A LOW-COMPLEXITY ALGORITHM FOR JOINT SPECTRUM AND SIGNAL COORDINATION IN UPSTREAM DSL TRANSMISSION

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ABSTRACT

Joint spectrum and signal coordination is a promising dynamic spectrum management (DSM) technique to significantly boost data rates in vectored DSL systems. This paper presents a novel low-complexity algorithm for joint spectrum and receiver signal coordination in upstream DSL transmission. The algorithm is referred to as MAC-DSB and can be used for both a MMSE-GDFE and a linear MMSE receiver. MAC-DSB consists of an improved dual decomposition based approach with an optimal gradient algorithm for updating the dual variables and a low-complexity iterative fixed point update approach for tackling the decoupled subproblems. Huge reductions in computational complexity and good performance are validated through practical simulation results.

Index Terms— Digital subscriber line (DSL), dynamic spectrum management (DSM), interference cancellation, vectoring.

1. INTRODUCTION

Digital subscriber line (DSL) technology is currently the most popular wireline broadband Internet access technology with a global market share of more than 60%, corresponding to more than 300 million DSL subscribers [1]. One of the major impairments that limits further improvement of DSL performance, is *crosstalk*, i.e., the electromagnetic interference amongst different lines in the same cable bundle. The presence of crosstalk transforms the DSL access network into a challenging interference environment where the transmission of one line can significantly degrade the data rate performance of the other lines.

One promising set of techniques to tackle this crosstalk problem and to significantly boost data rates, is referred to as *dynamic spectrum management* (DSM) [2]. DSM involves a set of multi-user techniques to coordinate the transmission of multiple lines within the same cable bundle. One typically distinguishes between two types of DSM coordination: *spectrum coordination* and *signal coordination*. In spectrum coordination, the users' transmit spectra are optimized so as to prevent the impact of crosstalk [3, 4, 5]. Signal coordination, also referred to as *vectored DSL* or *DSM Level 3*, consists of jointly processing the transmitted or received users' signals so as to actively cancel the impact of crosstalk [6, 7]. Recent application of signal coordination techniques in practical prototypes have demonstrated a huge potential, enabling even data rates of up to 300 Megabits per second over just two traditional DSL lines [8].

This paper focuses on joint spectrum and receiver signal coordination, where both types of coordination are combined so as to improve the performance of upstream DSL transmission. Related work on this topic has been proposed in [9, 10, 11, 12, 13]. In particular, in [10] efficient algorithms are proposed for the case of joint spectrum and receiver signal coordination, considering a minimum mean-square error generalized decision-feedback equalization (MMSE-GDFE) receiver. These algorithms are referred to as *multiple access channel optimal spectrum balancing (MAC-OSB)* and *multiple access channel iterative spectrum balancing (MAC-ISB)*. They consist of a dual decomposition approach to decompose the corresponding nonconvex optimization problem into multiple decoupled per-tone problems. Subgradient algorithms are then used to tackle the dual problem in combination with a discrete exhaustive search or an iterative discrete coordinate descent algorithm to tackle the nonconvex per-tone problems, for MAC-OSB and MAC-ISB, respectively. The disadvantage of these algorithms is the large computational complexity and also their discrete character which can have an impact on the convergence properties and the performance of these algorithms.

In this paper we focus on improving and extending

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the MAC-OSB solution approach so as to obtain a low-complexity algorithm that significantly reduces the computational complexity of solving the considered joint spectrum and receiver signal coordination problem, while considering both a MMSE-GDFE receiver as well as a linear MMSE (MMSE-LIN) receiver. More specifically, we propose an improved dual decomposition based algorithm, which consists of an optimal gradient based algorithm for the dual problem, in combination with a low-complexity iterative fixed point update approach for tackling the decoupled per-tone problems. Simulation results with a realistic DSL simulator demonstrate a huge reduction in computational complexity compared to MAC-OSB, as well as an improved performance.

The content of this paper is organized as follows. In Section 2 the system model is described. Section 3 introduces the problem statement of joint spectrum and receiver signal coordination, and briefly summarizes the existing MAC-OSB and MAC-ISB algorithms to tackle this problem. In Section 4 the novel low-complexity algorithm is proposed and discussed for different settings such as a non-zero SNR gap and different types of receivers (MMSE-GDFE and MMSE-LIN). Finally simulation results are presented in Section 5.

2. SYSTEM MODEL

We consider typical discrete multi-tone (DMT) based DSL systems. Under the standard assumption of perfect tone synchronization, the transmission for a cable bundle of a set $\mathcal{N} = \{1, \ldots, N\}$ of N users (i.e., modems or lines), using a frequency range of a set $\mathcal{K} = \{1, \ldots, K\}$ of K tones (carriers), can be modeled on each tone k by

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{z}_k, \qquad k \in \mathcal{K}. \tag{1}$$

The vector $\mathbf{x}_k = [x_k^1, x_k^2, \ldots, x_k^N]^T$ contains the transmitted signals on tone k for all N users. $\mathbf{z}_k$ is the vector of additive noise on tone k, containing thermal noise, alien crosstalk and RFI. The vector $\mathbf{y}_k$ contains the received symbols. To keep the formulae simple we assume that the noise is pre-whitened $E\{\mathbf{z}_k \mathbf{z}_k^H\} = \mathbf{I}$, with $\mathbf{I}$ referring to an $N \times N$ identity matrix. $[\mathbf{H}_k]_{n,m} = h_k^{n,m}$ is an $N \times N$ matrix containing the pre-whitened channel gains from transmitter m to receiver n. The diagonal elements are the direct channels, the off-diagonal elements are the crosstalk channels.

The transmit power is denoted by $s_k^n \triangleq \Delta_f E\{|x_k^n|^2\}$, with Δ_f being the tone spacing. The vector containing the transmit powers of all users for tone k is $\mathbf{s}_k \triangleq [s_k^1, s_k^2, \ldots, s_k^N]^T$.

We consider the upstream DSL channel which corresponds to a multi-carrier multiple access channel where N DSL signals are jointly coordinated at the receiver (at the central office (CO) side). The corresponding capacity for each tone k can then be expressed as

$$c_k = \log_2(\det(\mathbf{I} + \mathbf{H}_k \mathbf{S}_k \mathbf{H}_k^H)), \tag{2}$$

with $\mathbf{S}_k = \mathrm{diag}\{s_k^1, \ldots, s_k^N\}$, which is a diagonal matrix as no transmit signal coordination is assumed.

For conciseness (without loss of generality) we restrict some of the formulae to the 2-user case, i.e., $N = 2$. The system model (1) can be reformulated as

$$\mathbf{y}_k = \begin{bmatrix} \mathbf{h}_k^1 & \mathbf{h}_k^2 \end{bmatrix} \begin{bmatrix} x_k^1 \\ x_k^2 \end{bmatrix} + \mathbf{z}_k = \mathbf{h}_k^1 x_k^1 + \mathbf{h}_k^2 x_k^2 + \mathbf{z}_k.$$

Using this reformulation the capacity formula (2) can be dissected into the following *unweighted* capacity sum

$$\begin{aligned} c_k &= \log_2(\det(\mathbf{I} + \mathbf{h}_k^1 s_k^1 \mathbf{h}_k^{1,H} + \mathbf{h}_k^2 s_k^2 \mathbf{h}^{2,H})) \\ &= \overbrace{\log_2(\det(\mathbf{I} + (\mathbf{I} + \mathbf{h}_k^2 s_k^2 \mathbf{h}_k^{2,H})^{-1} \mathbf{h}_k^1 s_k^1 \mathbf{h}_k^{1,H}))}^{c_k^1} \\ &\qquad + \underbrace{\log_2(\det(\mathbf{I} + \mathbf{h}_k^2 s_k^2 \mathbf{h}_k^{2,H}))}_{c_k^2} \end{aligned} \tag{3}$$

The first term of (3) represents the capacity of user 1 when user 1 is detected under crosstalk from user 2. The second term represents the capacity of user 2 when the detection of user 2 is done after having removed the crosstalk from user 1. This exactly corresponds to the operation of the MMSE-GDFE receiver [13] and thus this receiver is (unweighted rate sum) optimal for given transmit powers.

A weighting of the user capacities is commonly [10] used to give more importance to some users with respect to other users as follows

$$\tilde{c}_k = w_1 c_k^1 + w_2 c_k^2, \qquad w_2 \geq w_1 \geq 0, \tag{4}$$

where c_k^1, c_k^2 are as given in (3). Note that the detection order is defined so that the users with the smallest weights are decoded first and users with the largest weights are decoded last [10]. We will implicitly assume this detection order for given weights in the remainder of this paper, i.e., $w_m \geq w_n$ if $m > n$.

The weighted capacity sum (4) can be worked into a weighted rate sum $\tilde{b}_k$ as follows, similarly as in [10],

$$\begin{aligned} \tilde{b}_k(\mathbf{s}_k) &= \sum_{n \in \mathcal{N}} w_n b_k^n(\mathbf{s}_k), \\ \text{with} \quad b_k^n(\mathbf{s}_k) &= \log_2(\det(\mathbf{I} + \frac{1}{\Gamma} \mathbf{G}_k^n)), \\ \mathbf{G}_k^n &= (\mathbf{M}_k^n)^{-1} \mathbf{h}_k^n s_k^n \mathbf{h}_k^{n,H}, \\ \mathbf{M}_k^n &= \mathbf{I} + \mathbf{J}_k^n, \\ \mathbf{J}_k^n &= \sum_{m \in \mathcal{J}_k^n} \mathbf{h}_k^m s_k^m \mathbf{h}_k^{m,H}. \end{aligned} \tag{5}$$

Note the inclusion of the SNR-gap to capacity Γ which takes into account practical QAM constellation mapping and is a function of the desired BER, coding gain and noise margin. We want to highlight here that by including the SNR-gap Γ, and when $\Gamma > 1$, the theoretical optimality of the MMSE-GDFE scheme is not guaranteed anymore. However, as shown in [10], it enables to obtain achievable rate regions that compare favorably with respect to existing schemes

such as the iterative waterfilling algorithm proposed in [13]. The set $\mathcal{J}_k^n$ indicates for user n on tone k the set of users that are decoded after user n, i.e., for MMSE-GDFE (with $w_N \geq w_{N-1} \geq \ldots \geq w_1$):

$$\mathcal{J}_k^n = \{n+1, \ldots, N\} \tag{6}$$

Finally the total data rate R^n and the total transmit power P^n of user n are expressed as follows, respectively,

$$R^n = \sum_{k \in \mathcal{K}} b_k^n(\mathbf{s}_k) \quad \text{and} \quad P^n = \sum_{k \in \mathcal{K}} s_k^n. \tag{7}$$

3. PROBLEM STATEMENT AND SOLUTION APPROACHES MAC-OSB/MAC-ISB

The problem of joint spectrum and receiver signal coordination comes down to the following optimization problem

$$\begin{aligned} \max_{\mathbf{s}_k \in \mathcal{S}_k, k \in \mathcal{K}} \quad & \sum_{n \in \mathcal{N}} w_n R^n \qquad \left(= \sum_{k \in \mathcal{K}} \tilde{b}_k(\mathbf{s}_k) \right) \\ \text{s.t.} \quad & \sum_{k \in \mathcal{K}} s_k^n \leq P^{n,\text{tot}}, \quad n \in \mathcal{N}, \end{aligned} \tag{8}$$

which consists of optimizing the transmit powers so as to maximize the weighted sum of data rates, subject to power constraints as defined by DSL standards. More specifically, $P^{n,\text{tot}}$ denotes the total power budget available to user n, $\mathbf{P}^{\text{tot}} = [P^{1,\text{tot}}, \ldots, P^{N,\text{tot}}]^T$, and $\mathcal{S}_k$ indicates the set of bound constraints, in which $s_k^{n,\text{mask}}$ denotes the spectral mask for user n on tone k, as follows

$$\mathcal{S}_k = \{\mathbf{s}_k \in \mathbb{R}^N : 0 \leq s_k^n \leq s_k^{n,\text{mask}}, n \in \mathcal{N}\}.$$

Note that if (8) is solved, and so the optimal transmit powers are known, one can determine the optimal MMSE-GDFE receiver structure using the simple closed-form expression of an MMSE equalization matrix.

The MAC-OSB and MAC-ISB solution approach [10] for (8) is to solve its dual formulation, which consists of a master dual problem as follows

$$\min_{\boldsymbol{\lambda} \geq 0} g(\boldsymbol{\lambda}), \tag{9}$$

with dual variables $\boldsymbol{\lambda} = [\lambda_1, \ldots, \lambda_N]^T$ and K decomposed subproblems (per-tone problems) as follows

$$g(\boldsymbol{\lambda}) := \left(\sum_{k \in \mathcal{K}} \max_{\mathbf{s}_k \in \mathcal{S}_k} (\tilde{b}_k(\mathbf{s}_k) - \boldsymbol{\lambda}^T \mathbf{s}_k) \right) + \boldsymbol{\lambda}^T \mathbf{P}^{\text{tot}}. \tag{10}$$

The above dual approach (9)(10) is commonly referred to as *dual decomposition*.

An iterative projected subgradient update approach is then standardly used to solve the master problem (9) as follows,

$$\boldsymbol{\lambda} = \left[\boldsymbol{\lambda} + \mu \left(\sum_{k \in \mathcal{K}} \mathbf{s}_k(\boldsymbol{\lambda}) - \mathbf{P}^{\text{tot}} \right) \right]^+, \tag{11}$$

with μ being the stepsize, $\mathbf{s}_k(\boldsymbol{\lambda})$ being the optimal transmit powers that solve (10) for given $\boldsymbol{\lambda}$, and $[x]^+ = \max(x, 0)$. For the subproblems (10), in [10] a solution is proposed based on an exhaustive discrete search and a discrete coordinate descent algorithm for MAC-OSB and MAC-ISB, respectively.

4. NOVEL LOW-COMPLEXITY ALGORITHM: MAC-DSB

Subgradient algorithms for solving (9) are however known to converge very slowly, i.e., with a convergence rate $\mathcal{O}(1/\epsilon^2)$, ϵ being the desired accuracy. Furthermore, the stepsize μ is very difficult to tune so as to guarantee fast convergence [14]. Also, the proposed solutions for the subproblems require a significant amount of computational complexity, especially for large-scale DSL scenarios.

In this section we propose an improved dual decomposition based algorithm for solving (8). Our approach is to focus on a smoothed approximation of the dual problem (9) as follows

$$\min_{\boldsymbol{\lambda} \geq 0} \bar{g}(\boldsymbol{\lambda}), \tag{12}$$

with K independent smoothed subproblems:

$$\bar{g}(\boldsymbol{\lambda}) = \left(\sum_{k \in \mathcal{K}} \max_{\mathbf{s}_k \in \mathcal{S}_k} (\tilde{b}_k(\mathbf{s}_k) - \boldsymbol{\lambda}^T \mathbf{s}_k \underbrace{- \frac{c}{2} \|\mathbf{s}_k\|^2)}_{(A)} \right) + \boldsymbol{\lambda}^T \mathbf{P}^{\text{tot}} \tag{13}$$

$$\text{and with } \; c = \frac{\epsilon}{1/2 \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} (s_k^{n,\text{mask}})^2}. \tag{14}$$

$\|.\|$ refers to the Euclidean norm and ϵ to the desired accuracy with which we want to solve our original problem (8). This approach is inspired by the results of [15] where it is shown that solving the smoothed approximation (12) results in the same solution as that of (8) within the desired accuracy ϵ. A similar approach is used in [14] to tackle the problem of spectrum coordination whereas here we will extend it for the problem of joint spectrum and signal coordination for different values of the SNR gap and for different receivers.

4.1. Zero SNR gap ($\Gamma = 1$) and MMSE-GDFE receiver

We first focus on the zero SNR gap case, i.e., $\Gamma = 1 (= 0\text{dB})$, and a MMSE-GDFE receiver, for which (8) becomes a *convex* optimization problem [16].

4.1.1. Solution for dual master problem (12)

For the considered convex problem setting, we can reuse the results from [15, 14] to show that the addition of a strongly concave term (A) in (13), results in a dual function $\bar{g}(\boldsymbol{\lambda})$ that is differentiable and has a Lipschitz continuous gradient.

This allows to apply an optimal gradient algorithm such as Nesterov's scheme [17] to solve the dual master problem (12), which converges much faster than the standard subgradient approach, and without increasing the computational complexity for each iteration [14]. We elaborate Nesterov's scheme for our concrete smoothed problem (12) in Algorithm 1. This algorithm iteratively updates the transmit powers $\mathbf{s}_k, k \in \mathcal{K}$, and the dual variables $\boldsymbol{\lambda}$. More specifically, line 6 corresponds to solving the subproblems (13), which will be discussed in Section 4.1.2. Line 7 and 8 correspond to a standard (sub)gradient update with stepsize $1/L_c$. In line 9 and 10 a weighted average is constructed of the present and past (sub)gradients. In line 11 a convex combination is taken of the averaged (sub)gradient and the present subgradient update. Finally, in line 14 the solution is computed as an average of the transmit powers over all iterations. The following theorem can be proven concerning the convergence rate of Algorithm 1.

Algorithm 1 MAC-DSB algorithm for solving (8)

1: $i := 0$, tmp $:= 0$, $\boldsymbol{\lambda}^0$
2: initialize required application accuracy ϵ
3: initialize $i_{\max} = \sqrt{K \sum_{k\in\mathcal{K}} \sum_{n\in\mathcal{N}} (s_k^{n,\text{mask}})^2/2 \frac{2}{\epsilon}} - 1$,
4: $c :=$(14) , $L_c := K/c$
5: **for** $i = 0 \dots i_{\max}$ **do**
6: $\quad \forall k : \mathbf{s}_k^{i+1} = \underset{\{\mathbf{s}_k \in \mathcal{S}_k\}}{\operatorname{argmax}} \left(\tilde{b}_k(\mathbf{s}_k) - \boldsymbol{\lambda}^T \mathbf{s}_k - c\|\mathbf{s}_k\|^2/2 \right)$
7: $\quad d\bar{g}^{i+1} = \sum_{k\in\mathcal{K}} \mathbf{s}_k^{i+1} - \mathbf{P}^{\text{tot}}$
8: $\quad \mathbf{u}^{i+1} = [\frac{d\bar{g}^{i+1}}{L_c} + \boldsymbol{\lambda}^i]^+$
9: $\quad \text{tmp} := \text{tmp} + \frac{i+1}{2} d\bar{g}^{i+1}$
10: $\quad \mathbf{v}^{i+1} = [\frac{\text{tmp}}{L_c}]^+$
11: $\quad \boldsymbol{\lambda}^{i+1} = \frac{i+1}{i+3}\mathbf{u}^{i+1} + \frac{2}{i+3}\mathbf{v}^{i+1}$
12: $\quad i := i + 1$
13: **end for**
14: $\hat{\boldsymbol{\lambda}} = \boldsymbol{\lambda}^{i_{\max}}$ and $\hat{\mathbf{s}}_k = \sum_{l=0}^{i_{\max}} \frac{2(l+1)}{(i_{\max}+1)(i_{\max}+2)} \mathbf{s}_k^{l+1}, k \in \mathcal{K}$

Theorem 1. *The solution of Algorithm 1, i.e., $\hat{\boldsymbol{\lambda}}, \{\hat{\mathbf{s}}_k, k \in \mathcal{K}\}$, approximates the optimal solution of (8) with a duality gap less than ϵ, i.e., $g(\hat{\boldsymbol{\lambda}}) - \sum_{k\in\mathcal{K}} \tilde{b}_k(\hat{\mathbf{s}}_k) \leq \epsilon$, after $i_{\max} = \sqrt{K \sum_{k\in\mathcal{K}} \sum_{n\in\mathcal{N}} (s_k^{n,\text{mask}})^2/2 \frac{2}{\epsilon}} - 1$ iterations, i.e., a convergence rate of $\mathcal{O}(1/\epsilon)$.*

Proof. The proof is similar to that of Theorem 2 in [14], with the only difference that it involves a different concave objective function, but which does not change the proof. □

Theorem 1 thus claims that Algorithm 1 is a dual method that improves the convergence rate with one order of magnitude compared to the standard subgradient approach used in [10], i.e., $\mathcal{O}(1/\epsilon)$ compared to $\mathcal{O}(1/\epsilon^2)$, without increasing the computational complexity per iteration.

4.1.2. Solution for smoothed subproblems (13)

We start from the following KKT stationarity condition of (13)

$$\frac{\partial}{\partial s_k^n}(\tilde{b}_k(\mathbf{s}_k)) - \lambda_n - c s_k^n = 0, \quad \forall n \in \mathcal{N}, k \in \mathcal{K}. \tag{15}$$

This can be reformulated as a fixed point update formula by extracting s_k^n to one side of the equation and taking the bounds into account by projection as follows,

$$s_k^n = \left[\frac{w_n}{\log(2)(\lambda_n + c s_k^n + p_k^n)} - \frac{\Gamma}{\mathbf{h}_k^{n,H} (\mathbf{M}_k^n)^{-1} \mathbf{h}_k^n} \right]_0^{s_k^{n,\text{mask}}} \tag{16}$$

with $p_k^n = \sum_{m=1, m\neq n}^{N} \frac{w_m}{\log(2)} \text{Tr}\left((\mathbf{D}_k^m)^{-1} (\mathbf{M}_k^m)^{-1} \frac{\partial \mathbf{J}_k^m}{\partial s_k^n} \mathbf{G}_k^m \frac{1}{\Gamma} \right)$
and $\mathbf{D}_k^m = \mathbf{I} + (\mathbf{M}_k^m)^{-1} \mathbf{h}_k^m \frac{s_k^m}{\Gamma} \mathbf{h}_k^{m,H}$.

Note that $\text{Tr}(\mathbf{X})$ refers to the trace of matrix $\mathbf{X}$ and $[x]_b^a = \max(\min(x, a), b)$. This transmit power update formula has a similar waterfilling-like structure as the transmit power update formula of the DSB algorithm proposed in [4]. However, update formula (16) is derived for a different problem setting and has therefore a different value for p_k^n. By iteratively updating the transmit powers s_k^n using (16) over all users in a Gauss-Seidel manner, it converges to the optimal solution of (13) as problem (8) is convex for the considered setting. A convergence analysis can be derived similarly as in [4]. It has been shown in [4] that this type of iterative fixed point update approach requires a very low complexity and converges up to a good accuracy in only 2-3 iterations over all users. Furthermore because of the waterfilling structure of (16), this results in a number of practical advantages (distributed implementation, fast implementations, etc.) as was highlighted in [4]. Because of the similarity of the proposed iterative fxed point update (16) approach with that of the DSB algorithm, we will refer to Algorithm 1 with the name *multiple access channel distributed spectrum balancing (MAC-DSB)*. We would also like to highlight that update formula (16) corresponds to *continuous* transmit powers and bits, in contrary to the discrete character of the MAC-OSB and MAC-ISB algorithms.

4.2. Non-zero SNR gap ($\Gamma > 1$) and MMSE-GDFE receiver

For a non-zero SNR gap, i.e., $\Gamma > 0$dB or $\Gamma > 1$, the convexity of (8) cannot be guaranteed anymore. As a consequence, the improvement of the convergence rate in Theorem 1 cannot be theoretically proven anymore. Furthermore, MAC-DSB does not necessarily converge to the globally optimal solution of (8). Although these theoretical results cannot be extended, one can observe similar speed-ups in practical simulation results. In [4] a similar observation was obtained for the case of pure spectrum coordination. The performance of MAC-DSB

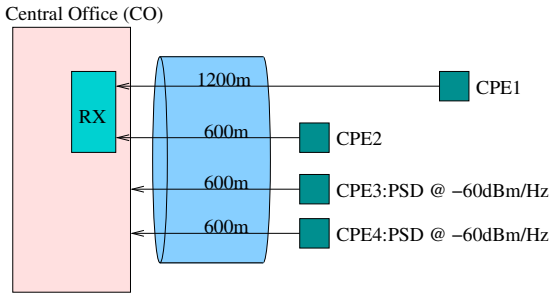

Fig. 1. 4-user VDSL upstream scenario with joint spectrum and signal coordination on two top most DSL lines

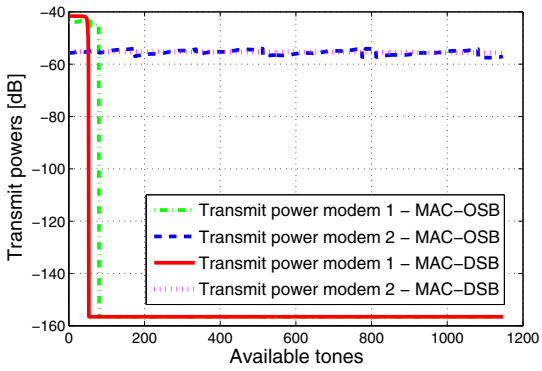

Fig. 2. Transmit spectra MAC-OSB and MAC-DSB

for non-zero SNR gap will be demonstrated in Section 5 and compared to that of MAC-OSB.

4.3. Non-zero SNR gap ($\Gamma > 1$) and linear MMSE receiver

MAC-DSB can also be used for a linear MMSE receiver instead of a MMSE-GDFE receiver. The only change is that in formulation (5), and consequently also (16), the definition of $\mathcal{J}_k^n$ (6) should be replaced by the following definition

$$\mathcal{J}_k^n = \{1, \ldots, n-1, n+1, \ldots, N\}. \qquad (17)$$

This demonstrates the generality of the proposed transmit power update formula (16) and also of the proposed MAC-DSB Algorithm 1.

5. SIMULATION RESULTS

Simulations are performed for the 4-user VDSL upstream scenario of Fig. 1, where the two top most DSL lines are jointly spectrum and signal coordinated. The two lower most lines are not-coordinated and can be regarded as alien crosstalkers. The following parameters are chosen for the DSL simulator. The twisted pair lines have a diameter of 0.5 mm (24 AWG). The maximum transmit power is 11.5 dBm. The SNR gap Γ

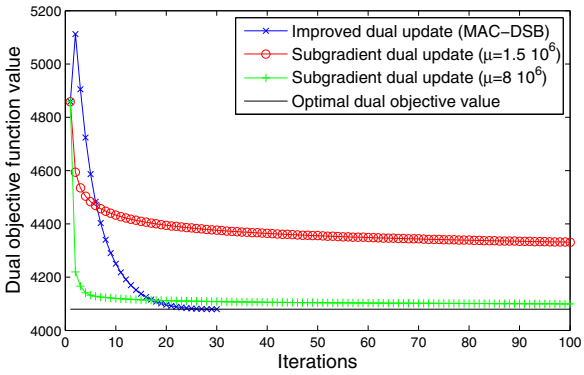

Fig. 3. Comparison of proposed improved approach for updating dual variables versus standard subgradient approach (11) with fixed stepsize μ.

is 12.9 dB, corresponding to a coding gain of 3 dB, a noise margin of 6 dB and a target symbol error probability of 10^7. The tone spacing is 4.3125 kHz. The DMT symbol rate is 4 kHz. The weights are fixed at 0.5 for all users.

We compare the performance of our improved dual decomposition algorithm (MAC-DSB) with that of the MAC-OSB algorithm [10], under the same conditions of a non-zero gap and assuming a MMSE-GDFE receiver structure. Both algorithms thus solve the same problem where MAC-OSB uses a discrete exhaustive search (with discrete (integer) bit loading) so as to find the (discrete) solution of (8), in contrary to the continuous bit loading targeted by MAC-DSB. The resulting transmit spectra are shown in Fig. 2. Both solutions are similar. In fact, the proposed MAC-DSB algorithm results in a slightly better performance compared to that of MAC-OSB, because it uses continuous rather than discrete bit loading. The increase in performance is however smaller than 1%. Furthermore, MAC-DSB requires only seconds to find the solution, whereas MAC-OSB requires 10 minutes. For more than 5 users, MAC-OSB requires more than a week of simulation time [10] whereas the proposed MAC-DSB algorithm requires only a few minutes. The computational complexity is thus reduced significantly.

Finally, in Fig. 3 we plot the convergence behaviour of the standard subgradient approach (11) with fixed stepsize μ versus the proposed improved dual update approach of the MAC-DSB algorithm as given in Algorithm 1 to solve the dual problem (9) up to a certain accuracy ϵ. We tried to tune the subgradient stepsizes μ as good as possible, where we plot the convergence behaviour for the values $\mu = 1.5 \times 10^6$ and $\mu = 8 \times 10^6$. For all approaches, we start from the same inital values for the dual variables. Note that the proposed improved dual update approach of the MAC-DSB algorithm does not require any tuning, and requires only 31 iterations to converge, whereas the subgradient approaches do not converge even after 100 iterations. This improved convergence behaviour is verified for other DSL scenarios too.

6. CONCLUSION

A novel improved dual decomposition based algorithm is proposed for joint spectrum and receiver signal coordination in upstream DSL transmission, and is referred to as MAC-DSB. The algorithm can be used for both a MMSE-GDFE receiver as well as linear MMSE receiver. MAC-DSB consists of an optimal gradient algorithm for updating the dual variables. In addition, low-complexity iterative fixed point updates are proposed for tackling the subproblems with fixed dual variables. Practical simulation results show that the computational complexity of the MAC-DSB algorithm is much lower than that of existing algorithms MAC-OSB and MAC-ISB, especially for large-scale DSL scenarios. Furthermore, it is shown that MAC-DSB can even result in an improved performance compared to that of the existing MAC-OSB algorithm, because of the continuous character of MAC-DSB.

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