# Optimal Time-Switching for MU MISO SWIPT employing OFDM with Power Harvesting Constraints


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Supplemental information is presented here that discusses applying the results of [1] to [2]. Appendix A describes the rationale for the *direction oracle* and Appendix B applies it to the problem considered in [2].

## Appendix A
## Direction Oracle Rationale

In this appendix, the relationship between $\alpha_i^{n*}$ and $\alpha_i^n$ will be examined for a single user, user $n$, through Fig. 1 and Fig. 2 to highlight the rationale behind the direction oracle. Both figures illustrate the Fixed-$\boldsymbol{\alpha}$ rate-power region (RPR) associated with $\alpha_i^n$, denoted with $\mathbf{RPR}_{\boldsymbol{\alpha}_i}$ (yellow line). The optimal time-switched point using $\alpha_i^n$ is labeled "Optimal Point" and is where the $\mathbf{RPR}_{\boldsymbol{\alpha}_i}$ intersects the Pareto edge of the Optimal RPR. The solution to the original problem considered in [2] given $\alpha_i^n$ is where the rate-power manifold (red line labeled $\mathcal{M}_A$ or $\mathcal{M}_B$) intersects the $\mathbf{RPR}_{\boldsymbol{\alpha}_i}$, denoted with $(1-\alpha_i^n)r_{x_i}^n, \alpha_i^n r_{y_i}^n$. If the problem is feasible, then $(1-\alpha_i^n)r_{x_i}^n$ will equal the harvesting constraint $c^n$. This rate-power manifold represents *all* of the time-switched points associated with the Utopia point $r_{x_i}^n, r_{y_i}^n$. When the optimal time-switching ratio $\alpha_i^{n*}$ is used to time-switch the Utopia point, it produces a point on the Pareto edge of the Optimal RPR, denoted with $(1-\alpha_i^{n*})r_{x_i}^n, \alpha_i^{n*} r_{y_i}^n$. This is the point where the rate-power manifold is tangent to the Optimal RPR. This point is also the intersection for the Fixed-$\boldsymbol{\alpha}$ RPR associated with $\alpha_i^{n*}$, $\mathbf{RPR}_{\boldsymbol{\alpha}_i^*}$ (green line), and the Pareto edge of the Optimal RPR.

Concerning the time-switching ratio, there are three possible relations:


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- $\alpha_i^{n*} > \alpha_i^n$
  Fig. 1 illustrates $\alpha_i^{n*} > \alpha_i^n$ for a single user. Notice that the optimal point on $\mathbf{RPR}_{\boldsymbol{\alpha}_i}$ achieves a higher harvested power ($\approx 0.65$) than the harvesting constraint $c^n$. This indicates that $\alpha_i^n$ should be *increased* to promote a higher data rate and move this intersection toward the constraint. Notice that this adjustment is in the direction of $\alpha_i^{n*}$ along the rate-power manifold $\mathcal{M}_A$.

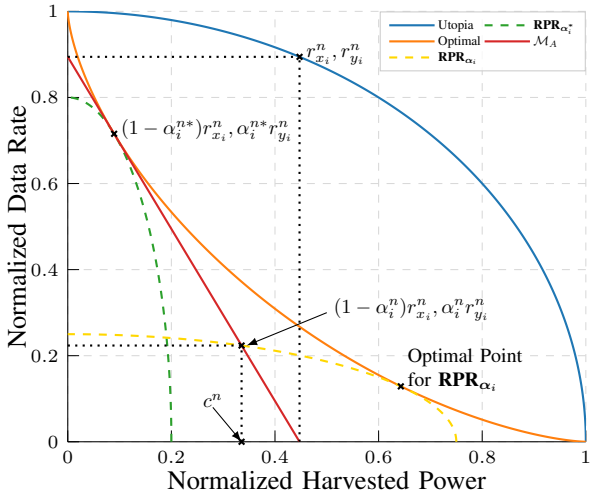


Fig. 1. The Utopia RPR, Optimal RPR, and Fixed-$\boldsymbol{\alpha}$ RPRs for $\alpha_i^n$ and $\alpha_i^{n*}$ are depicted. The Utopia point $\{r_{x_i}^n, r_{y_i}^n\}$ is associated with an optimal time-switching ratio $\alpha_i^{n*}$. Both $\alpha_i^n$ and $\alpha_i^{n*}$ are on the same rate-power manifold, given by trace $\mathcal{M}_A$. In this case $\alpha_i^{n*} > \alpha_i^n$, which indicates that *increasing* $\alpha_i^n$ improves its optimality.

- $\alpha_i^{n*} < \alpha_i^n$
  Fig. 2 illustrates $\alpha_i^{n*} < \alpha_i^n$ for a single user. Notice that the optimal point on $\mathbf{RPR}_{\boldsymbol{\alpha}_i}$ achieves a lower harvested power ($\approx 0.1$) than the harvested power constraint $c^n$. This indicates that $\alpha_i^n$ should be *decreased* to promote a higher harvested power and move this intersection toward the constraint. Notice that this adjustment is in the direction of $\alpha_i^{n*}$ on the rate-power manifold $\mathcal{M}_B$.

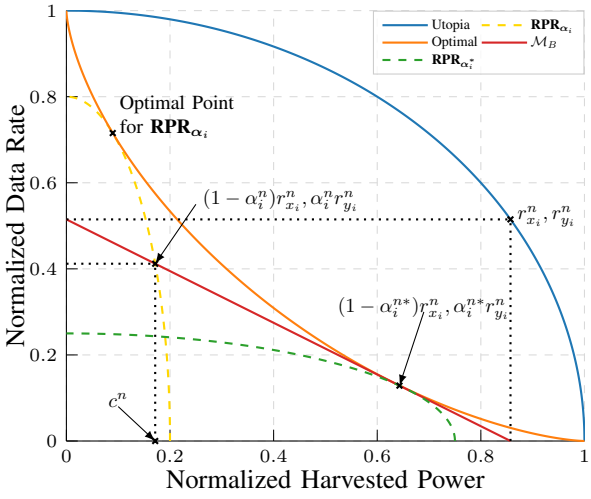


Fig. 2. The Utopia RPR, Optimal RPR, and Fixed-$\boldsymbol{\alpha}$ RPRs for $\alpha_i^n$ and $\alpha_i^{n*}$ are depicted. The Utopia point $\{r_{x_i}^n, r_{y_i}^n\}$ is associated with an optimal time-switching ratio $\alpha_i^{n*}$. Trace $\mathcal{M}_B$ is the associated rate-power manifold. In this case $\alpha_i^{n*} < \alpha_i^n$, which indicates that *decreasing* $\alpha_i^n$ improves its optimality.

- $\alpha_i^{n*} = \alpha_i^n$
  If $\alpha_i^{n*} = \alpha_i^n$ then $\mathbf{RPR}_{\boldsymbol{\alpha}_i}$ and $\mathbf{RPR}_{\boldsymbol{\alpha}_i^*}$ coincide and intersect the Pareto edge of the Optimal RPR at the same point. Hence, the time-switching ratio is optimal and *no adjustment* is needed and the optimal time-switching ratio is $\alpha_i^{n\ \mathrm{opt}} = \alpha_i^{n*}$.

Thus, the direction oracle defined in Appendix B may be used to update the time-switching ratio.

## Appendix B
## Joint Resource Allocation and Time-Switching Optimization with Power Harvesting Constraints

### A. Primal Problem Formulation

This appendix examines how to optimize the time-switching ratios for the problem considered in [2]. Including the time-switching ratios as optimization parameters reults in

$$\begin{aligned} \underset{\mathsf{T},\mathsf{s},\boldsymbol{\alpha}}{\text{maximize}} \ & \sum_{n=1}^{N} \omega^n \hat{r}_y^n \\ \text{s.t.} \ & \sum_{n=1}^{N} p^n \leq P_{\text{Total}} \\ & \hat{r}_x^n \geq c^n \quad \forall\, n \\ & \alpha^n \in [0,1] \ \forall\, n, \end{aligned} \tag{1}$$

where for user $n$, $\omega^n$ is a nonnegative weight used to prioritize the data rates, $p^n$ is the transmit power, $P_{\text{Total}}$ is the total transmit power constraint, and $c^n$ is the power harvesting constraint. The optimal point is chosen by selecting a set of precoder matrices $\mathsf{T}$, a set of symbol power vectors $\mathsf{s}$, *and* time-switching ratios $\boldsymbol{\alpha} = [\alpha^1, \ldots, \alpha^N]^\top$ that maximizes the weighted sum.

However, (1) is a challenging problem to solve, primarily because the time-switching ratios couple the subcarriers, making them inseparable and preventing the use of dual decomposition. The asymptotic computational complexity is exponential in the number of subcarriers $K$ without dual decomposition. Instead, we will optimize the time-switching ratios using the original problem considered in [2], given as

$$\begin{aligned} \underset{\mathsf{T},\mathsf{s}}{\text{maximize}} \ & \sum_{n=1}^{N} \omega^n \hat{r}_y^n \\ \text{s.t.} \ & \sum_{n=1}^{N} p^n \leq P_{\text{Total}} \\ & \hat{r}_x^n \geq c^n \ \forall\, n \\ & \alpha^n \in [0,1] \ \forall\, n, \end{aligned} \tag{2}$$

where the optimal point is chosen by selecting a set of precoder matrices $\mathsf{T}$ and a set of symbol power vectors $\mathsf{s}$ that maximize the weighted sum. Observe that both the Utopia RPR and the Fixed-$\boldsymbol{\alpha}$ RPRs seem to have concave shapes, which are conducive to maximizing over. A Pareto optimal point on the Optimal RPR may be found by maximizing a series of Fixed-$\boldsymbol{\alpha}$ RPRs for changing time-switching ratios until the optimal points are Pareto optimal on the Optimal RPR.

### B. Dual Problem Formulation

The Lagrangian for (2) is given by

$$\mathcal{L}(\boldsymbol{\alpha}, \mathsf{s}, \mathsf{T}, \boldsymbol{\lambda}) \triangleq \lambda^0 P_{\text{Total}} + \sum_{k=1}^{K} \mathcal{L}^k(\boldsymbol{\alpha}, \boldsymbol{\omega}, \boldsymbol{\lambda}, \boldsymbol{s}^k, \mathsf{T}^k), \tag{3}$$

where

$$\mathcal{L}^k(\boldsymbol{\alpha}, \boldsymbol{\omega}, \boldsymbol{\lambda}, \boldsymbol{s}^k, \mathsf{T}^k) = \sum_{n=1}^{N} \omega^n \hat{r}_y^n \ - \sum_{n=1}^{N} \boldsymbol{t}^{k,n\,\mathcal{H}} \Upsilon^k(\boldsymbol{\alpha}, \boldsymbol{\lambda}) \boldsymbol{t}^{k,n} s^{k,n} \tag{4}$$

and

$$\Upsilon^k(\boldsymbol{\alpha}, \boldsymbol{\lambda}) \triangleq \left( \lambda^0 I - \sum_{m=1}^{N} \lambda^m (1-\alpha^m) \boldsymbol{h}^{k,m} \boldsymbol{h}^{k,m\,\mathcal{H}} \right). \tag{5}$$

$\boldsymbol{\lambda} = [\lambda^0, \lambda^1, \ldots, \lambda^N]^\top$ is the $N+1 \times 1$ vector of Lagrange multipliers, where $\lambda^0$ is the Lagrange multiplier associated with the total transmit power constraint and $\lambda^n$ for $n \in [1, N]$ is the Lagrange multiplier associated with the power harvesting constraint $c^n$ of user $n$. The Lagrange dual problem is then given by

$$\begin{aligned} \underset{\lambda}{\text{minimize}} \ & q(\boldsymbol{\alpha}, \boldsymbol{\omega}, \boldsymbol{\lambda}) \\ \text{s.t.} \ & \boldsymbol{\lambda} \geq 0 \\ & \alpha^n \in [0,1] \quad \forall\, n \\ & \Upsilon^k(\boldsymbol{\alpha}, \boldsymbol{\lambda}) \succ 0 \quad \forall k, \end{aligned} \tag{6}$$

where $\boldsymbol{\lambda} \geq 0$ is element-wise and the constraint on $\Upsilon^k$ bounds the dual problem from below.

### C. Optimizing the Time-Switching Ratio

The results from [1] will now be utilized to create an algorithm that alternates between optimizing the time-switching ratio and, in turn, optimizing the precoding vectors and power allocation via (2) for fixed time-switching ratios, $\boldsymbol{\alpha}$.

Given fixed time-switching ratios, $\{\omega^n, \lambda^n\}_{\forall n}$ performs the same function as the time-switched weights $\{\hat{w}_x^n, \hat{w}_y^n\}_{\forall n}$ defined in [1], in that they define a hyperplane that is tangent to the Pareto edge of the Fixed-$\boldsymbol{\alpha}$ RPR at the solution i.e. the time-switched resource allocation $\{\hat{r}_x^n, \hat{r}_y^n\}_{\forall n}$. Hence the weights have the following relationship

$$\hat{w}_x^n = \lambda^n \quad \forall\, n \tag{7}$$

and

$$\hat{w}_y^n = \omega^n \quad \forall\, n, \tag{8}$$

for the same solution and Proposition 1 in [1] may be used to calculate the related optimal time-switching ratio $\alpha^{n*}$.

For the $i$-th iteration of time-switching ratios, the direction oracle given by

$$\boldsymbol{d}_i = \boldsymbol{\alpha}_i^* - \boldsymbol{\alpha}_i, \tag{9}$$

may be used to adjust the time-switching ratios through

$$\boldsymbol{\alpha}_{i+1} = \boldsymbol{\alpha}_i + \nu_i \boldsymbol{d}_i, \tag{10}$$

which scales the step $\boldsymbol{d}_i$ by $\nu_i \in (0, 1]$. The maximum scaling is $\nu_i = 1$ since it sets the next iterate $\boldsymbol{\alpha}_{i+1} = \boldsymbol{\alpha}_i^*$, which is the time-switching ratio that makes the current Utopia point Pareto optimal on the Optimal RPR. The rationale behind this is examined in Appendix A for a single user.

### *D. Convergence Postulate*

The rate-power manifold traces the position a particular Utopia point inhabits as $\boldsymbol{\alpha}$ changes. Conceptually, the direction oracle determines the direction and amount $\boldsymbol{\alpha}$ must change for the particular Utopia point to also become Pareto optimal on the Optimal RPR.

Given $\boldsymbol{\alpha}_i$, let the Lagrangian at the solution found by the dual problem be denoted with

$$\mathcal{L}^*(\boldsymbol{\alpha}_i) = \mathcal{L}(\boldsymbol{\alpha}_i, \mathsf{s}_i^*, \mathsf{T}_i^*, \boldsymbol{\lambda}_i^*), \tag{11}$$

which defines a Pareto optimal point on $\mathbf{RPR}_{\boldsymbol{\alpha}_i}$. Postulate 1 surmises that the direction oracle is an ascent direction at this point.

**Postulate 1.** The direction oracle defined in (9) is an ascent direction.

Given the $i$-th iteration of the time-switching ratio $\boldsymbol{\alpha_i}$, the sign of the direction oracle is given by

$$d_i^n = \alpha_i^{n*} - \alpha_i^n : \begin{cases} > 0 & \alpha_i^n < \alpha_i^{n*} \\ < 0 & \alpha_i^n > \alpha_i^{n*} \\ 0 & \alpha_i^n = \alpha_i^{n*} \end{cases} \quad \forall n. \tag{12}$$

When $\alpha_i^n < \alpha_i^{n*}$ then

$$\begin{aligned} \alpha_i^n &< \frac{\hat{w}_y^n \hat{r}_{y_i}^n}{\hat{w}_y^n \hat{r}_{y_i}^n + \hat{w}_x^n \hat{r}_{x_i}^n} \\ \alpha_i^n(\hat{w}_y^n \hat{r}_{y_i}^n + \hat{w}_x^n \hat{r}_{x_i}^n) &< \hat{w}_y^n \hat{r}_{y_i}^n \\ \frac{\alpha_i^n}{1-\alpha_i^n} &< \frac{\hat{w}_y^n \hat{r}_{y_i}^n}{\hat{w}_x^n \hat{r}_{x_i}^n} = \frac{\omega^n \alpha_i^n r_{y_i}^n}{\lambda_i^n (1-\alpha_i^n) r_{x_i}^n} \\ \frac{\alpha_i^n}{1-\alpha_i^n} \frac{1-\alpha_i^n}{\alpha_i^n} = 1 &< \frac{\omega^n r_{y_i}^n}{\lambda_i^n r_{x_i}^n}. \end{aligned} \tag{13}$$

The last two lines are due to

$$\{r_x^n, r_y^n\}_{\forall n} = \left\{ \frac{\hat{r}_x^n}{(1-\alpha^n)}, \frac{\hat{r}_y^n}{\alpha^n} \right\}_{\forall\, n} \tag{14}$$

from [1] as well as (7), and (8).

Similarly for $\alpha_i^n > \alpha_i^{n*}$

$$1 > \frac{\omega^n r_{y_i}^n}{\lambda_i^n r_{x_i}^n} \tag{15}$$

and for $\alpha_i^n = \alpha_i^{n*}$

$$1 = \frac{\omega^n r_{y_i}^n}{\lambda_i^n r_{x_i}^n}. \tag{16}$$

Given $\boldsymbol{\alpha}_i$, the gradient of $\mathcal{L}^*(\boldsymbol{\alpha}_i)$ is

$$\nabla_{\boldsymbol{\alpha}_i} \mathcal{L}^*(\boldsymbol{\alpha}_i) = \begin{bmatrix} \omega^1 r_{y_i}^{1*} - \lambda_i^{1*} r_{x_i}^{1*} \\ \vdots \\ \omega^N r_{y_i}^{N*} - \lambda_i^{N*} r_{x_i}^{N*} \end{bmatrix}, \tag{17}$$

where $\{\mathsf{s}_i^*, \mathsf{T}_i^*, \boldsymbol{\lambda}_i^*\}$ produced the Utopia point $\{r_{x_i}^{n*}, r_{y_i}^{n*}\}_{\forall n}$. The sign of the partial derivative of the dual problem with respect to (w.r.t.) user $n$ is then

$$\frac{\partial}{\partial \alpha_i^n} \mathcal{L}^*(\boldsymbol{\alpha}_i) : \begin{cases} > 0 & \alpha_i^n < \alpha_i^{n*} \\ < 0 & \alpha_i^n > \alpha_i^{n*} \\ 0 & \alpha_i^n = \alpha_i^{n*}. \end{cases} \tag{18}$$

Since the direction oracle and the partial derivative for user $n$ have the same sign and because $\mathcal{L}^*(\boldsymbol{\alpha}_i)$ is linear w.r.t. the time-switching ratio, $\boldsymbol{d}_i$ is an ascent direction.

Observe that any iteration that decreases the time-switching ratio from a feasible point w.r.t. $\{c^n\}_{\forall n}$ will also be feasible, but this is not true for increasing the time-switching ratio. If $\boldsymbol{\alpha}_{i+1}$ is too large, the problem becomes infeasible and the terms associated with the Lagrange multipliers will become negative since $(1-\alpha_{i+1}^n) r_{x_{i+1}}^n < c^n$ for user $n$, causing the Lagrangian to decrease in value. Hence, it is surmised that time-switching ratio iterates found through (10) will result in

$$\mathcal{L}^*(\boldsymbol{\alpha}_{i+1}) \geq \mathcal{L}^*(\boldsymbol{\alpha}_i) \tag{19}$$

for iteration $i$, provided the iterates are feasible. Since the optimization is upper-bounded by the total transmit power constraint, (19) leads to Postulate 2.

**Postulate 2.** Provided the iterates are feasible, the iteration defined in (10) will eventually converge.

A constant, backtracking, or diminishing step size may be used to set $\nu_i$. A routine utilizing this is implemented in Algorithm 1.

### *E. Algorithms*

*1) Optimizing the Time-Switching Ratio:* The process for optimizing the time-switching ratios in conjunction with (2) is defined in Algorithm 1. First, the time-switching ratios, $\boldsymbol{\alpha}$, are chosen, then a resource allocation is generated using the algorithm in [2] and checked for convergence. If it has not converged, the time-switching ratios are adjusted through (10) and the process is repeated. The direction oracle goes to 0

as the time-switching ratios approach the optimal values, so several types of step sizing can be used such as the diminishing step size $\nu_i = 2/2+i$ for iteration $i$, a constant step size that is small enough to increase the objective, or a step size found through backtracking line search.

**Algorithm 1** Resource Allocation for (1)

Set tolerance, $\epsilon_\alpha$.
Set data rate weights, $\boldsymbol{\omega}$.
Set harvesting constraints, $\boldsymbol{c}$.

1: $i = 0$
2: Initialize $\boldsymbol{\alpha}_i$.
3: **while** true **do**
4: $\{\mathsf{T}_i, \mathsf{s}_i, \boldsymbol{\lambda}_i\}$ = Ellipsoid($\boldsymbol{\omega}$,$\boldsymbol{c}$,$\boldsymbol{\alpha}_i$) ▷ From [2] and $\boldsymbol{d}_i$
5: $\nu_i = \frac{2}{2+i}$, $\boldsymbol{\alpha}_{i+1} = \boldsymbol{\alpha}_i + \nu_i \boldsymbol{d}_i$
6: **if** $\frac{\|\boldsymbol{\alpha}_{i+1} - \boldsymbol{\alpha}_i\|}{\|\boldsymbol{\alpha}_{i+1} + \boldsymbol{\alpha}_i\|} \leq \epsilon_\alpha$ **then**
7: **return** $\{\mathsf{T}_i, \mathsf{s}_i, \boldsymbol{\alpha}_i, \boldsymbol{w}_{x_i}, \boldsymbol{w}_{y_i}\}$ ▷ Exit with results.
8: $i = i + 1$

*2) Asymptotic Computational Complexity:* (2) has to meet specific power harvesting constraints in addition to the total power constraint. The Ellipsoid method was used here and in [2] to adjust the Lagrange multipliers to meet these constraints. The Ellipsoid method requires $O\big(2(N+1)(N+2)\ln\big(\lambda_{\text{UB}}\epsilon_E^{-1}\big)\big)$ iterations [3], where $\epsilon_E$ is the radius of the final ellipsoid, $\lambda_{\text{MAX}}$ is the maximum upper bound for the Lagrange multipliers defined in [2] , and $(N+1)$ is the number of constraints. Each iteration has $O\big((N+1)^2\big)$ complexity [4].

### F. Simulations

*1) Single User Rate-Power Region:* Fig. 3 illustrates the Utopia RPR and the corresponding Optimal RPR for a single user. Feasible operating points for three different harvested powers were also generated using

$$\alpha^n = 1 - \frac{c^n}{r_x^n}, \tag{20}$$

provided that $r_x^n \geq c^n$ . Notice that the optimal time-switching ratio results in operating points with the largest data rates compared to the feasible points, as expected. Also, note that power is always required to send data; although the power weights $w_x$ may be reduced to zero, $r_x$ must be nonzero for nonzero $r_y$. Hence there is a range of constraints (e.g. $[0, \sim 0.014]$ mW) that are not tightly obtainable on the Utopia RPR, so time-switching is required to attain these points tightly.

*2) Weighted Sum Rate: Optimizing the Time-Switching Ratio:* Fig. 4 depicts the paired iterates on the Optimal RPR and the Utopia RPR for a 2-user network and various power harvesting constraints. Algorithm 1 was used with $\nu_i = 2/2+i$ for iteration $i$ and with equal data rate weights. Some ringing occurs due to the diminishing step size, but all harvesting constraints are met since the scaling eventually becomes small enough for convergence. Fig. 5 illustrates Algorithm 1 converging using constant step size $\nu = 0.05$. The ringing is diminished and all points converged.

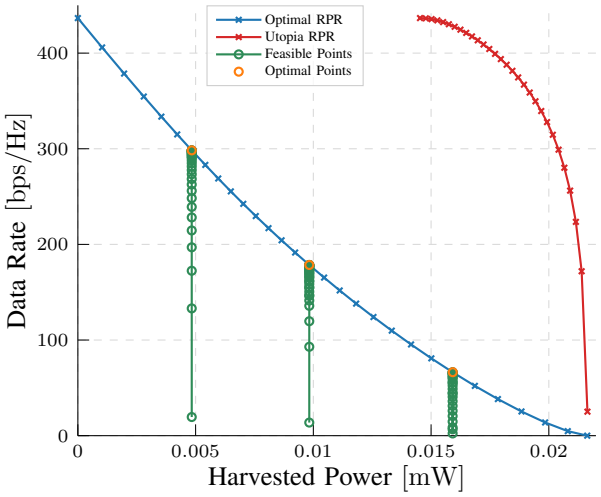


Fig. 3. The Utopia RPR and corresponding Optimal RPR for a single user. Feasible time-switched points are shown for three different power harvesting constraints. The optimal point for each constraint is the associated feasible point on the Optimal RPR.

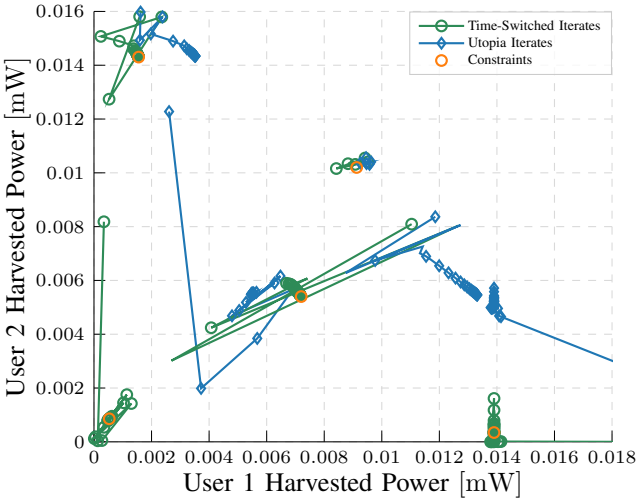


Fig. 4. Using equal data rate weighting, the Optimal RPR and Utopia RPR iterates for Algorithm 1 using a diminishing step size are shown to converge to selected constraints. Some ringing is observed as is expected.

Fig. 6 illustrates Algorithm 1 for 3 users using a constant step scaling $\nu = 0.1$. It shows that the feasible data rate iterates (e.g. $\boldsymbol{\alpha}_i^\top \boldsymbol{r}_{y_i}$) and the optimal data rate iterates (e.g. $\boldsymbol{\alpha}_i^{*\top} \boldsymbol{r}_{y_i}$) converge smoothly. Simultaneously with the data rate iterates, the harvested power iterates converge to the harvested power constraints. This is illustrated using the user harvesting power residual ratio which is given by

$$((1 - \alpha^{*n}) r_x^n - c^n)(c^n)^{-1} \tag{21}$$

for user $n$. Similarly, Fig. 7 illustrates Algorithm 1 for 4 users using a diminishing step size.

*3) Weighted Sum Rate: Grid Search:* A grid search can be performed for small networks to illustrate the near optimality of Algorithm 1. For a particular set of harvesting constraints,

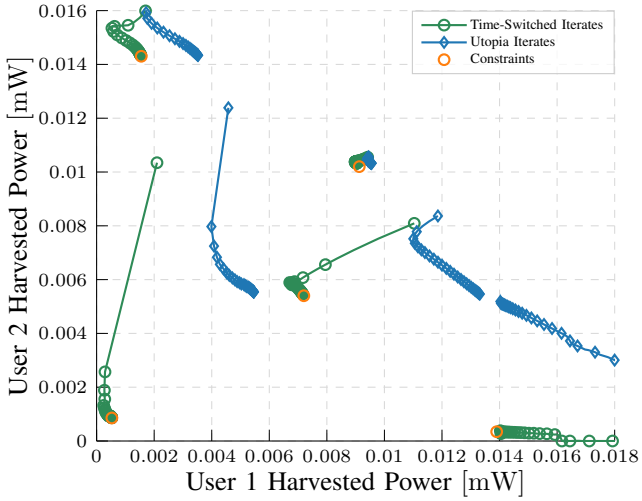


Fig. 5. Using equal data rate weighting, the Optimal RPR and Utopia RPR iterates for Algorithm 1 using a constant step size $\nu = 0.05$ are shown to converge to selected constraints.

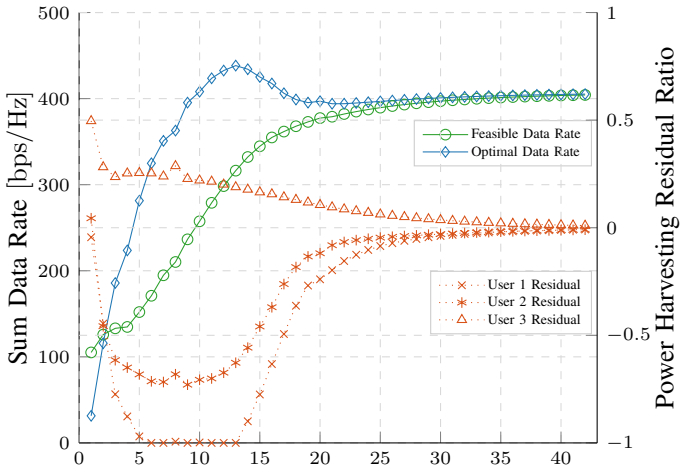


Fig. 6. An example of Algorithm 1 using a constant step size $\nu = 0.1$ for 3 users.

the ellipsoid-based algorithm from [2] is used to find a resource allocation for a range of time-switching ratios defined by a grid. Fig. 8 compares the achieved sum data rates between both algorithms, showing that Algorithm 1 produces a result that is very close to that of grid search. Note that the result from Algorithm 1 is likely improvable by using a tighter convergence threshold for the time-switching ratio.

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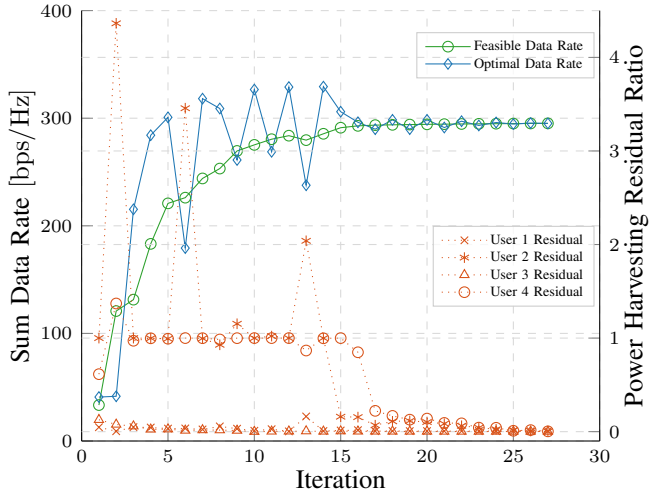


Fig. 7. An example of Algorithm 1 using a diminishing step size $\nu_i = 2/2+i$ for 4 users.

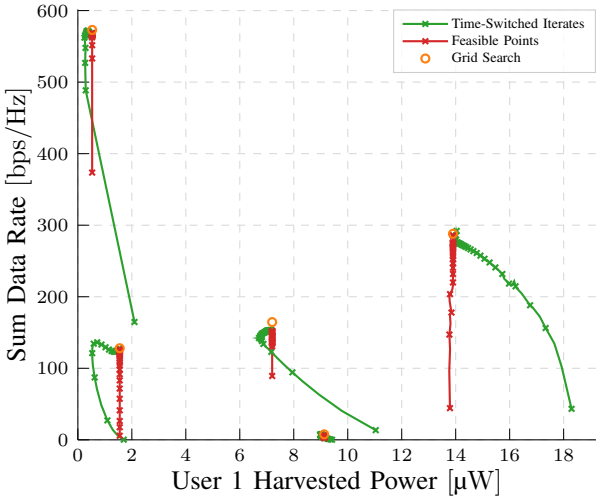


Fig. 8. The sum data rates for grid search compared to Algorithm 1 using a constant step size $\nu = 0.05$ for 2 users for 5 different constraints.


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