KU Leuven

Departement Elektrotechniek ESAT-SISTA/TR 12-161

Reduced Complexity Dynamic Spectrum Management Based on a Polar Coordinates Formulation [1]

Rodrigo B. Moraes, Paschalis Tsiaflakis and Marc Moonen[2]

2013
IEEE International Conference on Communications (ICC)

[1]This report is available by anonymous ftp from *ftp.esat.kuleuven.ac.be* in the directory *pub/sista/rmoraes/reports/12-161.pdf*

[2]K.U.Leuven, Dept. of Electrical Engineering (ESAT), Research group SISTA, Kastelpark Arenberg 10, 3001 Leuven, Belgium, Tel. 32/16/32 17 09, Fax 32/16/32 19 70, WWW: *http://www.esat.kuleuven.ac.be/sista*. E-mail: *rodrigo.moraes@esat.kuleuven.ac.be*. This research work was carried out at the ESAT Laboratory of the KU Leuven, in the frame of the KU Leuven Research Council: PFV/10/002 (OPTEC); Bilateral Scientific Cooperation between Tsinghua University & KU Leuven 2012-2014; FWO project G091213N 'Cross-layer optimization with realtime adaptive dynamic spectrum management for fourth generation broadband access networks'; Belgian Programme on Interuniversity Attraction Poles initiated by the Belgian Federal Science Policy Office: IUAP 'Belgian network on Stochastic modeling, analysis, design and optimization of communication systems' (BESTCOM) 2012-2017; and Concerted Research Action GOA-MaNet. P. Tsiaflakis is a postdoctoral fellow funded by the Research Foundation-Flanders (FWO).

Reduced Complexity Dynamic Spectrum Management Based on a Polar Coordinates Formulation

Rodrigo B. Moraes, Paschalis Tsiaflakis and Marc Moonen
KU Leuven, Dept. of Electrical Engineering - ESAT-SISTA
iMinds - Future Health Department
Leuven, Belgium
{rodrigo.moraes, paschalis.tsiaflakis, marc.moonen}@esat.kuleuven.be

***Abstract*—This paper deals with the problem of power allocation in a multi-user, multicarrier communication system. This problem arises, for example, in wireless OFDM or digital subscriber line networks. Users transmitting concurrently on the same sub-carriers cause interference to each other, and this interference is a serious limitation for system performance. We consider the problem of maximizing the weighted sum of users' data rates. The optimization variables are vectors containing the powers for all users on each tone. The problem is intrinsically difficult due to the non-concavity of the utility function. We propose to change the (cartesian) power vector by its polar coordinates vector equivalent. The main contribution of this paper is to show that at least for one dimension of the polar coordinates vector, the radius, the problem is concave and thus easy to solve. We develop an algorithm based in such a polar coordinates formulation to exploit such concavity. It is demonstrated that the algorithm we propose saves considerably on computational cost compared to previous algorithms.**

I. Introduction

Resource allocation is of fundamental importance in a high performance multi-user communication system. Multi-user systems that are limited by interference have as basic characteristic the sharing of the communication medium by different and independent users. It is the job of the system designer to use the available dimensions, be it in the domains of frequency, power, space, time, waveform or code, to counter the deleterious effects of interference.

This paper focuses on the dimensions of power and frequency. We focus on a multi-user, multicarrier communication system where power should be judiciously allocated for every user and every sub-carrier (i.e. tone) so that the interference is mitigated and the system utility is maximized. This problem arises in both wireless and wireline networks. For the latter, the continued research activities to find efficient and high performance solutions to this problem is referred to as dynamic spectrum management (DSM), and its focus is usually on digital subscriber line (DSL) networks.

For DSM, most often the utility function to be optimized takes the form of the maximization of the weighted sum of the users' data rates subject to per-user power constraints. The optimization variables are vectors containing the power allocation for all users on each tone. The problem is challenging because the utility function is not concave, which means that finding an optimal solution is a difficult endeavor. The focus in the recent past has been mostly on high quality and efficient algorithms. An optimal solution to this problem still eludes researchers.

In this paper, we focus on this problem from a different perspective in comparison to all previous work. In our approach, the (cartesian) power vector is replaced by its polar coordinates vector equivalent. For example, for a case with 2 users, instead of using a power vector $\mathbf{p} = [p_1 \quad p_2]^{\mathrm{T}}$, we re-write the problem as a function of a radius and an angle. The fundamental advantage of this formulation is that, for one of the dimensions, the radius, the utility function is always concave, which means that it can be solved quickly and unambiguously. We propose an algorithm that exploits this characteristic. Experimental results show a considerable saving in computational cost in comparison to similar previously proposed algorithms.

In this paper, we use lower-case boldface letters to denote vectors, upper-case boldface letters for matrices and calligraphic letters for sets (for example, $\mathbf{a}$, $\mathbf{A}$ and $\mathcal{A}$). We use $(\cdot)^*$ as the complex conjugate, $\left\|\cdot\right\|$ as the ℓ^2-norm and $\mathrm{E}\left[\cdot\right]$ as expectation.

This paper is organized as follows. Section II presents the system model and the problem of interest. Section III describes the change of variables from cartesian to polar coordinates and Section IV presents the resulting algorithm. Simulation results are presented in Section V and final conclusions are drawn in Section VI.

II. System Model and Previous Work

Consider an N user DMT system with K Δ_f-spaced tones. We define the set of users as $\mathcal{N} = \{1,\ 2,\ \ldots,\ N\}$ and the set of tones as $\mathcal{K} = \{1,\ 2,\ \ldots,\ K\}$. Let $\mathbf{P} = \{p_n^k\} \in \mathbb{R}^{K\times N}$ be a matrix in which p_n^k is the transmit power of user n on tone k. We also define $\mathbf{p}^k \in \mathbb{R}^N$ as the vector containing the powers of all users on tone k, i.e. $\mathbf{p}^k = \begin{bmatrix} p_1^k & p_2^k & \ldots & p_N^k \end{bmatrix}^{\mathrm{T}}$. Let $h_{n,i}^k$ be the channel gain between the transmitter of user i and receiver of user n at tone k. The received signal for user

n on tone k is given by

$$y_n^k = h_{n,n}x_n^k + \sum_{\substack{j\in\mathcal{N}\\ j\neq n}} h_{n,j}x_j^k + z_n^k. \tag{1}$$

Here, x_n^k and y_n^k are respectively the transmitted and received symbols for user n on tone k and z_n^k represents complex zero mean Gaussian noise. In (1), we have $p_n^k = \mathrm{E}\left[x_n^k(x_n^k)^*\right]$ and we define $\tilde{\sigma}_n^k = \mathrm{E}\left[z_n^k(z_n^k)^*\right]$ as the Gaussian noise power.

In this paper, we consider all interference to be Gaussian noise—an approximation that becomes more accurate as the number of users increases. The data rate for user n on tone k is given by

$$b_n^k = \log_2\left(1 + \frac{p_n^k}{\sigma_n^k + \sum_{\substack{j\in\mathcal{N}\\ j\neq n}} \alpha_{n,j}^k p_j^k}\right), \tag{2}$$

where $\alpha_{n,j}^k = \Gamma|h_{n,j}^k|^2|h_{n,n}^k|^{-2}$ is the normalized interference channel gain from user i to user n on tone k and $\sigma_n^k = \Gamma\tilde{\sigma}_n^k|h_{n,n}^k|^{-2}$ is the normalized Gaussian noise power. Also Γ accounts for the Shannon gap to capacity, the noise margin and the coding gain. The data rate of user n is given by $R_n = \sum_{k\in\mathcal{K}} b_n^k$.

The problem we focus on is that of maximizing the weighted rate sum of the participating users in the network while respecting their per-user power constraints. Mathematically, we write

$$\begin{aligned} \max_{\mathbf{P}} \quad & \sum_{n\in\mathcal{N}}\sum_{k\in\mathcal{K}} u_n b_n^k \\ \text{subject to} \quad & \sum_{k\in\mathcal{K}} p_n^k \leq P_n^{\max} \;\; n\in\mathcal{N} \\ & p_n^k \geq 0 \;\; n\in\mathcal{N},\;\; k\in\mathcal{K} \end{aligned} \tag{3}$$

Here, $P_n^{\max}$ is the power budget for user n and u_n are weights assigned to each user. We call (3) the *DSM problem*. It can be easily shown that this problem is not concave.

As mentioned in the introduction, several papers have attempted to solve the DSM problem. Most of these have focused on efficient, good quality and low complexity algorithms, e.g. [1]–[7]. These algorithms usually depend on some kind of convex approximation of the non-concave utility function in (3). As a consequence, they can at best reach local optimal points of the DSM problem.

The OSB algorithm [8] is of particular interest to this paper. In [8], the DSM problem is solved through the definition of the Lagrangian dual function, which is then solved on a per-tone basis. This formulation profits from the fact that the duality gap of the DSM problem vanishes as the number of tones increases [9], [10]. For each tone, an optimal power vector $\mathbf{p}^k$ is found with an exhaustive grid search. Experimental results present strong evidence that the OSB provides the best results so far. There are, however two disadvantages to it. First, because of the per-tone exhaustive search, the computational cost of OSB grows exponentially with the number of users. It has been reported that this algorithm becomes prohibitively complex already for a case of 3 users. There are some alternatives

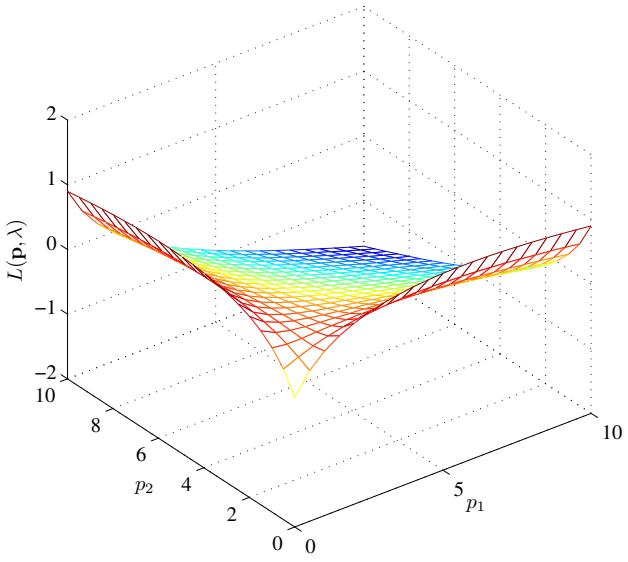

Fig. 1. Illustration of non-concavity of $L(\mathbf{p},\boldsymbol{\lambda})$ for a two user case. The index k is not shown for simplicity. We choose $\alpha_{1,2} = \alpha_{2,1} = 1$, $\sigma_1 = \sigma_2 = 1$, $\lambda_1 = \lambda_2 = 0.15$.

to the exhaustive search in $\mathbf{p}^k$. For example, in [11], [12] branch and bound algorithms substitute it with large savings in complexity. However, even these techniques can have worst case complexity that grows exponentially with N. The second disadvantage is that the OSB is not guaranteed to deliver the optimal solution. In [13], the duality gap of the DSM problem is estimated to be proportional to $1/\sqrt{K}$. For the DSL case, where the number of tones varies from a couple of hundreds to a couple of thousands, the estimate of [13] seems to imply that the duality gap could still be considerable. Therefore, the OSB cannot strictly claim optimality.

III. Polar coordinates formulation

As in [8], we write the Lagrangian of (3) as

$$L(\mathbf{P},\boldsymbol{\lambda}) = \sum_{n\in\mathcal{N}}\sum_{k\in\mathcal{K}} u_n b_n^k - \sum_{n\in\mathcal{N}} \lambda_n\Big(\sum_{k\in\mathcal{K}} p_n^k - P_n^{\max}\Big). \tag{4}$$

Here $\boldsymbol{\lambda} = \begin{bmatrix}\lambda_1 & \lambda_2 & \dots & \lambda_N\end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^N$ is the vector of Lagrange multipliers associated with the power budgets. We can approximately (because the duality gap can be considerable) solve (3) by solving the unconstrained problem

$$\max_{\mathbf{P}} \quad L(\mathbf{P},\boldsymbol{\lambda}), \tag{5}$$

where appropriate values for $\boldsymbol{\lambda}$ should be searched for so that the power budgets are respected.

By re-writing (4) as

$$L(\mathbf{P},\boldsymbol{\lambda}) = \sum_{k\in\mathcal{K}} L^k(\mathbf{p}^k,\boldsymbol{\lambda}) + \sum_{n\in\mathcal{N}} \lambda_n P_n^{\max},$$

where $L^k(\mathbf{p}^k,\boldsymbol{\lambda}) = \sum_{n\in\mathcal{N}} u_n b_n^k - \sum_{n\in\mathcal{N}} \lambda_n p_n^k$ we can see that the per-tone maximization of $L^k(\mathbf{p}^k,\boldsymbol{\lambda})$ also leads to the maximization of (5). We can thus focus on the per-tone maximization of $L^k(\mathbf{p}^k,\boldsymbol{\lambda})$.

As it is widely known, $L^k(\mathbf{p}^k, \boldsymbol{\lambda})$ is not concave in $\mathbf{p}^k$ for all situations. When the interference coefficients $\alpha^k_{n,j}$ are sufficiently small, $L^k(\mathbf{p}^k, \boldsymbol{\lambda})$ can be concave, but this is certainly not always the case. We illustrate this in Fig. 1 for a case with two users. The optimal $\mathbf{p}^k$ for Fig. 1 is either $\mathbf{p}^k = \begin{bmatrix} 0 & 5.66 \end{bmatrix}^{\mathrm{T}}$ or $\mathbf{p}^k = \begin{bmatrix} 5.66 & 0 \end{bmatrix}^{\mathrm{T}}$. These values can be found by an exhaustive search in a two dimensional grid. Throughout this paper we often come back to this 2 user example.

Now consider the representation of $\mathbf{p}^k$ in polar coordinates. We write this vector as

$$\mathbf{p}^k = \rho^k \mathbf{d}^k, \quad \rho^k \geq 0, \ \left\|\mathbf{d}^k\right\| = 1, \tag{6}$$

where ρ^k is the radius and the vector $\mathbf{d}^k = \begin{bmatrix} d^k_{[1]} & \dots & d^k_{[N]} \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^N$, with $d^k_{[n]} \geq 0 \ \forall n$, points to a direction. Here we make two remarks. First, notice that ρ^k and the $d^k_{[n]}$'s are not exactly associated with users. If we for example change the value of ρ^k, all users' powers (in the cartesian vector) change. That is why we do not use a subscript for ρ^k and why we add the brackets for the subscripts in $\mathbf{d}^k$. In $d^k_{[n]}$, the bracketed subscripts denote directions, not users. Second, although $\mathbf{d}^k$ is of size N, there are only $N-1$ free variables in this vector. That is a direct consequence of the norm constraint in (6). For the two user case exemplified in Fig. 1, we would have $\mathbf{d}^k = \begin{bmatrix} \cos\theta^k & \sin\theta^k \end{bmatrix}^{\mathrm{T}}$, with $0 \leq \theta^k \leq \pi/2$. For a case with three users we would have

$$\mathbf{d}^k = \begin{bmatrix} \cos\theta^k \cos\phi^k \\ \cos\theta^k \sin\phi^k \\ \sin\theta^k \end{bmatrix},$$

with $\theta^k, \ \phi^k \in [0, \pi/2]$.

We now re-write (2) and $L^k(\mathbf{p}^k, \boldsymbol{\lambda})$ in polar coordinates respectively as

$$b^k_n = \log_2\left(1 + \frac{\rho^k d^k_{[n]}}{\sigma^k_n + \sum_{\substack{j\in\mathcal{N} \\ j\neq n}} \alpha^k_{n,j} \rho^k d^k_{[j]}}\right)$$

$$L^k(\rho^k, \mathbf{d}^k, \boldsymbol{\lambda}) = \sum_{n\in\mathcal{N}} u_n b^k_n - \sum_{n\in\mathcal{N}} \lambda_n \rho^k d^k_{[n]} \tag{7}$$

The fundamental advantage of the formulation in (7) is that, for a fixed direction $\mathbf{d}^k$ (with $\left\|\mathbf{d}^k\right\| = 1$ and $d^k_{[n]} \geq 0$), $L^k(\rho^k, \mathbf{d}^k, \boldsymbol{\lambda})$ is concave in ρ^k. This is illustrated in Fig. 2. Again for a two user case, we plot $L^k(\rho^k, \boldsymbol{\lambda})$ for fixed directions, which are denoted by the angle θ.

To show concavity more rigorously and for the general N user case, we calculate the second derivative of $L^k(\rho^k, \mathbf{d}^k, \boldsymbol{\lambda})$ in ρ^k as

$$\frac{\partial^2 L^k(\rho^k, \mathbf{d}^k, \boldsymbol{\lambda})}{\partial(\rho^k)^2} = -\sum_{n\in\mathcal{N}} \frac{u_n d^k_{[n]} \sigma^k_n \left(2(I^k_n)^2 \rho^k + 2I^k_n\left(d^k_{[n]}\rho^k + \sigma^k_n\right) + d^k_{[n]}\sigma^k_n\right)}{\log(2)\left(I^k_n \rho^k + \sigma^k_n\right)^2 \left(\rho^k(I^k_n + d^k_{[n]}) + \sigma^k_n\right)^2}. \tag{8}$$

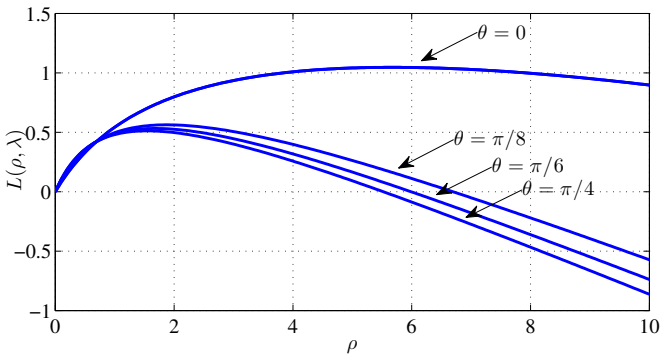

Fig. 2. Illustration of concavity of $L(\rho, \boldsymbol{\lambda})$ for fixed $\mathbf{d}$ for a two user case. The index k is not shown for simplicity. We represent $\mathbf{d}$ as the angle θ. We choose $\alpha_{1,2} = \alpha_{2,1} = 1$, $\sigma_1 = \sigma_2 = 1$, $\lambda_1 = \lambda_2 = 0.15$.

Here

$$I^k_n = \sum_{\substack{j\in\mathcal{N} \\ j\neq n}} \alpha^k_{n,j} d^k_{[j]}$$

and $\log(\cdot)$ denotes the natural logarithm. Clearly, since $d^k_{[n]} \geq 0 \ \forall n$, all terms of the quotient in (8) are non-negative. Hence each term of the summation in n is non-negative. Because $\left\|\mathbf{d}^k\right\| = 1$, at least one term in the sum is strictly positive. Hence the sum is strictly positive. Because of the minus sign, the second derivative is, for $\rho^k \geq 0$, always negative, from which it follows that $L^k(\rho^k \mathbf{d}^k, \boldsymbol{\lambda})$ is strictly concave in ρ^k for fixed $\mathbf{d}^k$.

As a consequence of such concavity, we can save a lot in computational cost. For the remaining of this section we consider a 2 user case, as in Fig 1. For such a situation, we mentioned that the OSB performs an exhaustive grid search in 2 dimensions to find the optimal point(s). If we sample each of the cartesian axes with Q points, i.e. $\{0, \ {}^{P^{\max}_n}\!/_Q, \ {}^{2P^{\max}_n}\!/_Q, \ \dots, \ P^{\max}_n\}$, the exhaustive search needs to evaluate $L^k(\mathbf{p}^k, \boldsymbol{\lambda})$ in a total of Q^2 points and then pick the maximum.

This is in contrast with the formulation with the polar coordinates, where we would need only a one dimensional search. With this formulation, the exhaustive search is done only in θ^k. We sample this axis with Q points, i.e. $\{0, \ {}^{\pi}\!/_{2Q}, \ {}^{\pi}\!/_{Q}, \ \dots, \ {}^{\pi}\!/_{2}\}$. For each fixed θ^k, $L^k(\rho^k, \mathbf{d}^k, \boldsymbol{\lambda})$ is concave in ρ^k and hence its solution is easy and unambiguous. For fixed θ^k, we solve for ρ^k with a simple procedure, such as the Newton method. Hence, of the two dimensions of the problem, i.e. θ^k and ρ^k, one of them is solved practically for free.

If we consider the example given in Fig. 1, the search in θ^k would return two equivalent optimal values, 0 and ${}^{\pi}\!/_{2}$. For both of these directions, the optimal ρ^k equals 5.66. With (6) we can transform back to cartesian coordinates.

IV. Algorithm and Complexity

We can exploit the radial concavity to solve (3) in an algorithm, which we call *OSB with polar coordinates*. We

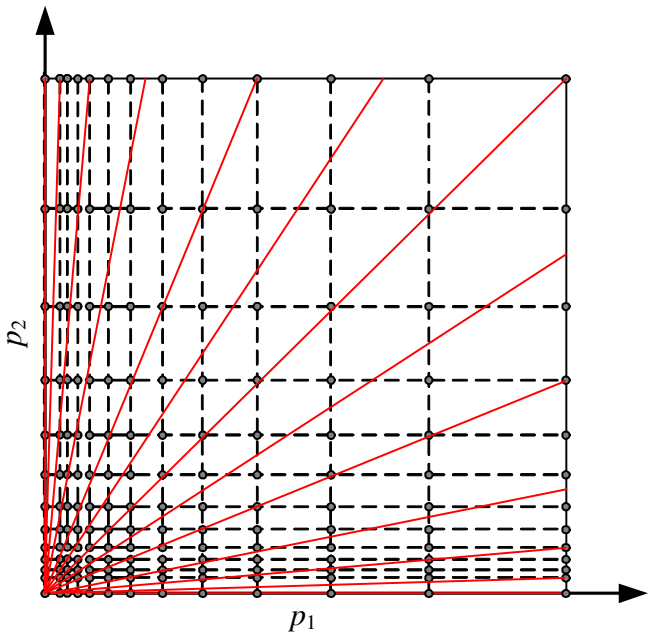

Fig. 3. Example with two users of the sampling for the cartesian coordinates domain and in the polar coordinates domain. We use $Q = 13$. For the cartesian coordinates, there are $13^2 = 169$ points. These points are shown as the dots that intersect the dashed lines. For the polar coordinates, the sampling of the angles are shown as the solid red lines. There are 13 lines.

describe the algorithm for the general N user case.

The algorithm contains one main loop, wherein the Lagrange multipliers are adjusted. Before going into this loop, we define $\mathcal{D}$ as sampled $\mathbf{d}^k$-space, with $\left\|\mathbf{d}^k\right\| = 1$. Because $\mathbf{d}^k$ has only $N-1$ free variables, there are in total Q^{N-1} points in this set. Because of the logarithmic dependence between power and data rate, it is better to sample this space nonuniformly. In the original OSB the cartesian axes are also sampled nonuniformly. We illustrate how we do such sampling with Fig. 3. The example deals with a case with two users. We depict the sampling of the cartesian axes, each having $Q = 13$ points. This makes a total of 169 points available for the original OSB, shown as the dots on the intersection of the dashed lines. With the OSB with polar coordinates, the sampling of the angle dimension is done also with $Q = 13$ points and it is shown as the solid red lines. Notice that both samplings have more density towards lower values of power. We remark that here the OSB with polar coordinates has an additional advantage: because the angle dimension is continuous, it can use all points on the red solid lines. Unlike the original OSB, we cannot count how many points are available.

Inside the loop, in line 4 in the algorithm, we calculate the optimal radius,, denoted by $\hat{\rho}^k(\mathbf{d}^k)$, for each fixed direction $\mathbf{d}^k \in \mathcal{D}$ and as a function of the fixed Lagrange multiplier $\boldsymbol{\lambda}$. This can be obtained by solving K concave optimization problems. We use the Newton method to accomplish that.

The next step is to calculate $L^k(\hat{\rho}^k(\mathbf{d}^k), \mathbf{d}^k, \boldsymbol{\lambda})$ for every direction $\mathbf{d}^k \in \mathcal{D}$ and pick the maximum, assigned to $\hat{\mathbf{d}}^k$.

Algorithm 1: OSB with polar coordinates

1 Initialize $\boldsymbol{\lambda}$;
2 $\mathcal{D} \leftarrow$ sampled $\mathbf{d}^k$-space, with $\left\|\mathbf{d}\right\| = 1$;
3 **repeat**
4 Obtain $\hat{\rho}^k(\mathbf{d}^k)$, $\mathbf{d}^k \in \mathcal{D},\ k \in \mathcal{K}$;
5 $\hat{\mathbf{d}}^k = \text{argmax}_{\mathbf{d}^k \in \mathcal{D}}\ L^k(\hat{\rho}^k(\mathbf{d}^k), \mathbf{d}^k, \boldsymbol{\lambda})\ k \in \mathcal{K}$;
6 $\mathbf{p}_k = \hat{\rho}^k \hat{\mathbf{d}}^k\ , k \in \mathcal{K}$;
7 $\lambda_n = \max\left[\lambda_n + \epsilon(\sum_{k\in\mathcal{K}} p_n^k - P_n^{\max}), 0\right],\ n \in \mathcal{N}$;
8 **until** *until convergence*

The combination of $\hat{\rho}^k$ and $\hat{\mathbf{d}}^k$ solves the problem for a given $\boldsymbol{\lambda}$. The process should repeat until appropriate Lagrange multipliers are found. In line 7 in the algorithm, we use a sub-gradient method for this search. Other methods are also possible.

For each $\mathbf{d}^k$, solving for ρ^k implies solving K concave optimization problems. The complexity of each concave problem increases quadratically with the number of users. This should be repeated for every direction in $\mathcal{D}$. Since there are Q^{N-1} directions in $\mathcal{D}$, the total computational cost of the OSB with polar coordinates is $O(KN^2Q^{N-1})$. The original OSB has computational cost of $O(KQ^N)$.

We remark again that, for a DSL scenario with hundreds or thousands of tones, we cannot claim global optimality with the proposed algorithm. Just as the case of the original OSB in [8], there is a duality gap that may be considerable. We re-emphasize that, although optimality cannot be certified, experimental results clearly show that the original OSB (or the equivalent but less complex OSB in polar coordinates) have better performance than any other proposed algorithm.

In the next section, we compare complexity of these two methods.

V. Experiments

We perform experiments in an ADSL downstream scenario. We simulate a system with 0.5 mm (24 AWG) cables. Each user has a total power budget of 20.4 dBm. We want to illustrate the gain in computational cost achieved when using the formulation with polar coordinates. We run the OSB from [8] and the OSB with polar coordinates described in this paper for a 2 user case with varying Q's and compare the run time complexity of the two. The scenario is that shown in Fig. 4. We use $Q = \{150,\ 300,\ 450,\ 600\}$ for both algorithms. The larger the Q, the more precise the final power allocation and the power budget are. For downstream ADSL, $K = 224$. We set the users weights $u_n = 1/N$ for all users.

As can be seen in Fig. 5, the OSB with polar coordinates is several times faster than the original OSB. For $Q = 150$, the original OSB takes approximately 5 minutes to converge. The OSB with the polar coordinates takes about 30 seconds. For $Q = 600$, the original OSB needs almost 86 minutes to converge. The OSB with polar coordinates converges in approximately 1 minute. Both algorithms converge to the same solution.

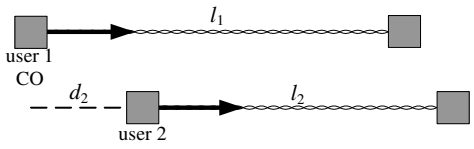

Fig. 4. Downstream ADSL scenario for the simulations. We set $l_1 = 5$ km, $l_2 = 3$ km and $d_2 = 4$ km.

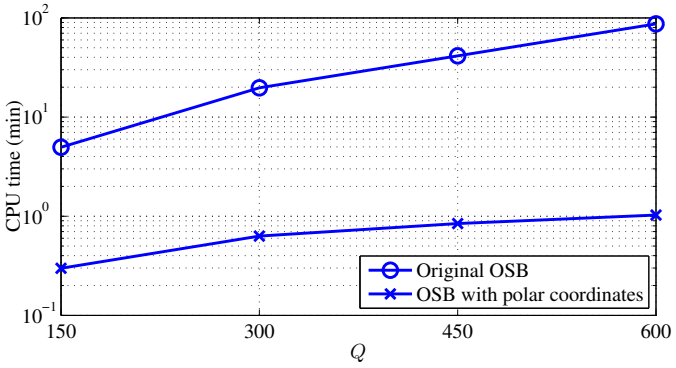

Fig. 5. CPU time for different Q's.

We also run the OSB with polar coordinates for a 3 users case. In this scenario, we add a third user to the scenario in Fig. 4 with $l_3 = 4$ km and $d_3 = 2$ km. With the OSB with polar coordinates and for $Q = 150$, approximately 1.5 hours are necessary for convergence. The original OSB would be extremely complex in this situation. We estimate that it would take around 70 hours (almost 3 days) to converge.

VI. Conclusion

In this paper, we have proposed a new formulation for the DSM problem. This problem occurs both in wireless and wireline networks and it is intrinsically difficult because of the non-concavity of the utility function.

We have shown that, by replacing the cartesian vector of powers by its polar coordinates equivalent, we obtain one dimension, the radius, where the utility function is concave and thus easy to solve. We propose an algorithm based on this property that saves considerably in computational complexity in comparison to the original OSB algorithm.

Acknowledgment

This research work was carried out at the ESAT Laboratory of the KU Leuven, in the frame of the KU Leuven Research Council: PFV/10/002 (OPTEC); Bilateral Scientific Cooperation between Tsinghua University & KU Leuven 2012-2014; FWO project G091213N 'Cross-layer optimization with real-time adaptive dynamic spectrum management for fourth generation broadband access networks'; Belgian Programme on Interuniversity Attraction Poles initiated by the Belgian Federal Science Policy Office: IUAP 'Belgian network on Stochastic modeling, analysis, design and optimization of communication systems' (BESTCOM) 2012-2017; and Concerted Research Action GOA-MaNet. P. Tsiaflakis is a postdoctoral fellow funded by the Research Foundation-Flanders (FWO). The scientific responsibility is assumed by the authors.

References

[1] W. Yu, "Multiuser water-filling in the presence of crosstalk," in *Inf. Theory and Appl. Workshop*, San Diego, USA, 2007.

[2] W. Yu, G. Ginis, and J. M. Cioffi, "Distributed multiuser power control for digital subscriber lines," *IEEE J. Sel. Areas of Commun.*, vol. 20, no. 5, pp. 1105–1115, 2002.

[3] J. Papandriopoulos and J. S. Evans, "SCALE: a low-complexity distributed protocol for spectrum balancing in multiuser DSL networks," *IEEE Trans. Inf. Theory*, vol. 55, no. 8, pp. 3711–3724, 2009.

[4] Q. Shi, M. Razaviyayn, Z.-Q. Luo, and C. He, "An iteratively weighted MMSE approach to distributed sum-utility maximization for a MIMO interfering broadcast channel," *IEEE Trans. Signal Process.*, vol. 59, no. 9, pp. 4331–4340, 2011.

[5] P. Tsiaflakis, M. Diehl, and M. Moonen, "Distributed spectrum management algorithms for multiuser DSL networks," *IEEE Trans. Signal Process.*, vol. 56, no. 10, pp. 4825–4843, 2008.

[6] R. Cendrillon and M. Moonen, "Iterative spectrum management for digital subscriber lines," in *IEEE Int. Conf. Commun.*, Seoul, Korea, 2005.

[7] R. B. Moraes, B. Dortschy, A. Klautau, and J. R. i Riu, "Semiblind spectrum balancing for DSL," *IEEE Trans. Signal Process.*, vol. 58, no. 7, pp. 3717–3727, 2010.

[8] R. Cendrillon, W. Yu, M. Moonen, J. Verlinden, and T. Bostoen, "Optimal multiuser spectrum balancing for digital subscriber lines," *IEEE Trans. Commun.*, vol. 54, no. 5, pp. 922–933, 2006.

[9] Z.-Q. Luo and S. Zhang, "Dynamic spectrum management: Complexity and duality," *IEEE Trans. Signal Process.*, vol. 2, no. 1, pp. 57–73, 2009.

[10] W. Yu and R. Lui, "Dual methods for nonconvex spectrum optimization of multicarrier systems," *IEEE Trans. Commun.*, vol. 54, no. 7, pp. 1310–1322, 2006.

[11] Y. Xu, T. Le-Ngoc, and S. Panigrahi, "Global concave minimization for optimal spectrum balancing in multi-user DSL networks," *IEEE Trans. Signal Process.*, vol. 56, no. 7, pp. 2875 – 2885, 2008.

[12] P. Tsiaflakis, J. Vangorp, M. Moonen, and J. Verdilen, "A low complexity branch and bound approach to optimal spectrum balancing for digital subscriber lines," in *IEEE Global Telecomm. Conf.*, San Francisco, USA, 2006.

[13] Z.-Q. Luo and S. Zhang, "Duality gap estimation and polynomial time approximation for optimal spectrum management," *IEEE Trans. Signal Process.*, vol. 57, no. 7, pp. 2675 – 2689, 2009.