Katholieke Universiteit Leuven

Departement Elektrotechniek ESAT-SISTA/TR 10-236

Dynamic Spectrum management in DSL with Asynchronous Crosstalk [1]

Rodrigo B. Moraes, Paschalis Tsiaflakis and Marc Moonen[2]

October 2010
Accepted for publication in the Proceedings of the 2011 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), Prague, Czech Republic, May 2011

[1]This report is available by anonymous ftp from *ftp.esat.kuleuven.ac.be* in the directory *pub/sista/rmoraes/reports/10-236.pdf*

[2]K.U.Leuven, Dept. of Electrical Engineering (ESAT), Research group SISTA, Kastelpark Arenberg 10, 3001 Leuven, Belgium, Tel. 32/16/32 17 09, Fax 32/16/32 19 70, WWW: *http://www.esat.kuleuven.ac.be/sista*. E-mail: *rodrigo.moraes@esat.kuleuven.ac.be*. This research work was carried out at the ESAT Laboratory of Katholieke Universiteit Leuven, in the frame of K.U.Leuven Research Council CoE EF/05/006 Optimization in Engineering (OPTEC), Concerted Research Action GOA-MaNet, the Belgian Programme on Interuniversity Attraction Poles initiated by the Belgian Federal Science Policy Office IUAP P6/04 (DYSCO, Dynamical systems, control and optimization, 2007-2011), Research Project FWO nr.G.0235.07(Design and evaluation of DSL systems with common mode signal exploitation) and IWT Project PHANTER: PHysical layer and Access Node TEchnology Revolutions: enabling the next generation broadband network.

DYNAMIC SPECTRUM MANAGEMENT IN DSL WITH ASYNCHRONOUS CROSSTALK

Rodrigo B. Moraes, Paschalis Tsiaflakis and Marc Moonen

Dept. of Electrical Engineering (ESAT-SCD) – Katholieke Universiteit Leuven, 3001 Leuven, Belgium
E-mails: {rodrigo.moraes, paschalis.tsiaflakis , marc.moonen}@esat.kuleuven.be

ABSTRACT

Dynamic spectrum management (DSM) aims to optimally allocate per-user transmit spectra so that the effect of multi-user interference (crosstalk) is minimized and the capabilities of the network are maximized. Work on DSM has progressed considerably over the past decade. Most of this work, however, deals with the idealized situation of perfect synchronization amongst the multiple users. This work deals with a more realistic albeit more difficult situation, i.e we consider the DSM problem with asynchronous crosstalk. We propose a novel solution for this problem, one that is mainly based on the approximate summability of the crosstalk damage ratio (CDR). Performance of the algorithm is assessed through numerical experiments and is shown to be better than the performance of state-of-the-art methods.

Index Terms— Digital subscriber lines, dynamic spectrum management, inter-carrier interference.

1. INTRODUCTION

Digital Subscriber Line (DSL) is today the main technology for high speed broadband internet access. For about a decade now, there has been strong activity in the research community to deal with DSL's main problems and to try to expand its lifetime as much as possible. One such area of research is focused on the optimal allocation of per-user transmit spectra so that the impact of multi-user interference (*crosstalk*), the main source of performance degradation for DSL, is minimized and the capabilities of the network are mazimized. This is referred to as dynamic spectrum management (DSM).

Work on DSM has gone a long way in the past decade, see e.g. [1, 2, 3, 4, 5]. Most of this work considered a synchronous discrete multi-tone (DMT) model, one in which all users have their data blocks perfectly synchronized (i.e. all users data blocks are aligned in time). This leads to synchronous crosstalk, i.e. crosstalk that can be dealt with on a per-tone basis, and this simplifies the optimization problem significantly. However, perfectly synchronized DSL users may not be very realistic in practice. Situations in which interfering users belong to different service providers or where transmitters are not co-located are especially troublesome. This work focuses on the DSM problem with *asynchronous* crosstalk [2, 3, 6].

Although similar at a first glance, the synchronous and asynchronous DMT DSM problems are quite different, the latter being much more challenging. The consequence of the time offset of data blocks from different users is inter-carrier interference (ICI). Hence, one tone of an interferering user will affect not only the corresponding tone of a victim user, but all neighboring tones too. Solutions that take the dual decomposition approach, like all the good synchronous algorithm proposed so far, are no longer directly applicable.

In this paper, we present a novel solution for the asynchronous DMT DSM problem. The proposed algorithm is based on approximately orthogonalizing the problem, i.e our approach can be interpreted as approximately transforming the asynchronous problem into a synchronous one. We profit from the summability property of the crosstalk damage ratio (CDR) [5] to re-write the highly coupled original problem into one that is simpler. We show that performance of the proposed algorithm compares favorably with those of existing state-of-the-art methods.

This paper is organized as follows: Section 2 presents the problem of interest, Section 3 introduces the novel solution fot the asynchronous DMT DSM problem, Section 4 contains experimental results, and Section 5 presents final remarks.

2. PROBLEM STATEMENT

Consider a V user DMT system with K Δ_f-spaced tones and let $\mathbf{P} = \{p_v^k\} \in \mathbb{R}^{K\times V}$ be a matrix in which p_v^k is the transmit power of the vth user on tone k. The vth column $\mathbf{p}_v = \begin{bmatrix} p_v^1 & \cdots & p_v^K \end{bmatrix}^T$ represents the power allocation of user v in all tones and the kth row $\mathbf{p}^k = \begin{bmatrix} p_1^k & \cdots & p_V^k \end{bmatrix}$ represents the power allocation of all users in a given tone. Let $\tilde{\sigma}_v^k$ be the background noise's power observed by the vth user on tone k, $h_{i,v}^k$ be the transfer function between

This research work was carried out at the ESAT Laboratory of Katholieke Universiteit Leuven, in the frame of K.U.Leuven Research Council CoE EF/05/006 Optimization in Engineering (OPTEC), Concerted Research Action GOA-MaNet, the Belgian Programme on Interuniversity Attraction Poles initiated by the Belgian Federal Science Policy Office IUAP P6/04 (DYSCO, Dynamical systems, control and optimization, 2007-2011), Research Project FWO nr.G.0235.07(Design and evaluation of DSL systems with common mode signal exploitation) and IWT Project PHANTER: PHysical layer and Access Node TEchnology Revolutions: enabling the next generation broadband network. The scientific responsibility is assumed by its authors.

transmitter i and receiver v at tone k and Γ be the SNR gap to capacity. Now let $\sigma_v^k = \Gamma\tilde{\sigma}_v^k(|h_{v,v}^k|^2)^{-1}$ and $\alpha_{i,v}^{j,k} = \Gamma|h_{i,v}^j|^2(|h_{v,v}^k|^2)^{-1}\ \forall i \neq v$. The bit loading for user v on tone k in the asynchronous case is a function of the whole matrix $\mathbf{P}$—and that is in contrast with the synchronous case, in which bit loading is only a function of $\mathbf{p}^k$. The bit loading for user v, tone k, is defined as

$$b_v^k = \log\left(1 + \frac{p_v^k}{\sigma_v^k + \mathrm{XT}_v^k}\right), \tag{1}$$

where

$$\mathrm{XT}_v^k = \sum_{i\neq v}^{V}\sum_{j=0}^{K} \mathrm{xt}_{i,v}^{j,k} \tag{2}$$

and

$$\mathrm{xt}_{i,v}^{j,k} = \gamma_{i,v}^{j,k}\alpha_{i,v}^{j,k}p_i^j. \tag{3}$$

We will use $\log$ to denote base two logarithm. In (2) and (3), $\gamma_{i,v}^{j,k}$ and $\mathrm{xt}_{i,v}^{j,k}$ (lowercase) are respectively the ICI coefficient and the resulting normalized crosstalk specifically from user i to user v, and from tone j to tone k. Notice that XT_v^k (uppercase) in (2) is the total crosstalk for user v, tone k. For the ICI coefficients, we follow the model in [6]. For the synchronous case, $\gamma_{i,v}^{j,k} = 1$ for $k = j$ and otherwise zero for all users and tones. The total rate for the vth user is given by

$$R_v = f_s\sum_k b_v^k, \tag{4}$$

where f_s is the symbol rate. Throughout this text we always consider continuous bit loading.

The problem of interest is that of finding a matrix $\hat{\mathbf{P}}$ that maximizes the weighted data rates sum (WRS) in the network given a power budget for each user.

$$\begin{aligned}\hat{\mathbf{P}} = \arg\max_{\mathbf{P}} &\sum_v w_v R_v \\ \text{subject to}\quad & P_v^{\mathrm{tot}} \leq P_v^{\max}\ \forall v \\ & R_v \geq R_v^{\mathrm{tar}}\ \forall v > 1.\end{aligned} \tag{5}$$

Here $P_v^{\mathrm{tot}} = \sum_k p_v^k$. In (5), P_v^{tot} and $P_v^{\max}$ are the total actually allocated power and maximum power available to user v, respectively; and R_v and R_v^{tar} are the achieved and target rate for user v, respectively. The weights w_v can be interpreted as priorities needed for each user to achieve their respective target rates. In (5), we have target rates for all users except user 1. We consider that the w_v's sum to one, so $w_1 = 1 - \sum_{v>1} w_v$.

The asynchronous DMT DSM problem is quite challenging. The kind of per-tone de-coupling used for the solutions to the synchronous DMT DSM problem has no direct application, since the ICI coefficients re-couple tones and users much more strongly. To the best of the authors' knowledge, three solutions were proposed so far [2, 3, 6]. The solution in [2], called modified water-filling (MIW), has been shown to outperform the other two solutions and will be used for benchmarking in this paper.

3. NOVEL SOLUTION

This section contains the derivation of the proposed algorithm for the solution of the asynchronous DMT DSM problem. For the sake of simplicity, we will focus on a two user case. Extensions to the general V user case are straightforward.

We will apply an iterative approach. Instead of solving (5) directly, we will optimize the transmit spectra of each user individually when all other users have their transmit spectra fixed. For the two user case, we will compute $\mathbf{p}_1$ while keeping $\mathbf{p}_2$ fixed, then $\mathbf{p}_2$ while keeping $\mathbf{p}_1$ fixed and so on. The problem to be solved by one given user, say 2, is

$$\begin{aligned}\hat{\mathbf{p}}_2 = \arg\max_{\mathbf{p}_2} &\sum_k w_2 b_2^k + \sum_k w_1 b_1^k \\ \text{subject to}\quad & P_2^{\mathrm{tot}} \leq P_2^{\max} \\ & R_2 \geq R_2^{\mathrm{tar}}.\end{aligned} \tag{6}$$

Moreover, consider the set

$$\mathcal{P} = \left\{\mathbf{p}_2 | \sum_k p^k \leq P_2^{\max} \quad \text{and} \quad R_2 \geq R_2^{\mathrm{tar}}\right\}. \tag{7}$$

For the sake of an easy exposition, in the following discussion we will avoid the explicit mention of the constraints in (6). Instead, we will write that the optimization problem is only valid on the set $\mathcal{P}$.

The definition of crosstalk damage ratio (CDR) [5] is given by

$$\mathrm{CDR}_v^k = 1 - \frac{b_v^k}{b_v^k(\sigma)}. \tag{8}$$

Here b_v^k is given by (1) and $b_v^k(\sigma)$ is similar, but it only takes into account the background noise—no crosstalk is considered, i.e. $b_v^k(\sigma) = \log\left(1 + p_v^k(\sigma_v^k)^{-1}\right)$. The CDR is a measure of the impact of crosstalk damage on bit loading. It ranges in a continuum from zero to 1: The closer it is to zero the smaller the damage inflicted by crosstalk; and the closer it is to 1 the larger the damage. We will also need to introduce a slight variant of (8),

$$\mathrm{CDR}_v^{j,k} = 1 - \frac{b_v^{j,k}}{b_v^k(\sigma)}. \tag{9}$$

Here, $b_v^k(\sigma)$ is the same as before and $b_v^{j,k}$ is the bit loading when considering the specific damage from all the interferers on tone j to the victim v on tone k. To write $b_v^{j,k}$ in the same way as (1), we only need to substitute XT_v^k by $\mathrm{xt}_v^{j,k}$, i.e. $b_v^{j,k} = \log\left(1 + p_v^k(\sigma_v^k + \mathrm{xt}_v^{j,k})^{-1}\right)$. We define $\mathrm{xt}_v^{j,k} = \sum_{i\neq v}\mathrm{xt}_{i,v}^{j,k}$, i.e. it represents crosstalk from all interferers on tone j to the victim v on tone k.

Define the set $\mathcal{K}_v$ as the set of active tones of user v, i.e. $\mathcal{K}_v = \{k \mid p_v^k \neq 0\}$. We will use this set for summations of quantities of user 1. Using (8), we can write (6) as

$$\hat{\mathbf{p}}_2 = \arg\max_{\mathbf{p_2}\in\mathcal{P}} \sum_k w_2 b_2^k - w_1\sum_{k\in\mathcal{K}_1}\left(\mathrm{CDR}_1^k b_1^k(\sigma) - b_1^k(\sigma)\right). \tag{10}$$

Since the term $b_1^k(\sigma)$ does not depend on the vector $\mathbf{p}$, it can be removed. Observe in (10) that, because of the ICI effect, the problem is highly coupled in frequency. Hence, we cannot use the dual decomposition approach of [1] to decouple the problem and solve it for every tone separately.

Approximation: We will assume that $p_v^k + \sigma_v^k \gg \mathrm{xt}_v^{j,k}$, so that we can write

$$p_v^k + \sigma_v^k \approx p_v^k + \sigma_v^k + \mathrm{xt}_v^{j,k} \; \forall v, j, k. \tag{11}$$

This should be only valid when p_v^k is nonzero, i.e. only for $k \in \mathcal{K}_v$.

We write the approximation for the general V user case, not for the two user case we are considering. The approximation can be said to be mild, and it can be verified to hold true for the typical DSM solution at convergence. Notice that the crosstalk considered originates from only one interferer tone. Hence we use the lowercase crosstalk, not the uppercase one.

Now consider the sum in j of all $\mathrm{CDR}_1^{j,k}$. We can write this sum as

$$\begin{aligned}
&\sum_j \mathrm{CDR}_1^{j,k} = \mathrm{CDR}_1^{1,k} + \cdots + \mathrm{CDR}_1^{K,k} \\
&\approx \frac{1}{b_1^k(\sigma)} \left[\log\left(\frac{\mathrm{xt}_1^{1,k} + \sigma_1^k}{\sigma_1^k} \right) + \cdots + \log\left(\frac{\mathrm{xt}_1^{K,k} + \sigma_1^k}{\sigma_1^k} \right) \right] \\
&= \frac{1}{b_1^k(\sigma)} \log\left(\frac{\prod_j (\mathrm{xt}_1^{j,k} + \sigma_1^k)}{(\sigma_1^k)^K} \right) \\
&= \frac{1}{b_1^k(\sigma)} \log\left(\frac{(\sigma_1^k)^K + (\sigma_1^k)^{K-1} \sum_j \mathrm{xt}_1^{j,k}}{(\sigma_1^k)^K} + \epsilon_1^k \right) \\
&= \frac{1}{b_1^k(\sigma)} \log\left(\frac{\sigma_1^k + \mathrm{XT}_1^k}{\sigma_1^k} + \epsilon_1^k \right), \forall k \in \mathcal{K}_1
\end{aligned} \tag{12}$$

Here, we use the approximation (11) from the first to the second line. Note that (12) is only valid for the tones on the set $\mathcal{K}_1$. If $k \notin \mathcal{K}_1$, $\mathrm{CDR}_1^{j,k}$ should be zero for all j, because, since no power is loaded on tone k, no crosstalk damage can be caused. In (12),

$$\epsilon_1^k = (\sigma_1^k)^{-2} \mathrm{XXT}_1^k + (\sigma_1^k)^{-3} \mathrm{XXXT}_1^k + \cdots, \tag{13}$$

where

$$\mathrm{XXT}_1^k = \sum_{j=1}^{K} \sum_{q=j+1}^{K} \mathrm{xt}_1^{j,k} \mathrm{xt}_1^{q,k} \tag{14}$$

$$\mathrm{XXXT}_1^k = \sum_{j=1}^{K} \sum_{q=j+1}^{K} \sum_{m=q+1}^{K} \mathrm{xt}_1^{j,k} \mathrm{xt}_1^{q,k} \mathrm{xt}_1^{m,k}, \tag{15}$$

and likewise for higher orders, i.e. XXXXT_1^k and etc.

Since ϵ_1^k contains the cross-multiplications of the interference terms, it is quite small. We will ignore this term on the following development. We can thus write (12) as

$$\sum_j \mathrm{CDR}_1^{j,k} \approx \mathrm{CDR}_1^k, \tag{16}$$

i.e. CDR is approximately "summable" [5]. Taking this into account, we can re-write (10) as

$$\hat{\mathbf{p}}_2 = \arg\max_{\mathbf{p}_2 \in \mathcal{P}} \sum_k w_2 b_2^k - \sum_{k \in \mathcal{K}_1} \sum_j w_1 \mathrm{CDR}_1^{j,k} b_1^k(\sigma). \tag{17}$$

and, by inverting the order of the summation for the term with $\mathrm{CDR}_1^{j,k}$, we get

$$\hat{\mathbf{p}}_2 = \arg\max_{\mathbf{p}_2 \in \mathcal{P}} \sum_k w_2 b_2^k - \sum_j \sum_{k \in \mathcal{K}_1} w_1 \mathrm{CDR}_1^{j,k} b_1^k(\sigma). \tag{18}$$

Now, we choose all terms that depend on the power loaded on one tone, say q. This entails getting $k = q$ from the first summation in (18) and $j = q$ from the second summation in (18). We do this and also use (9) to obtain

$$\hat{p}_2^q = \arg\max_{p_2^q} w_2 b_2^q + \sum_{k \in \mathcal{K}_1} w_1 b_1^{q,k}. \tag{19}$$

Notice that, taking into account approximation (11) and ignoring ϵ_1^k in (12), we have transformed (10), which depended on the vector $\mathbf{p}$, to (19), an equivalent problem which is decoupled in frequency. Eq. (19) should be solved for every tone separately.

With (19) in hands, we can now move towards a solution. We now write the Lagrangean dual of (19), take the derivative in relation to p_i^q and then solve for p_i^q [4, 2]. After doing this, we obtain

$$p_i^q = \frac{w_i}{\lambda_i + t_i^q} - (\sigma_i^q + \mathrm{XT}_i^q), \tag{20}$$

where

$$t_i^q = \sum_{v \neq i} w_v \left[\sum_{k \in \mathcal{K}_v} \alpha_{i,v}^{q,k} \gamma_{i,v}^{q,k} \left(\frac{1}{\mathrm{int}_{i,v}^{q,k}} - \frac{1}{\mathrm{rec}_{i,v}^{q,k}} \right) \right] \tag{21}$$

$$\mathrm{int}_{i,v}^{q,k} = \sigma_v^k + \mathrm{xt}_{i,v}^{q,k} \tag{22}$$

$$\mathrm{rec}_{i,v}^{q,k} = \mathrm{int}_{i,v}^{q,k} + p_v^k. \tag{23}$$

The power loading formula is a type of waterfilling formula. The variable t_i^q is a per-tone penalty that distorts the waterfilling level so that damage to other users is considered. This variable should be large for a tone q if there is potential for large crosstalk damage to other users by loading power on tone q—accordingly, [3] calls a similar power loading strategy frequency selective waterfilling and [2] calls t_i^q a taxation term. Eq. (20) and (21) should be applied iteratively until convergence.

In the next section we shall compare the performance of the proposed algorithm with that of the MIW algorithm. We shall see that this comparison will be favorable to the proposed algorithm not only in terms of performance in data rate, but also in the total consumed power to reach a certain point in the rate region.

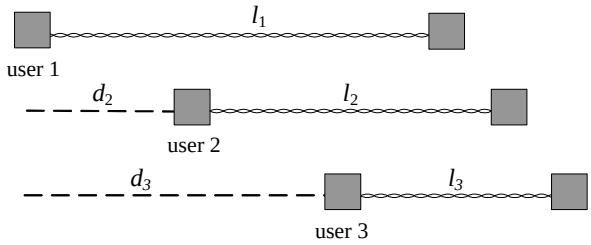

Fig. 1. Scenario for the experiment.

4. EXPERIMENTS

All simulations in this section consider downstream ADSL. Cables of 0.5 mm cables (AWG 24 cable) are used and a SNR gap of 12.8 dB is considered. The values for Δ_f and f_s are set to 4.3125 kHz and 4 kHz, respectively. Modems have at their disposal a maximum power of 20.4 dBm. For each line, noise model ANSI A is adopted. The scenario of interest is depicted in Fig. 1. This kind of near-far scenario is one of the most important testing grounds for DSM solutions. Referring to the figure, we set $l_1 = 5$ km, $l_2 = 4$ km, $d_2 = 2$ km, $l_3 = 3$ km and $d_3 = 4$ km. Hence, in this situation users 3 and 2 should protect user 1 from excessive crosstalk. In Fig. 2 we plot the rate regions for the MIW algorithm and for the proposed algorithm. In all points of the plots, the target rate for user 2 is 2 Mbps. It can be seen that the proposed solution does better, but that is not the whole picture. For the asynchronous problem, it is common for modems not to use full power in most points of the rate region. Because of the ICI effect, the rate maximization problem of (5) involves some degree of power minimization too. In Fig. 3, we plot how much power was used for user 3 to achieve each point of the rate region in Fig. 2. The proposed algorithm always loads less power than the MIW algorithm. Hence, we conclude that not only the proposed algorithm reaches better rates than the MIW, but also that it needs a smaller amount of total power to do so.

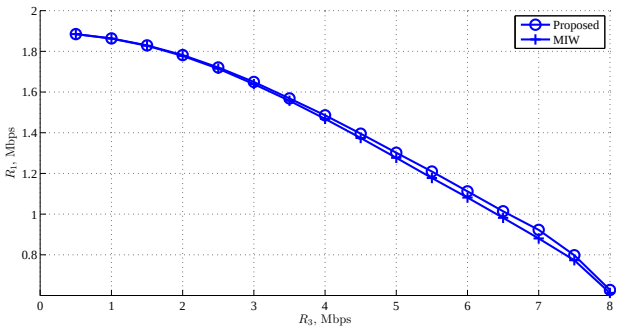

Fig. 2. Rate regions.

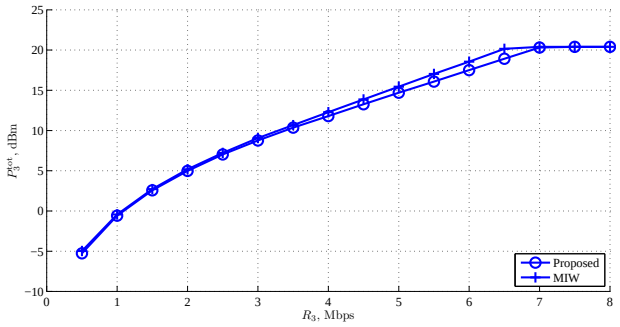

Fig. 3. Plot of rate vs. power for user 3.

5. FINAL REMARKS

We have derived a new solution for the asynchronous DMT DSM problem. Our proposal is mainly based on the approximate summability of the CDR. In the end, we have reached a power loading formula that is a type of waterfilling with an extra per-tone penalty. It was shown through numerical results that, for a simple scenario, the proposed algorithm achieves slightly higher rates and loads less total power than the MIW, presumably the most effective algorithm proposed to date.

6. REFERENCES

[1] R. Cendrillon, W. Yu, M. Moonen, J. Verlinden, and T. Bostoen, "Optimal multiuser spectrum balancing for digital subscriber lines," *IEEE Trans. Commun.*, vol. 54, no. 5, pp. 922–933, 2006.

[2] W. Yu, "Multiuser water-filling in the presence of crosstalk," in *Inf. Theory and Appl. Workshop*, La Jolla, USA, 2007.

[3] R. Cendrillon, J. Huang, M. Chiang, and M. Moonen, "Autonomous spectrum balancing for digital subscriber lines," *IEEE Trans. Signal Process.*, vol. 55, no. 8, pp. 4241–4257, 2007.

[4] P. Tsiaflakis, M. Diehl, and M. Moonen, "Distributed spectrum management algorithms for multiuser DSL networks," *IEEE Trans. Signal Process.*, vol. 56, no. 10, pp. 4825–4843, 2008.

[5] R. B. Moraes, B. Dortschy, A. Klautau, and J. R. i Riu, "Semiblind spectrum balancing for DSL," *IEEE Trans. Signal Process.*, vol. 58, no. 7, pp. 3717–3727, 2010.

[6] V. M. K. Chan and W. Yu, "Multiuser spectrum optimization for discrete multitone systems with asynchronous crosstalk," *IEEE Trans. Signal Process.*, vol. 55, no. 11, pp. 5425–5435, 2007.