the six users over the 1147 upstream tones of the VDSL upstream scenario of Fig. 5. Transmit power of each user is indicated by a different number. Transmit powers of users 4, 5, and 6 coincide.

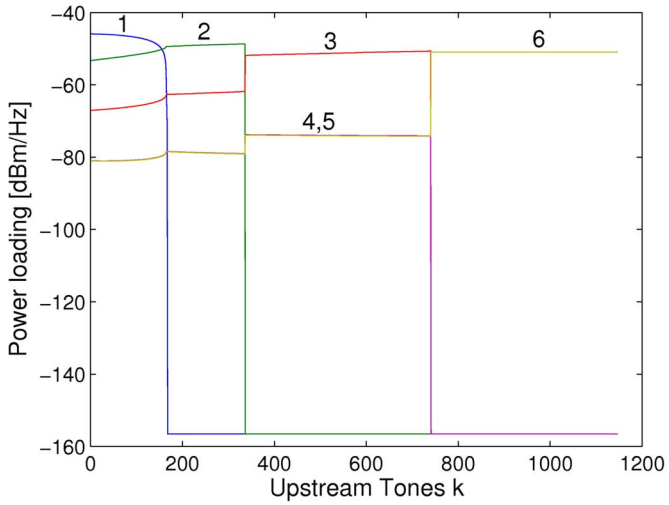


Fig. 7. Transmit power loading obtained by the DSB algorithm for the 6 users over the 1147 upstream tones of the VDSL upstream scenario of Fig. 5. Transmit power of each user is indicated by a different number. Transmit powers of users 4 and 5 coincide.

converge to a FDMA solution where only one modem is active in each tone interval. This fact was also recently proved in [23].

VII. Conclusion

In this paper, it is shown that state-of-the-art distributed spectrum management algorithms can be viewed from a similar framework. This framework consists of choosing an (iterative) (non)convex approximation of the spectrum management problem and deriving a distributed solution based on the KKT conditions of the chosen approximation. Based on this unifying perspective novel distributed spectrum management algorithms are proposed. A first algorithm ASB-2 is a variant of the ASB algorithm. It is fully autonomous and can be simply extended to include multiple reference lines without a significant complexity increase. A second algorithm CA-DSB involves an iterative convex approximation approach. A third algorithm DSB is based on the KKT conditions of the spectrum management problem. The fourth algorithm MS-DSB is a multiple starting point extension of the third algorithm which leads to a better performance for scenarios with multiple locally optimal solutions. CA-DSB, DSB and MS-DSB are locally optimal algorithms. They consist of water-filling-like algorithms running in the individual modems and controlled by a spectrum management center to improve the overall performance of the network. A convergence and complexity analysis is provided for the proposed algorithms. Extensive simulation results for several DSL scenarios show that these algorithms perform very well in mitigating the effect of crosstalk.

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