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General Framework and Algorithm for Data Rate Maximization in DSL Networks [1]

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Abstract

In this paper, we treat the combined signal and spectrum coordination problem in digital subscriber line (DSL) networks with linear design for transmitters and receivers. The transmission is modeled as a multitone MIMO system with coordination between sets of users on the transmitter and on the receiver side. We consider both synchronous and asynchronous transmission schemes. For the latter, the transmission of DMT blocks for different users is not aligned in time, which gives rise to inter-carrier interference. Our objective is the maximization of the weighted sum of users' data rates subject to power constraints. Although this problem is well known in the literature, we notice that previous works have always based their designs on strong assumptions about the network infrastructure. In this paper, we propose a general framework and algorithm that apply for any network infrastructure, including any number of users, any number of transceivers, any number of tones, any kind of coordination on both the transmitter and on the receiver sides, and synchronous or asynchronous transmission. We also do not assume any special structure of the channel matrix. Our algorithm is seen to perform very well and with reasonable computational complexity.

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Index Terms

DSL, Crosstalk, Optimization, MIMO.

I. Introduction

Multi-input, multiple-output (MIMO) processing has constituted a paradigm shift in the way communication systems are designed. It has captured the attention of researchers and the telecommunication industry since the 90's, when it was first studied. The technology is a success story, and within a decade it has evolved from a theoretical concept to practical implementation [1].

The initial research efforts in MIMO communications were focused mainly on single-user systems, but the focus quickly evolved to multiuser scenarios [2]. In these scenarios, the extra spatial dimensions serve the purpose of spatial separation of the different users, providing the means for interference mitigation and thus an improved channel utilization. Multiuser MIMO is classically divided into three situations, depending on how much coordination there is on the receiver and on the transmitter sides. A system with full transmitter coordination is called a broadcast channel (BC), whereas a system with full receiver coordination is called a multiple access channel (MAC). When there is no inter-user coordination neither on the transmitter nor on the receiver side, the scenario is referred to as an interference channel (IC).

Most of the research and standardization activities when it comes to MIMO technology have focused on wireless transmission. However, the same paradigm can be applied to any multi-transceiver scenario where there are cross channel gains between all transmitters and all receivers.[1] A digital subscriber line (DSL) binder fits such a description. In DSL, multiple users transmit over closely packed copper pairs. Because of electromagnetic radiation, a signal transmitted in a given pair leaks to the neighboring pairs, a phenomenon referred to as crosstalk. Crosstalk has been traditionally identified as the main source of performance degradation for such systems. However, with MIMO processing, crosstalk can be used as a means to improve performance. After all, crosstalk contains signal energy that can be detected on the other side of the network [3]. It is the processing of such signals that defines whether crosstalk is beneficial or detrimental to performance.

By far the world's favorite means of wireline broadband access, DSL counts more than 400 million subscribers and a market share of more than 70% [4]. Although it is predicted that DSL will eventually

[1]We define a transceiver as a generalization of the concept of DSL line. A transceiver is connected to a physical channel and can be either a direct mode or common mode of a copper pair or one wire of a split wire signaling. In wireless parlance, 'antenna' is a similar concept.

be replaced with optical fiber even for home use, it is expected that DSL will be around as an important market technology for decades. To expand its lifetime as much as possible and to keep competitive, DSL technology has been evolving in two main directions. First, as a result of the expansion of the optical fiber network, copper loops are getting shorter. In the future, it is foreseen that DSL will be responsible for bridging the last couple of hundred meters from the fiber-fed last distribution point to the customers. Accordingly, standardization bodies have been working on new types of DSL better suited for shorter loops. For example, future generations standards include the G.fast, which will go into the market around 2016 and is designed to work on copper lines that are at most a couple of hundred meters long. The second direction of evolution results from the decade-long research activities that aim at amplifying DSL's advantages from a signal processing perspective. This body of work is called dynamic spectrum management (DSM) and its main goal is to deal with crosstalk interference—in the DSL context, DSM that involves MIMO processing is often called *vectoring* or DSM level 3 [5]. MIMO processing in DSL also encompasses the possibility of the utilization of common-mode transmission or of split wire signaling [3], [6]. In the latter, the concept of the twisted pair is given up and transmission is done on each wire separately. While the first direction of evolution is about the physical infrastructure of the access network (i.e. bringing the network hardware closer to the end user), DSM techniques rely on intelligent coordination and processing of signals (i.e. expanding the functionality of the network software). It has been repeatedly shown in the literature that applying DSM leads to formidable gains.

Given a DSL network, depending on the kind of coordination on the transmitter and on the receiver sides, i.e. receiver-only, transmitter-only, etc., a BC, a MAC, an IC or a combination of them can be used as a model. Several papers have focused on the design of the transmission and reception strategies for such scenarios, e.g. [5], [7]–[15]. However, it is observed that these references base their designs on strong assumptions about the network infrastructure. We give four examples of these assumptions.

- First, most previous work considers a synchronous transmission case, i.e. the situation when all users' DMT blocks are aligned (i.e. time synchronized) at the receivers. This is not necessarily the case. Users who are in different physical locations or who belong to different service providers are difficult to synchronize. The result of asynchronous DTM transmission is inter-carrier interference [16]–[19], which complicates the problem significantly.
- Some previous references [5], [7], [8], [12] assume that in DSL coordination is only possible on the system provider side. In other words, there is either a BC scenario for downstream transmission or a MAC scenario for upstream transmission. Most likely this does not always hold. It can be that

a number of DSL lines arrive at the same box on the customer side if the connection serves, for example, a large residential building. This allows for some limited coordination on the customer side as well. Plus, if it can be envisaged that a copper pair that arrives on the customer premises uses split wire signaling, then there is a two transceiver system that can be coordinated on the customer side.

- Another assumption is that every user will use only one line (or one transceiver). However, this is not always the case. There are places where it is not uncommon that there are two DSL lines connected to a user. Pair bonding or common mode transmission can then be used.
- A final common assumption is that that DSL channels have the so-called property of column-wise or row-wise diagonal dominance [7], [8], [12]. However, this only holds true for the case when sources of noise other than crosstalk are spatially white.

To the best of our knowledge, no work up to now has developed a general framework and algorithm that applies to the general scenario. This is the goal of this paper. We develop a general system framework that includes MAC, BC and IC and any combination of them as a special case (we consider only linear transmission schemes). We propose an algorithm similar to the one in [20] that works for all cases, including any number of users, any number of transceivers, any number of tones, any kind of coordination on both the transmitter and on the receiver sides, and synchronous or asynchronous transmission. Through extensive numerical simulations, the algorithm is seen to perform very well and has reasonable computational complexity.

We organize this paper building up complexity as it progresses. Section II deals with the synchronous problem with the focus on different kinds of coordination on the transmitter and receiver sides. In Section II-A we formalize the problem of interest and present the notation. In Section II-B we present our proposed approach. Section III focuses on the asynchronous transmission problem. At this point we present a general algorithm that includes all cited assumptions as a special case. Section IV contains some simulation results and Section V presents final remarks.

We use lower-case boldface letters to denote vectors, upper-case boldface letters for matrices and calligraphic letters for sets (for example, $\mathbf{a}$, $\mathbf{A}$ and $\mathcal{A}$). We use $\mathbf{I}_A$ as the identity matrix of size A, $\mathbf{0}_{A\times B}$ as the $A \times B$ matrix of zeros, $(\cdot)^{\mathrm{T}}$ as the transpose, $(\cdot)^{\mathrm{H}}$ as the Hermitian transpose, $(\cdot)^*$ as the complex conjugate, $\mathrm{E}[\cdot]$ as expectation, $\mathrm{tr}\{\cdot\}$ as trace, $|\cdot|$ as determinant and $\mathrm{diag}\{\mathbf{a}\}$ as the matrix with a vector $\mathbf{a}$ on the main diagonal.

II. SYNCHRONOUS TRANSMISSION

A. System model and problem statement

We consider an N user DSL system with discrete multitone (DMT) modulation with K Δ_f-spaced tones. Here we focus on the situation when transmission for all users is synchronized, which means that interference is dealt with on a per-tone basis. We consider a system where upstream and downstream transmission are separated (with, for example, time or frequency division duplexing), hence the crosstalk we consider is far-end crosstalk (FEXT).[2] We denote the set of users by $\mathcal{N} = \{1, \ldots, N\}$ and the set of tones by $\mathcal{K} = \{1, \ldots, K\}$. We let p_n^k be the transmit power of user n on tone k and we organize these values in the matrix $\mathbf{P} \in \mathbb{R}^{K \times N}$. The nth column of $\mathbf{P}$, denoted by $\mathbf{p}_n = \begin{bmatrix} p_n^1 & \ldots & p_n^K \end{bmatrix}^{\mathrm{T}}$, contains the power allocation of user n in all tones. The kth row of $\mathbf{P}$, $\mathbf{p}^k = \begin{bmatrix} p_1^k & \ldots & p_N^k \end{bmatrix}$, represents the power allocation of all users in tone k. User n has A_n transceivers. Every user belongs to a group both on the transmitter side and on the receiver side. Inside a group, users can apply coordinated MIMO processing. We define each group as a set. For the grouping on the transmitter side, we define $\mathcal{T}_i$, $i = 1, \ldots, I \leq N$. For the grouping on the receiver sides, we similarly define $\mathcal{R}_j$, $j = 1, \ldots, J \leq N$. Here I and J denote the number of groups on the transmitter and on the receiver sides, respectively. A user can only be in a single group both on the transmitter and on the receiver side, i.e. if $n \in \mathcal{T}_i$ then $n \notin \mathcal{T}_j,\ j \neq i$ and if $n \in \mathcal{R}_i$ then $n \notin \mathcal{R}_j,\ j \neq i$. We also define the number of transceivers per group on the transmitter and on the receiver sides respectively as

$$A_{\mathcal{T}_i} = \sum_{n \in \mathcal{T}_i} A_n \tag{1}$$

$$A_{\mathcal{R}_i} = \sum_{n \in \mathcal{R}_i} A_n \tag{2}$$

As already mentioned, throughout this paper we focus on a linear design for both transmitters and receivers and treat interference as noise. All channel gains are considered perfectly known. Taking this into account, we obtain the received signal for group $\mathcal{R}_j$ on tone k as

$$\mathbf{y}_{\mathcal{R}_j}^k = \widetilde{\mathbf{H}}_{n,n}^k \mathbf{T}_n^k \mathbf{x}_n^k + \sum_{\substack{u \in \mathcal{N} \\ u \neq n}} \widetilde{\mathbf{H}}_{n,u}^k \mathbf{T}_u^k \mathbf{x}_u^k + \mathbf{z}_{\mathcal{R}_j}^k,\ n \in \mathcal{R}_j. \tag{3}$$

Here $\mathbf{y}_{\mathcal{R}_j}^k \in \mathbb{C}^{A_{\mathcal{R}_j}}$ is the received signal vector for group $\mathcal{R}_j$ on tone k; $\mathbf{x}_n^k \in \mathbb{C}^{A_n}$ is the transmitted signal vector for user n on tone k; $\mathbf{T}_n^k \in \mathbb{C}^{A_{\mathcal{T}_i} \times A_n}$, where $n \in \mathcal{T}_i$, is the transmit matrix for user n on

[2]The system model can be straightforwardly generalized to a situation where upstream and downstream transmission are jointly optimized, eventually taking near-end crosstalk (NEXT) into account.

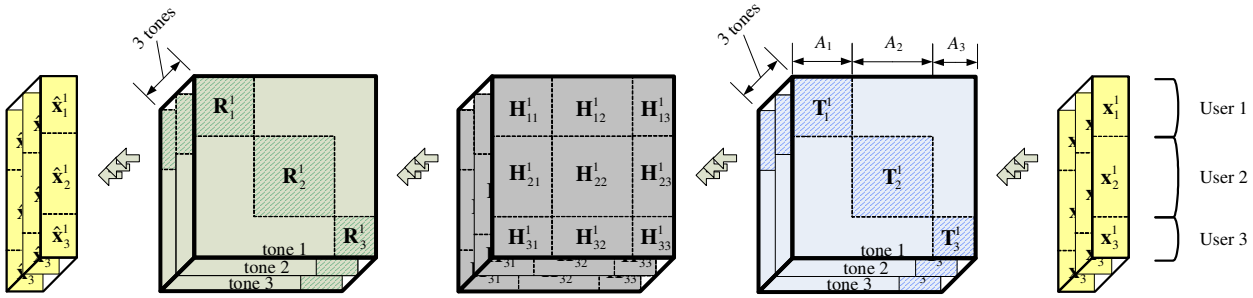

Fig. 1. Illustration of a MIMO IC. For this case, there is only one user on every group in both the transmitter and on the receiver sides. Here, we have $J = I = N = 3$ and $\mathcal{T}_n = \mathcal{R}_n = \{n\}$, $n = 1, \ldots, 3$.

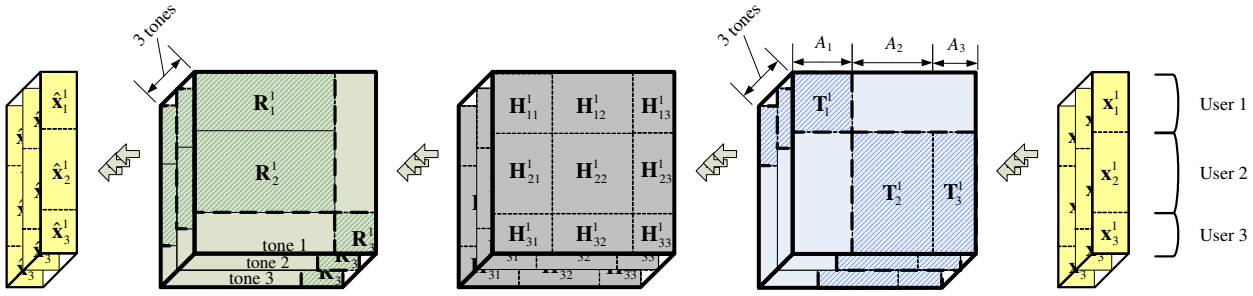

Fig. 2. Illustration of a hybrid MAC, BC and IC. For this case, we have on the transmitter side $\mathcal{T}_1 = \{1\}$ and $\mathcal{T}_2 = \{2, 3\}$; and on the receiver side $\mathcal{R}_1 = \{1, 2\}$ and $\mathcal{R}_2 = \{3\}$.

tone k; and $\widetilde{\mathbf{H}}_{n,u}^k \in \mathbb{C}^{A_{\mathcal{R}_j} \times A_{\mathcal{T}_i}}$, where j and i are such that $n \in \mathcal{R}_j$ and $u \in \mathcal{T}_i$, is the channel matrix on tone k between the group to which user u belongs on the transmitter side to the group to which user n belongs on the receiver side. We later show that $\widetilde{\mathbf{H}}_{n,u}^k$ depends on the type of coordination on both sides of the channel and that it can be viewed as a concatenation of matrices relating to the MIMO IC, that we define as $\mathbf{H}_{n,u}^k \in \mathbb{C}^{A_n \times A_u}$. Three examples given later in this section make this clearer.

In (3), we assume that user n has A_n parallel data streams, but some of these streams can have rate of zero. We also assume $\mathrm{E}\left[\mathbf{x}_n^k (\mathbf{x}_n^k)^{\mathrm{H}}\right] = \mathbf{I}_{A_n}$, and hence $\mathrm{tr}\{\mathbf{T}_n^k (\mathbf{T}_n^k)^{\mathrm{H}}\} = p_n^k$. Still in (3), the noise vector $\mathbf{z}_{\mathcal{R}_j}^k \in \mathbb{C}^{A_{\mathcal{R}_j}}$ is zero mean complex Gaussian and, without loss of generality, is assumed to be spatially white with covariance matrix $\mathrm{E}\left[\mathbf{z}_{\mathcal{R}_j}^k (\mathbf{z}_{\mathcal{R}_j}^k)^{\mathrm{H}}\right] = \mathbf{I}_{A_{\mathcal{R}_j}}$, $j = 1 \ldots, J$. The estimated signal vector for user n on tone k is given by

$$\hat{\mathbf{x}}_n^k = \mathbf{R}_n^k \mathbf{y}_{\mathcal{R}_j}^k, \tag{4}$$

where $\mathbf{R}_n^k \in \mathbb{C}^{A_n \times A_{\mathcal{R}_j}}$, $n \in \mathcal{R}_j$, is the receive matrix for user n on tone k. The receive matrix we use is the linear minimum mean squared error (MMSE). For a given set of transmit matrices, we write

$$\mathbf{R}_n^k = (\mathbf{T}_n^k)^{\mathrm{H}} (\widetilde{\mathbf{H}}_{n,n}^k)^{\mathrm{H}} \left(\mathbf{M}_n^k + \widetilde{\mathbf{H}}_{n,n}^k \mathbf{T}_n^k (\mathbf{T}_n^k)^{\mathrm{H}} (\widetilde{\mathbf{H}}_{n,n}^k)^{\mathrm{H}} \right)^{-1}, \tag{5}$$

where, for the synchronous case,

$$\mathbf{M}_n^k = \sum_{\substack{u\in\mathcal{N}\\ u\neq n}} \widetilde{\mathbf{H}}_{n,u}^k \mathbf{T}_u^k (\mathbf{T}_u^k)^{\mathrm{H}} (\widetilde{\mathbf{H}}_{n,u}^k)^{\mathrm{H}} + \mathbf{I}_{A_{\mathcal{R}_j}},\ n \in A_{\mathcal{R}_j} \tag{6}$$

is the noise plus interference covariance matrix for user n on tone k. We remark that, although we do not write it explicitly, this matrix may be normalized by a capacity gap Γ.

We give three examples to better explain the notation and to explain how to form $\widetilde{\mathbf{H}}_{n,u}^k$ using the $\mathbf{H}_{n,u}^k$ as building blocks.

The **first example** is depicted in Fig. 1. This is a system with three users and three tones. This is a pure MIMO IC, i.e. every group contains a single user. This is the situation where there is a minimum amount of coordination. We have $J = I = N = 3$ and $\mathcal{T}_n = \mathcal{R}_n = \{n\}$, $n \in \mathcal{N}$. In this case, for all tones we have $\widetilde{\mathbf{H}}_{n,u}^k = \mathbf{H}_{n,u}^k$, $n, u \in \mathcal{N}$; $A_{\mathcal{T}_n} = A_{\mathcal{R}_n} = A_n$, $n \in \mathcal{N}$; and $\mathbf{T}_n^k, \mathbf{R}_n^k \in \mathbb{C}^{A_n \times A_n}$.

The **second example** is depicted in Fig. 2. This is a system with three users, with, say $A_1 = 2$, $A_2 = 3$ and $A_3 = 1$, and three tones. There are two groups on both transmitter and receiver side, i.e. $J = I = 2$. We have $\mathcal{T}_1 = \{1\}$ and $\mathcal{T}_2 = \{2, 3\}$; and on the receiver side $\mathcal{R}_1 = \{1, 2\}$ and $\mathcal{R}_2 = \{3\}$. Using (1) and (2), we obtain $A_{\mathcal{T}_1} = 2$, $A_{\mathcal{T}_2} = 4$, $A_{\mathcal{R}_1} = 5$ and $A_{\mathcal{R}_2} = 1$. As a consequence, for all tones we have $\mathbf{T}_1^k \in \mathbb{C}^{2\times 2}$, $\mathbf{T}_2^k \in \mathbb{C}^{4\times 3}$, etc. Notice that this system does not fit exactly neither the MAC, BC or IC scenario. This is a scenario with elements of BC, MAC and IC.

To calculate the equivalent channel matrices, we take into account the matrices $\mathbf{H}_{n,u}^k \in \mathbb{C}^{A_n \times A_u}$, $n, u \in \mathcal{N}$ and $k \in \mathcal{K}$. As mentioned in the previous paragraph, these would be the channel matrices for the case of a pure MIMO IC. The received signal for, say, group 1 is given by

$$\mathbf{y}_{\mathcal{R}_1}^k = \begin{bmatrix} \mathbf{H}_{1,1}^k \\ \mathbf{H}_{2,1}^k \end{bmatrix} \mathbf{T}_1^k \mathbf{x}_1^k + \begin{bmatrix} \mathbf{H}_{1,2}^k & \mathbf{H}_{1,3}^k \\ \mathbf{H}_{2,2}^k & \mathbf{H}_{2,3}^k \end{bmatrix} \mathbf{T}_2^k \mathbf{x}_2^k + \begin{bmatrix} \mathbf{H}_{1,2}^k & \mathbf{H}_{1,3}^k \\ \mathbf{H}_{2,2}^k & \mathbf{H}_{2,3}^k \end{bmatrix} \mathbf{T}_3^k \mathbf{x}_3^k + \begin{bmatrix} \mathbf{z}_1^k \\ \mathbf{z}_2^k \end{bmatrix}. \tag{7}$$

The estimated signal for, say, user 2 is given by applying (4), i.e.

$$\hat{\mathbf{x}}_2^k = \mathbf{R}_2^k \underbrace{\begin{bmatrix} \mathbf{H}_{1,1}^k \\ \mathbf{H}_{2,1}^k \end{bmatrix}}_{\triangleq \widetilde{\mathbf{H}}_{2,1}^k} \mathbf{T}_1^k \mathbf{x}_1^k + \mathbf{R}_2^k \underbrace{\begin{bmatrix} \mathbf{H}_{1,2}^k & \mathbf{H}_{1,3}^k \\ \mathbf{H}_{2,2}^k & \mathbf{H}_{2,3}^k \end{bmatrix}}_{\triangleq \widetilde{\mathbf{H}}_{2,2}^k} \mathbf{T}_2^k \mathbf{x}_2^k + \mathbf{R}_2^k \underbrace{\begin{bmatrix} \mathbf{H}_{1,2}^k & \mathbf{H}_{1,3}^k \\ \mathbf{H}_{2,2}^k & \mathbf{H}_{2,3}^k \end{bmatrix}}_{\triangleq \widetilde{\mathbf{H}}_{2,3}^k} \mathbf{T}_3^k \mathbf{x}_3^k + \mathbf{R}_2^k \underbrace{\begin{bmatrix} \mathbf{z}_1^k \\ \mathbf{z}_2^k \end{bmatrix}}_{\triangleq \mathbf{z}_{\mathcal{R}_1}^k}. \quad (8)$$

Hence we define the $\widetilde{\mathbf{H}}_{n,u}^k$, $n, u \in \mathcal{N}$ as the channel matrix that comes between $\mathbf{R}_n^k$ and $\mathbf{T}_u^k$. It is a concatenation of the matrices $\mathbf{H}_{n,u}^k$. The concatenation depends on the grouping on both the receiver and transmitter sides.

The **third example** is that of a three user MIMO BC. This system is represented by $\mathcal{T}_1 = \{1, 2, 3\}$, i.e. $I = 1$, and $R_n = \{n\}$, $n \in \mathcal{N}$, i.e. $J = 3$. If $A_n = 2\ \forall n$, then $A_{\mathcal{T}_1} = 6$ and $A_{\mathcal{R}_j} = 2$, $j = 1, \ldots, 3$. For this case, we have

$$\widetilde{\mathbf{H}}_{1,n}^k = \begin{bmatrix} \mathbf{H}_{1,1}^k & \mathbf{H}_{1,2}^k & \mathbf{H}_{1,3}^k \end{bmatrix},\ n \in \mathcal{N} \quad (9)$$

$$\widetilde{\mathbf{H}}_{2,n}^k = \begin{bmatrix} \mathbf{H}_{2,1}^k & \mathbf{H}_{2,2}^k & \mathbf{H}_{2,3}^k \end{bmatrix},\ n \in \mathcal{N} \quad (10)$$

$$\widetilde{\mathbf{H}}_{3,n}^k = \begin{bmatrix} \mathbf{H}_{3,1}^k & \mathbf{H}_{3,2}^k & \mathbf{H}_{3,3}^k \end{bmatrix},\ n \in \mathcal{N} \quad (11)$$

Because of the structure with groups and because the $\widetilde{\mathbf{H}}_{n,u}^k$ are defined as functions of the $\mathbf{H}_{n,u}^k$, the proposed system model includes any kind of transmitter and receiver coordination. On one extreme, we have a MIMO IC, as explained in the first example. On the other extreme, there is only one group both on the transmitter and on the receiver side, i.e. $\mathcal{T}_1 = \mathcal{R}_1 = \{1, \ldots, N\}$. This is the case with full two-sided coordination. It is often called a point-to-point system. With our formulation, every case between (and including) these two extremes is possible.

As a rule of thumb, we remark that coordination on the transmitter side makes $\widetilde{\mathbf{H}}_{n,u}^k$ 'wider' (more columns) than $\mathbf{H}_{n,u}^k$, and that coordination on the receiver side makes $\widetilde{\mathbf{H}}_{n,u}^k$ 'taller' (more rows) than $\mathbf{H}_{n,u}^k$.

The optimization problem we consider is the maximization of the weighted rate sum of the participating users in the network subject to per-user power constraints. Consider $\mathbb{T} = \{\mathbf{T}_n^k | n \in \mathcal{N},\ k \in \mathcal{K}\}$ and

TABLE I

COMPARISON OF PREVIOUS SOLUTIONS—SYNCHRONOUS CASE

Solution [reference]†	Description	Restrictions‡
IWF, OSB, SCALE, DSB, 2SB, MIW [21]–[26]	solutions to SISO spectrum management	$A_n = 1\ \forall n$, IC
ZF-GDFE [5]	ZF, (non-linear) GDFE and waterfilling	$A_n = 1\ \forall n$; BC or MAC
ZF [7], [8]	linear ZF plus waterfilling	$A_n = 1\ \forall n$; BC or MAC; diagonally dominant channel matrix
Low complexity linear ZF [12]	linear ZF, lower complexity version	$A_n = 1\ \forall n$; BC or MAC; diagonally dominant channel matrix
MAC-OSB [27]	MMSE-GDFE plus OSB	MAC, $A_n = 1\ \forall n$
SVD [9], [10]	SVD applied to channel matrix	point-to-point
Vectoring with common mode [11]	SVD applied to channel matrix of each user, inter-user interference with GDFE	IC
IC-MAC OSB/ IC-BC OSB [14], [15]	MMSE-GDFE plus OSB	Partial MAC ($I = N$, $J < N$, with $\mathcal{T}_n = \{n\},\ n \in \mathcal{N}$ and J groups on receiver) or partial BC ($I < N$, $J = N$, with $\mathcal{R}_n = \{n\},\ n \in \mathcal{N}$ and I groups on transmitter) with $A_n = 1\ \forall n$
WMMSE-GDSB [20]	WMMSE solution for signal coordination plus generalized DSB for spectrum coordination	IC

† A list of the acronyms is given as follows: optimal spectrum balancing (OSB), single input, single output (SISO), generalized decision feedback equalization (GDFE), zero forcing (ZF), minimum mean squared error (MMSE), weighted MMSE (WMMSE) and singular value decomposition (SVD).

‡ In the notation of this paper, a BC is characterized by $I = 1$, $J = N$, $\mathcal{T}_1 = \{1, \ldots, N\}$ and $\mathcal{R}_n = \{n\},\ n \in \mathcal{N}$; a MAC is characterized by $I = N$, $J = 1$, $\mathcal{T}_n = \{n\},\ n \in \mathcal{N}$ and $\mathcal{R}_1 = \{1, \ldots, N\}$; a point-to-point system is characterized by $I = J = 1$, $\mathcal{T}_1 = \mathcal{R}_1 = \{1, \ldots, N\}$; and an IC is characterized by $I = J = N$ and $\mathcal{T}_n = \mathcal{R}_n = \{n\}, n \in \mathcal{N}$.

$\mathbb{R} = \{\mathbf{R}_n^k | n \in \mathcal{N},\ k \in \mathcal{K}\}$. The optimization problem is then given as

$$\begin{aligned} \max_{\mathbb{R},\mathbb{T},\mathbf{P}} \quad & \sum_{n\in\mathcal{N}} \sum_{k\in\mathcal{K}} u_n b_n^k \\ \text{subject to} \quad & \mathrm{tr}\big\{\mathbf{T}_n^k (\mathbf{T}_n^k)^{\mathrm{H}}\big\} = p_n^k,\ k \in \mathcal{K},\ n \in \mathcal{N} \\ & \sum_{k\in\mathcal{K}} p_n^k \le P_n^{\max},\ n \in \mathcal{N} \end{aligned} \tag{12}$$

where, when using the linear MMSE (LMMSE) receiver, data rate for user n and tone k is given by

$$b_n^k = \log\left|\mathbf{I}_{A_n} + (\mathbf{T}_n^k)^{\mathrm{H}}(\widetilde{\mathbf{H}}_{n,n}^k)^{\mathrm{H}}(\mathbf{M}_n^k)^{-1}\widetilde{\mathbf{H}}_{n,n}^k\mathbf{T}_n^k\right| \tag{13}$$

with $\mathbf{M}_n^k$ given by (6). The variables u_n are weights or priorities given to each user and $P_n^{\max}$ is the total power constraint for user n. We remark that in [28] a similar problem is treated with per-transceiver power constraints, whereas here a per-user power constraint is considered. The situation with the per-transceiver power constraints adds complexity to the problem, and for simplicity is not considered in this paper.

The challenge in (12) is twofold. On the one hand, the transmit matrices, i.e. $\mathbb{T}$, should be designed so that the desired signal is easy to identify and the undesired signals are easy to cancel. On the other hand, power should be carefully allocated for every user and tone. These two challenges have a strong interplay with each other. The choice of $\mathbb{T}$ determines the spatial separation or spatial multiplexing that is characteristic of MIMO systems. How this separation occurs depends on the kind of coordination present on both sides of the network. Notice that more transmitter and receiver coordination makes the channel matrices 'wider' or 'taller', which means that there are more dimensions in which this separation can take place. For a point-to-point system, perfect spatial separation is possible. In this particular case, the singular value decomposition (SVD) of the channel matrix separates the channel in independent eigenmodes, one for each data stream (e.g. [29]). For anything with less coordination, there will always be some residual interference that every user has to withstand. Users should avoid excessive residual interference to each other by choosing a smart power loading through frequency, i.e. by choosing $\mathbf{P}$.

Table I reviews previous solutions to the problem. We give preference to papers dealing with DSL. As we already mentioned, all previous work has focused on special assumptions on the network infrastructure. As we briefly describe some previous solutions in Table I, we always use the framework just discussed.

B. Proposed Solution

In [20], a solution for the DMT MIMO IC is proposed that achieves good results with reasonable computational cost. The problem is divided in signal and spectrum coordination parts and these two parts are solved iteratively and independently. The algorithm in [20] is of special interest here, because it forms the basis of the general algorithm that is presented in this paper.

The solution of [20] departs from an equation of the received signal similar to (3). The difference is that for the general case with any kind of groupings on both sides of the network the channel matrix $\widetilde{\mathbf{H}}_{n,u}^k$ is (or can be) a concatenation of the matrices $\mathbf{H}_{n,u}^k$, whereas in [20] there is a restriction that

$\widetilde{\mathbf{H}}_{n,u}^k = \mathbf{H}_{n,u}^k$, $n, u \in \mathcal{N}$. The main insight of this paper is that independently of what channel matrix is used between $\mathbf{R}_n^k$ and $\mathbf{T}_n^k$, the solution of [20] applies. In the following, we derive the main parts of the algorithm while emphasizing the main differences between the current problem and that of [20].

We begin by writing the Lagrangian of (12)

$$L(\mathbb{R}, \mathbb{T}, \mathbf{P}, \boldsymbol{\mu}, \boldsymbol{\lambda}) = \sum_{n\in\mathcal{N}} \sum_{k\in\mathcal{K}} u_n b_n^k - \sum_{n\in\mathcal{N}} \lambda_n \big(\sum_{k\in\mathcal{K}} p_n^k - P_n^{\max} \big) - \sum_{n\in\mathcal{N}} \sum_{k\in\mathcal{K}} \mu_n^k \big(\mathrm{tr}\big\{ \mathbf{T}_n^k (\mathbf{T}_n^k)^{\mathrm{H}} \big\} - p_n^k \big) \quad (14)$$

Here $\boldsymbol{\lambda} = \begin{bmatrix} \lambda_1 & \dots & \lambda_N \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^N$ and $\boldsymbol{\mu} = \begin{bmatrix} \mu_1^1 & \dots & \mu_1^k & \mu_2^1 \dots \mu_N^K \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{NK}$ are Lagrange multipliers associated respectively with the per-user power constraints and the per-user and per-tone trace constraint on the transmit matrices.

We separate the problem in two parts. First, we solve for the transmit and receive matrices while keeping the power loading matrix fixed, then we solve for the power loading matrix while keeping the transmit and receive matrices fixed. Towards this end, we write (14) in two parts

$$L(\mathbb{R}, \mathbb{T}, \boldsymbol{\mu}) = \sum_{n\in\mathcal{N}} \sum_{k\in\mathcal{K}} u_n b_n^k - \sum_{n\in\mathcal{N}} \sum_{k\in\mathcal{K}} \mu_n^k \big(\mathrm{tr}\big\{ \mathbf{T}_n^k (\mathbf{T}_n^k)^{\mathrm{H}} \big\} - p_n^k \big) \quad (15)$$

$$L(\mathbf{P}, \boldsymbol{\lambda}) = \sum_{n\in\mathcal{N}} \sum_{k\in\mathcal{K}} u_n b_n^k - \sum_{n\in\mathcal{N}} \lambda_n \big(\sum_{k\in\mathcal{K}} p_n^k - P_n^{\max} \big) \quad (16)$$

For the following argument, we decompose the transmit matrices as $\mathbf{T}_n^k = \sqrt{p_n^k}\overline{\mathbf{T}}_n^k$, where $\mathrm{tr}\big\{ \overline{\mathbf{T}}_n^k (\overline{\mathbf{T}}_n^k)^{\mathrm{H}} \big\} = 1$. While solving for $\mathbf{P}$ we keep $\overline{\mathbf{T}}_n^k$ for all n and k fixed and while solving for $\mathbb{T}$ we keep $\mathbf{P}$ fixed.

The first thing to notice about (15) is that it is now decoupled through the tones. Each tone now has its power constraint fixed for all the users, and as a consequence the solution to (15) involves solving K (one for each tone) independent rate maximization problems. The important thing about (16) is that it is a pure power allocation problem, just like [21]–[25], [30]. It is, however, a MIMO spectrum coordination problem. All the references just cited treat a single input, single output (SISO) power allocation problem. The problem in $\mathbf{P}$ is coupled through frequency by the total power constraint.

1) Solving for $\mathbb{T}$: To solve (15), we follow the weighted MMSE (WMMSE) approach [20], [31], [32]. This approach establishes a way to maximize rate by minimizing the weighted mean squared error (MSE)

of symbol detection. To use this method, for all users and tones we first calculate (5) and then do

$$\mathbf{W}_n^k = u_n(\mathbf{E}_n^k)^{-1} \tag{17}$$

$$\mathbf{T}_n^k = \big(\sum_{u\in\mathcal{N}}(\widetilde{\mathbf{H}}_{u,n}^k)^{\mathrm{H}}(\mathbf{R}_u^k)^{\mathrm{H}}\mathbf{W}_u^k\mathbf{R}_u^k\widetilde{\mathbf{H}}_{u,n}^k + \lambda_n\mathbf{I}_{A_{\mathcal{T}_i}}\big)^{-1} \times (\widetilde{\mathbf{H}}_{n,n}^k)^{\mathrm{H}}(\mathbf{R}_n^k)^{\mathrm{H}}\mathbf{W}_n^k,\ n\in\mathcal{T}_i. \tag{18}$$

In (17),

$$\mathbf{E}_n^k = \big((\mathbf{T}_n^k)^{\mathrm{H}}(\widetilde{\mathbf{H}}_{n,n}^k)^{\mathrm{H}}(\mathbf{M}_n^k)^{-1}\widetilde{\mathbf{H}}_{n,n}^k\mathbf{T}_n^k + \mathbf{I}_{A_n}\big)^{-1} \tag{19}$$

is the MSE matrix. Also, $\mathbf{W}_n^k \in \mathbb{C}^{A_n\times A_n}$ is the key to making the connection between the rate maximization and the MSE minimization. In (18), the Lagrange multiplier λ_n should be chosen such that the power constraint is respected.

The main difference between (17) and (18) and the equivalent equations in [20] is that the channel matrices $\widetilde{\mathbf{H}}_{n,u}^k$ replaces the MIMO IC matrices $\mathbf{H}_{n,u}^k$. Notice also that the transmit matrices $\mathbf{T}_n^k$ are larger or at least as large as the ones in [20], i.e. there are more dimensions in which to spatially separate the transmission of each user.

2) Solving for $\mathbf{P}$*:* To solve for $\mathbf{P}$, we take a per-user approach, i.e. we solve for each user separately while keeping power for all other users fixed. To solve for user n in (16), we first notice that $L(\mathbf{p}_n,\lambda_n)$ has a difference of concave (DC) programming structure, i.e. it is the difference of convex functions in p_n^k: while b_n^k is concave in p_n^k, b_u^k, $u\neq n$, is convex.[3] To more easily solve the problem, we first approximate b_u^k, $u\neq n$, by its first order Taylor expansion. After such approximation, we write

$$L_{\mathrm{approx}}(\mathbf{p}_n,\lambda_n) = \sum_{k\in\mathcal{K}} w_n b_n^k + \sum_{\substack{u\in\mathcal{N}\\u\neq n}}\sum_{k\in\mathcal{N}}(p_n^k - \bar{p}_n^k)\frac{\partial b_u^k}{\partial p_n^k} - \lambda_n p_n^k, \tag{20}$$

which is now concave in p_n^k. Here $\bar{p}_n^k$ represents p_n^k from a previous iteration. To solve this concave problem, we take the derivative in p_n^k and set it to zero. We obtain

$$\frac{\partial L_{\mathrm{approx}}(\mathbf{p}_n,\lambda_n)}{\partial p_n^k} = \mathrm{tr}\big\{\big(p_n^k\mathbf{S}_n^k + \mathbf{I}_{A_n}\big)^{-1}\mathbf{S}_n^k\big\} - \lambda_n - \tau_n^k = 0, \tag{21}$$

where $\mathbf{S}_n^k = (\overline{\mathbf{T}}_n^k)^{\mathrm{H}}(\widetilde{\mathbf{H}}_{n,n}^k)^{\mathrm{H}}(\mathbf{M}_n^k)^{-1}\widetilde{\mathbf{H}}_{n,n}^k\overline{\mathbf{T}}_n^k$ and τ_n^k is given by

$$\tau_n^k = \sum_{\substack{u\in\mathcal{N}\\u\neq n}} w_u \mathrm{tr}\Big\{\mathbf{E}_u^k(\mathbf{T}_u^k)^{\mathrm{H}}(\mathbf{H}_{u,u}^k)^{\mathrm{H}}(\mathbf{M}_u^k)^{-1}\mathbf{H}_{u,n}^k\overline{\mathbf{T}}_n^k \times(\overline{\mathbf{T}}_n^k)^{\mathrm{H}}(\mathbf{H}_{u,n}^k)^{\mathrm{H}}(\mathbf{M}_u^k)^{-1}\mathbf{H}_{u,u}^k\mathbf{T}_u^k\Big\} \tag{22}$$

[3]There is one exception to this, namely the case of the MAC with equal weights for all users. See e.g. [33]

Here $\mathbf{E}_u^k$ is given by (19).

In (21), τ_n^k is a price for excessive interference. This variable should be large if there is potential for large rate losses for other users in case user n loads too much power on tone k. It can be shown that (21) has at most one nonnegative root and that this equation can be simplified to a polynomial form [20]. For solving it, we need but to solve the polynomial and pick the largest root.

The polynomial is of degree $A_{\mathcal{T}_i}$, where $n \in \mathcal{T}_i$. When $A_{\mathcal{T}_i}$ is large, it may be too costly to solve the polynomial and pick the largest root. Since $A_{\mathcal{T}_i} - 1$ roots should be discarded anyway, in [20] the power method has been proposed to obtain the largest root of the polynomial. Here we exploit the fact that (21) is the derivative of a concave function, and thus it can have only one zero. To find its zero, we can use the Newton method.

The algorithm should be organized such that the signal and spectrum coordination parts are solved iteratively and independently. The algorithm to be presented in the next section focusing on asynchronous transmission includes the synchronous algorithm as a special case. More details are therefore provided in the next section.

III. Asynchronous transmission

A. System model and problem statement

An asynchronous transmission scenario occurs when the DMT blocks of the different users are not aligned in time. We demonstrate this with the example of Fig. 3, where two users with two transceivers interfere with each other. Their respective DMT blocks are offset by η, $0 \leq \eta \leq 1$, as shown in the figure. Such a situation gives rise to inter-carrier interference (ICI), which complicates the problem significantly. With ICI, transmission on a given tone k of an interferer influences not only the corresponding tone k of a victim, but all neighboring tones as well.

Three alternative solutions have been proposed to the asynchronous problem [16], [17], [26], all of them considering the SISO IC situation; see Table II. Some references have also focused on the modeling of the ICI effect [16], [18], [19], [34]. In this section we provide both an accurate characterization of the MIMO asynchronous transmission and a solution for the signal and spectrum coordination problem.

The bulk of the system model described in Section II-A continues to be valid for the asynchronous case, including the definition of groups on the transmitter and receiver sides. In the following, we assume that all the transceivers of inside a group are synchronized. The received signal for group $\mathcal{R}_j$ on tone k

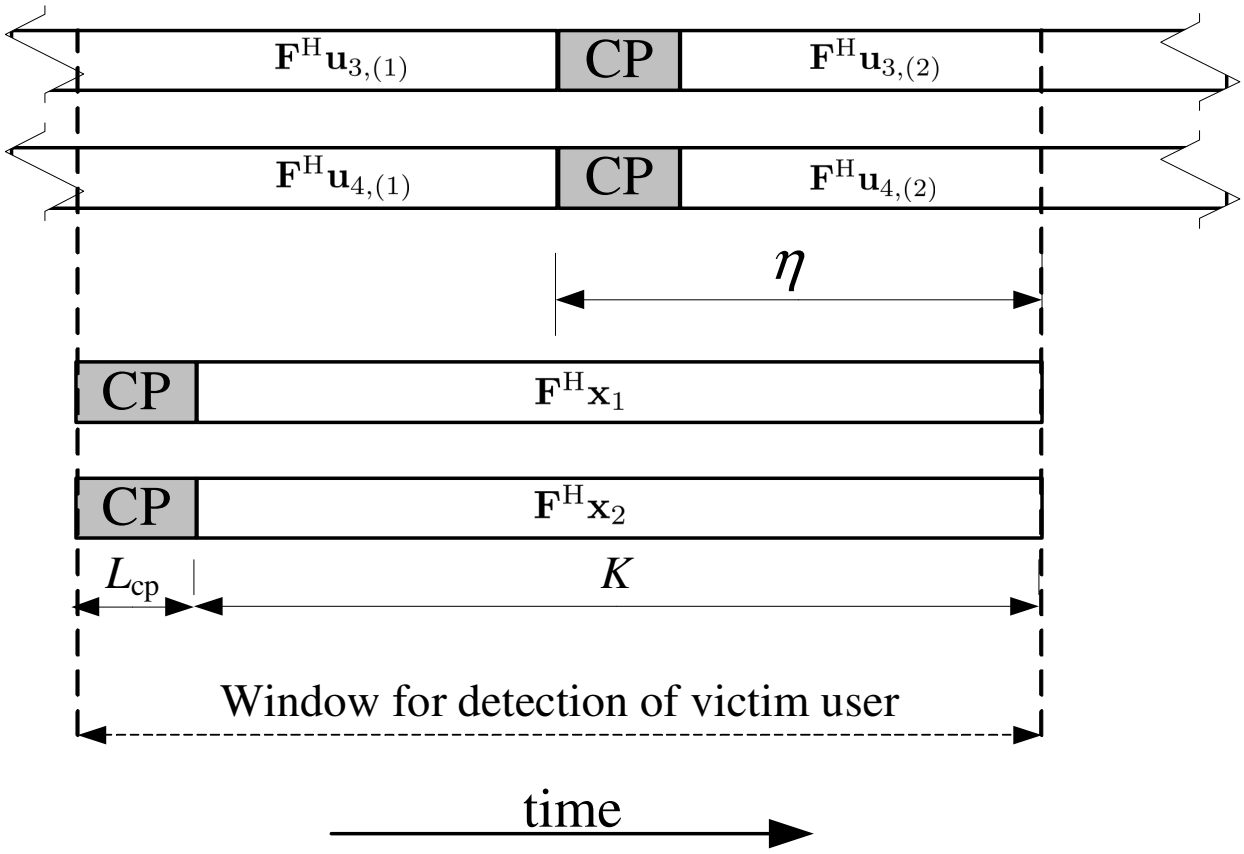

Fig. 3. Model for asynchronous transmission. In this example, two users, each with two transceivers, interfere with one another. Here the victim user has a DMT block for its ith transceiver given by $\mathbf{x}_i$, while the DMT block of the interferer user is denoted by $\mathbf{u}_i$. Symbols from two times instants of the interferer user affect the victim. These instants are denoted by the subscripts (1) and (2). The offset on reception is given by η, as defined in the figure.

is given by

$$\mathbf{y}_{\mathcal{R}_j}^k = \widetilde{\mathbf{H}}_{n,n}^k \mathbf{T}_n^k \mathbf{x}_n^k + \sum_{\substack{u\in\mathcal{N}\\ u\neq n}} \sum_{q\in\mathcal{K}} \mathbf{A}_{n,u}^{k,q} \mathbf{T}_u^q \mathbf{x}_u^q + \mathbf{B}_{n,u}^{k,q} \mathbf{T}_u^q \bar{\mathbf{x}}_u^q + \mathbf{z}_{\mathcal{R}_j}^k. \quad (23)$$

Here $\mathbf{x}_u^q$, $\mathbf{y}_u^q$ are the DMT symbols of user u on tone q that interfere with the reception of the DMT symbol for user n. Symbol $\mathbf{x}_u^q$ comes before (in time) the reception of user n and symbol $\bar{\mathbf{x}}_u^q$ comes after. The matrices $\mathbf{A}_{n,u}^{k,q}$, $\mathbf{B}_{n,u}^{k,q}$ account for the ICI effect. They represent the channel transfer function from user u to user n and from tone q to tone k. If $\eta = 0$ or $\eta = 1$, then there is no ICI. If $\eta = 0$, then $\mathbf{A}_{n,u}^{k,k} = \widetilde{\mathbf{H}}_{n,u}^k$, $\mathbf{A}_{n,u}^{k,q} = \mathbf{0}$ for $k \neq q$ and $\mathbf{B}_{n,u}^{k,q} = \mathbf{0}$ for all q, k. If $\eta = 1$, then $\mathbf{B}_{n,u}^{k,k} = \widetilde{\mathbf{H}}_{n,u}^k$, $\mathbf{B}_{n,u}^{k,q} = \mathbf{0}$ for

TABLE II

COMPARISON OF PREVIOUS SOLUTIONS—ASYNCHRONOUS CASE

Solution [reference]†	Description	Restrictions‡
MIW, ASB-A1, ASB-A2, GPS/GPA [16], [17], [26],	solutions to SISO spectrum management	$A_n = 1\ \forall n$, IC

† single input, single output (SISO); ‡ In the notation of this paper, an IC is characterized by $I = J = N$ and $\mathcal{T}_n = \mathcal{R}_n = \{n\}, n \in \mathcal{N}$.

$k \neq q$ and $\mathbf{A}_{n,u}^{k,q} = \mathbf{0}$ for all q, k. We detail how to accurately calculate the matrices $\mathbf{A}_{n,u}^{k,q}$ and $\mathbf{B}_{n,u}^{k,q}$ as a function of the offset η for the general case in the Appendix.

The problem continues to be the one expressed in (12). However, the ICI changes the data rate calculation in that it changes the calculation of the noise plus interference covariance matrix. Considering $\mathrm{E}\left[\mathbf{x}_u^q(\mathbf{x}_u^q)^{\mathrm{H}}\right] = \mathrm{E}\left[\bar{\mathbf{x}}_u^q(\bar{\mathbf{x}}_u^q)^{\mathrm{H}}\right] = \mathbf{I}_{\mathcal{R}_j},\ u \in \mathcal{R}_j$, we now have

$$\mathbf{M}_n^k = \sum_{\substack{u \in \mathcal{N} \\ u \neq n}} \sum_{q \in \mathcal{K}} \mathbf{A}_{n,u}^{k,q} \mathbf{T}_u^q (\mathbf{T}_u^q)^{\mathrm{H}} (\mathbf{A}_{n,u}^{k,q})^{\mathrm{H}} + \mathbf{B}_{n,u}^{k,q} \mathbf{T}_u^q (\mathbf{T}_u^q)^{\mathrm{H}} (\mathbf{B}_{n,u}^{k,q})^{\mathrm{H}} + \mathbf{I}_{A_{\mathcal{R}_j}}. \tag{24}$$

We follow the same general principle of the algorithm described for the synchronous case. We first solve for $\mathbb{T}$ while keeping $\mathbf{P}$ fixed. Eqs. (5) and (17) are still valid for the asynchronous case given that the $\mathbf{M}_n^k$ are calculated with (24). The transmit matrices are given by (25) on the top of the next page.

Solving for $\mathbf{P}$ while $\overline{\mathbf{T}}_n^k$ is held fixed for all n, k, we notice that (21) still holds. The per-tone penalties are now different, and are given by (26).

B. Proposed solution

We are now ready to write a general algorithm that applies to any number of users, to any number of transceivers, to any number of tones, to any kind of coordination on both the transmitter and on the receiver sides, and to synchronous/asynchronous transmission. Because of the general framework containing the BC, MAC, IC or any combination of them, the WMMSE approach to solve the signal coordination problem and the generalized DSB [20], [24] approach to solve the spectrum coordination part, we call it general framework WMMSE-GDSB (GF-WMMSE-GDSB).

$$\mathbf{T}_n^k = \Big((\widetilde{\mathbf{H}}_{n,n}^k)^{\mathrm{H}} (\mathbf{R}_n^k)^{\mathrm{H}} \mathbf{W}_n^k \mathbf{R}_n^k \widetilde{\mathbf{H}}_{n,n}^k + \sum_{\substack{u \in \mathcal{N} \\ u \neq n}} \sum_{q \in \mathcal{K}} (\mathbf{A}_{u,n}^{q,k})^{\mathrm{H}} (\mathbf{R}_u^q)^{\mathrm{H}} \mathbf{W}_u^q \mathbf{R}_u^q \mathbf{A}_{u,n}^{q,k} + (\mathbf{B}_{u,n}^{q,k})^{\mathrm{H}} (\mathbf{R}_u^q)^{\mathrm{H}} \mathbf{W}_u^q \mathbf{R}_u^q \mathbf{B}_{u,n}^{q,k} + \lambda_n \mathbf{I}_{A_{\mathcal{T}_i}} \Big)^{-1} \times (\widetilde{\mathbf{H}}_{n,n}^k)^{\mathrm{H}} (\mathbf{R}_n^k)^{\mathrm{H}} \mathbf{W}_n^k, \ n \in \mathcal{T}_i \quad (25)$$

$$\tau_n^k = \sum_{\substack{u \in \mathcal{N} \\ u \neq n}} \sum_{q \in \mathcal{K}} w_u \mathrm{tr} \Big\{ \mathbf{E}_u^q (\mathbf{T}_u^q)^{\mathrm{H}} (\mathbf{H}_{u,u}^q)^{\mathrm{H}} (\mathbf{M}_u^q)^{-1} \Big(\mathbf{A}_{u,n}^{q,k} \overline{\mathbf{T}}_n^k (\overline{\mathbf{T}}_n^k)^{\mathrm{H}} (\mathbf{A}_{u,n}^{q,k})^{\mathrm{H}} + \mathbf{B}_{u,n}^{q,k} \overline{\mathbf{T}}_n^k (\overline{\mathbf{T}}_n^k)^{\mathrm{H}} (\mathbf{B}_{u,n}^{q,k})^{\mathrm{H}} \Big) (\mathbf{M}_u^q)^{-1} \mathbf{H}_{u,u}^q \mathbf{T}_u^q \Big\} \quad (26)$$

In the algorithm, we first solve for the power matrix and then for the transmit matrices. In lines 3 and 4 we calculate the per-tone penalties and the $\mathbf{S}_n^k$ matrix for all tones and users. The interference plus noise covariance matrix is given by (24). As mentioned in Section III, for the synchronous case either $\mathbf{A}_{n,u}^{k,k} = \widetilde{\mathbf{H}}_{n,u}^k$, $\mathbf{A}_{n,u}^{k,q} = \mathbf{0}$ for $k \neq q$ and $\mathbf{B}_{n,u}^{k,q} = \mathbf{0}$ for all q, k; or $\mathbf{B}_{n,u}^{k,k} = \widetilde{\mathbf{H}}_{n,u}^k$, $\mathbf{B}_{n,u}^{k,q} = \mathbf{0}$ for $k \neq q$ and $\mathbf{A}_{n,u}^{k,q} = \mathbf{0}$ for all q, k. So calculating τ_n^k with (26) and $\mathbf{M}_n^k$ with (24) includes the synchronous case as a special case.

In line 8 we solve for p_n^k for a given Lagrange multiplier. As mentioned in [20] and also earlier in this paper, this can be done by solving a polynomial and picking the largest root. Other possibilities include using the power method or the Newton method. The next step in the algorithm is to adjust the Lagrange multiplier so that the power constraints are satisfied.

The next step is to solve for the matrices $\mathbf{T}_n^k$ for fixed $\mathbf{P}$. As mentioned before, this implies solving K independent problems, one for each tone. We solve first for $\mathbf{R}_n^k$ and for $\mathbf{W}_n^k$ in lines 14-15. The loop that follows calculates $\mathbf{T}_n^k$ with (25) and finds the appropriate Lagrange multiplier. It is again clear that calculating $\mathbf{T}_n^k$ with (25) includes the synchronous case as a special case.

In general the computational complexity of the proposed algorithm is given by $O(K^2 N^2 \max_i A_{\mathcal{T}_i}^3)$. For the special case of synchronous transmission, this is reduced to $O(K N^2 A_{\mathcal{T}_i}^3)$. The algorithm has been extensively experimented with and has always been observed to produce a monotonically increasing objective function. It has always been observed to converge.

GF-WMMSE-GDSB

Initialize $\mathbf{T}_n^k \forall n,k$ and $\mathbf{P}$;
repeat
  Calculate τ_n^k with (26) $\forall n,k$;
  Calculate $\mathbf{S}_n^k \triangleq (\overline{\mathbf{T}}_n^k)^{\mathrm{H}}(\widetilde{\mathbf{H}}_{n,n}^k)^{\mathrm{H}}(\mathbf{M}_n^k)^{-1}\widetilde{\mathbf{H}}_{n,n}^k\overline{\mathbf{T}}_n^k \ \forall n,k$;
  for $n = 1,\ldots,N$ **do**
    repeat
      $\nu_n^k = \tau_n^k + \lambda_n$;
      Solve (21) to obtain p_n^k;
      if $\sum_k p_n^k > P_n^{\max}$ **then**
        increase λ_n;
      else
        decrease λ_n;
    until $\lambda_n\left|\sum_k p_n^k - P_n^{\max}\right|/P_n^{\max} < \epsilon$
  Calculate $\mathbf{R}_n^k$ with (5) $\forall n,k$;
  Calculate $\mathbf{W}_n^k$ with (17) $\forall n,k$;
  for $n = 1,\ldots,N$ **do**
    for $k = 1,\ldots,K$ **do**
      repeat
        Calculate $\mathbf{T}_n^k$ with (25);
        if $\mathrm{tr}\{\mathbf{T}_n^k(\mathbf{T}_n^k)^{\mathrm{H}}\} > p_n^k$ **then**
          increase μ_n^k;
        else
          decrease μ_n^k;
      until $\left|\mathrm{tr}\{\mathbf{T}_n^k(\mathbf{T}_n^k)^{\mathrm{H}}\} - p_n^k\right|/p_n^k < \epsilon$
until *until convergence*

IV. Simulation results

A. Downstream ADSL

The first of our simulation scenarios is the ADSL downstream near-far scenario shown in Fig. 4. There are two users, each with two transceivers (two direct modes), i.e. $A_n = 2 \ \forall n$. In this scenario the user with the short lines has to avoid excessive interference to the user with the long lines. The channel gains are calculated according to the model of [35]. The system has cables of 0.5 mm (AWG 24) and an SNR gap of 12.8 dB. Carrier spacing is given by $\Delta_f = 4.3125$ kHz and the symbol rate is 4 KHz. Each user has a total power constraint of 23.4 dBm. For each line, noise model ANSI A is adopted.

We simulate a MIMO IC, i.e. $\mathcal{T}_n = \mathcal{R}_n = \{n\},\ n = 1,2$ with both synchronous and asynchronous transmission. For the latter, we use an offset $\eta = 0.5$. The two rate regions found with the GF-WMMSE-GDSB can be found in Fig. 5. We notice that the asynchronous transmission incurs a rate loss of about

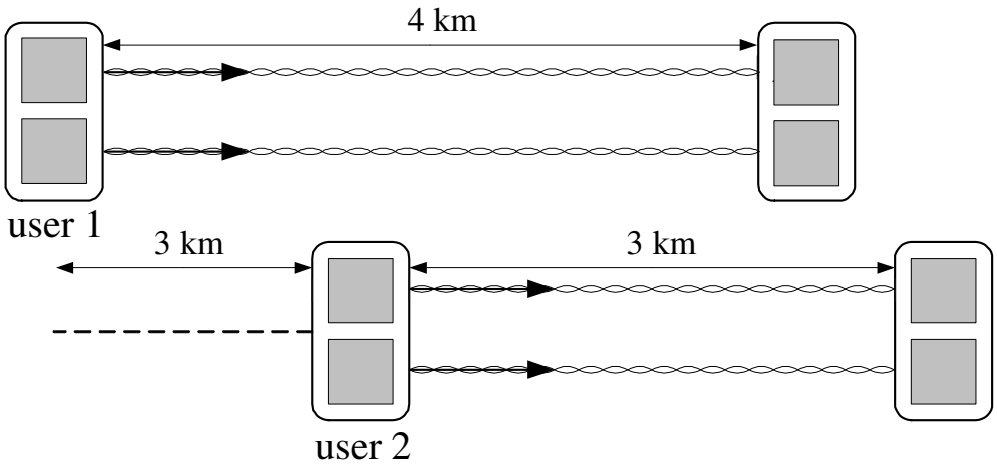

Fig. 4. Downstream ADSL scenario.

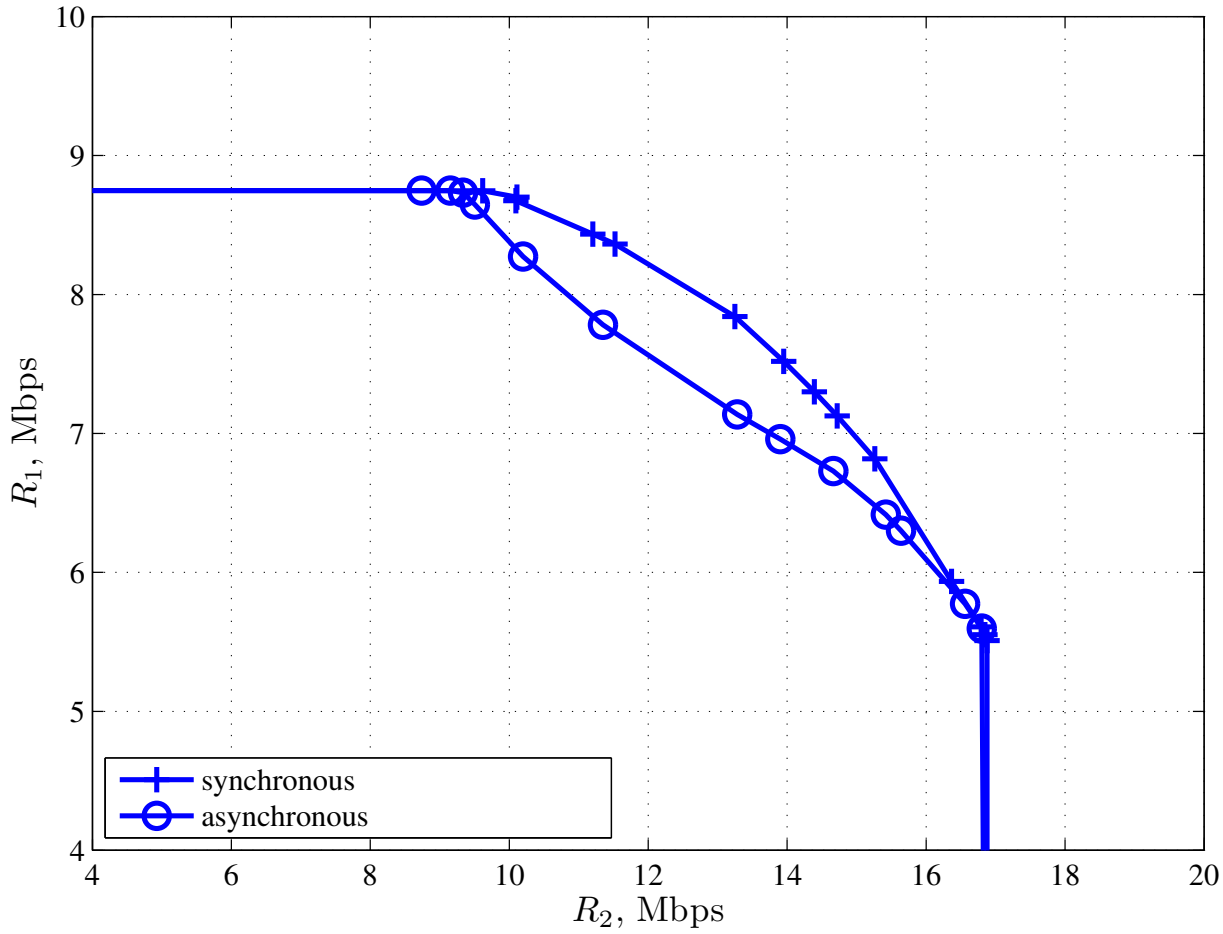

Fig. 5. Rate region for the downstream ADSL scenario.

$10 - 20$ %.

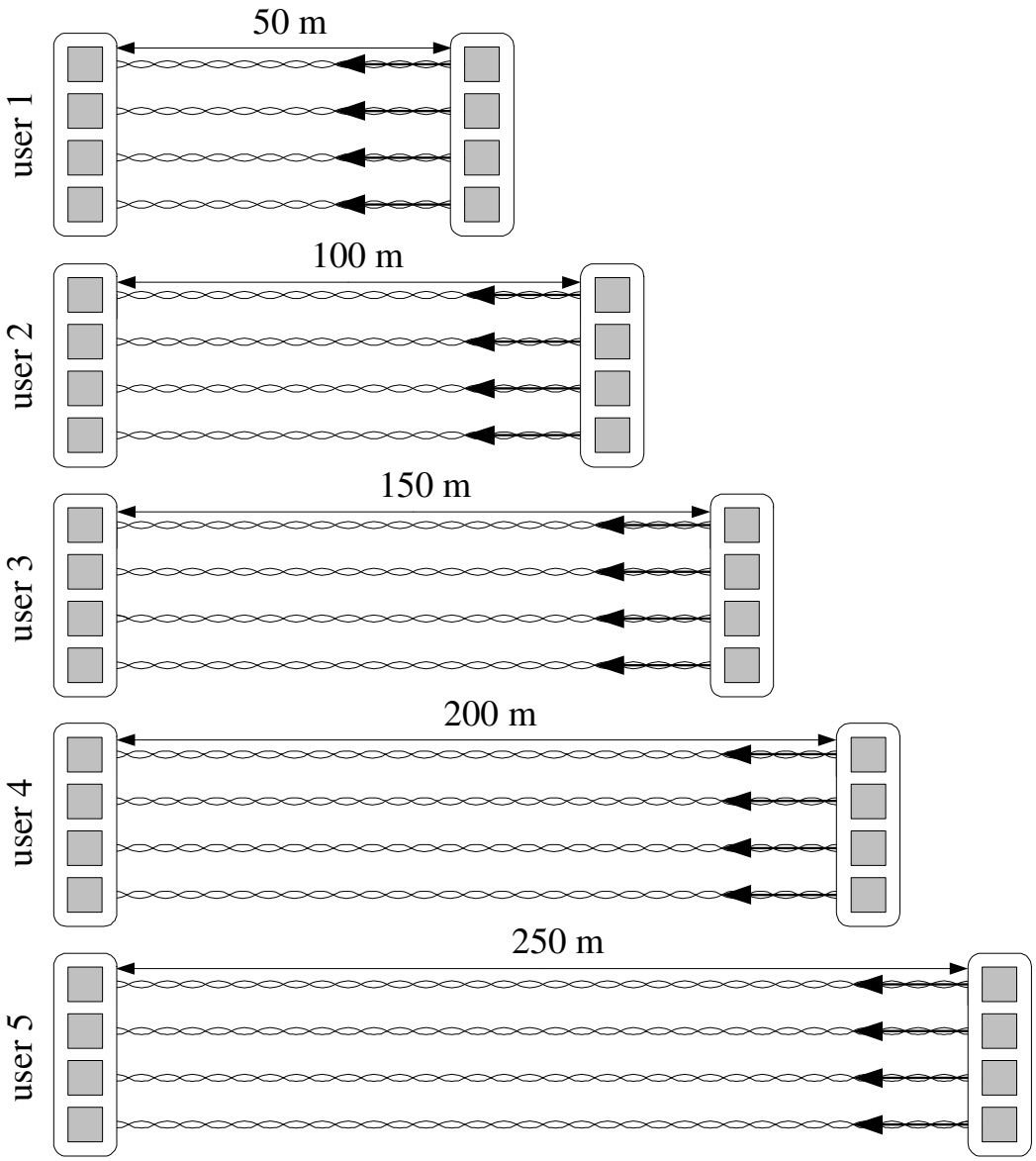

Fig. 6. Upstream G.fast scenario.

B. Upstream G.fast

The second situation is the upstream G.fast scenario depicted in Fig. 6. Here, there are 5 users, each with 4 transceivers (4 direct modes). We use a maximum power of 4 dBm for each transceiver, the SNR gap is set to 6 dB, carrier spacing is 51.75 KHz and the symbol rate is 48 KHz. The bandwidth of the G.fast standard starts at 2.2 MHz and goes up to 106 MHz—there is a total of $K = 2047$ tones. For this scenario, we used measured channels.

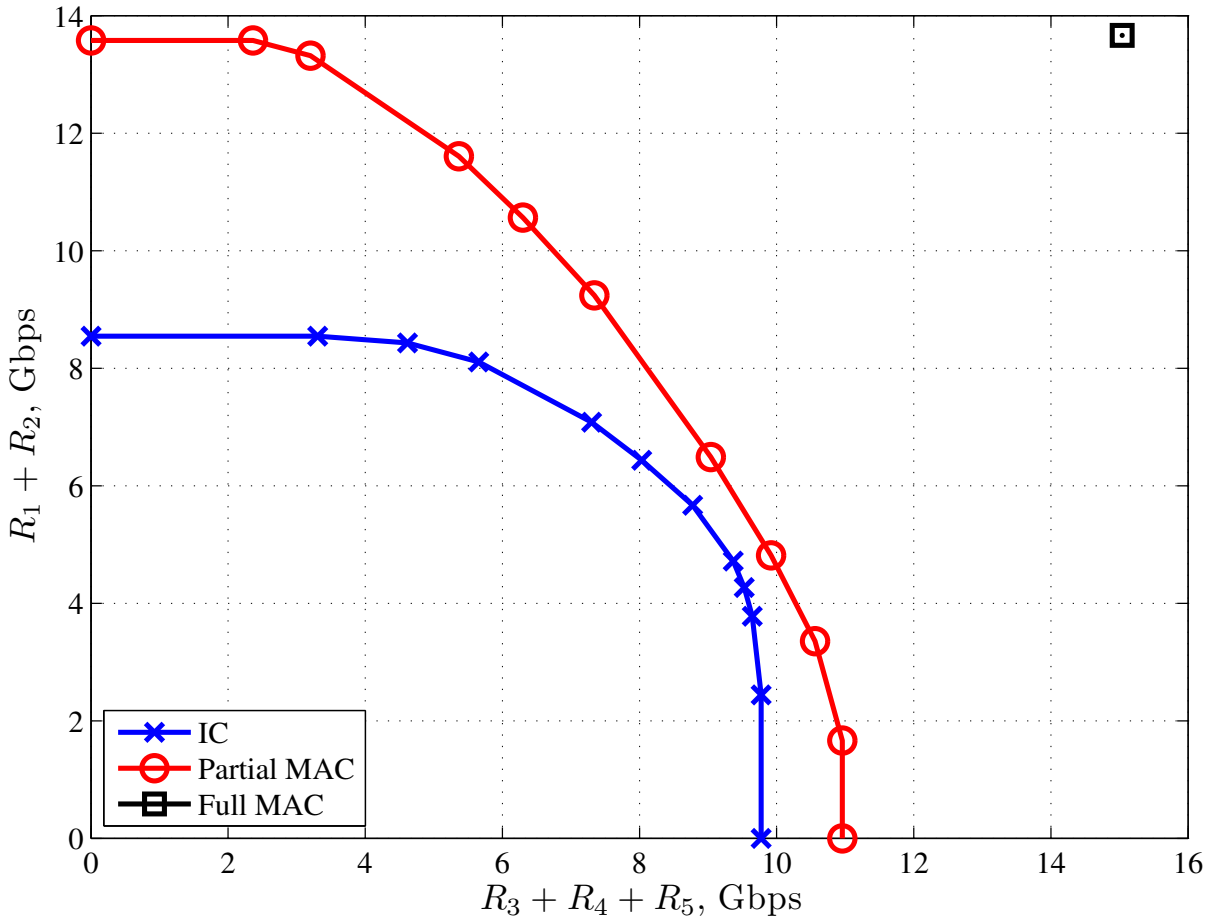

Fig. 7. Rate region for the upstream G.fast scenario.

For this experiment, we are interested in assessing the impact of added coordination on the receiver side. There is already some coordination on the transmitter side between the different transceivers of the users. We first simulate a MIMO IC, i.e. the situation when there is only one user in every group on both sides of the network. As mentioned before, this is the situation with the minimum amount of inter-user coordination. We further simulate a Partial MAC, where there are three groups on the receiver side, as follows: $\mathcal{R}_1 = \{1, 2\}$, $\mathcal{R}_2 = \{3\}$ and $\mathcal{R}_1 = \{4, 5\}$. This grouping could for example indicate that the grouped users belong to the same service provider. Another possibility would be a sub-loop unbundled scenario, where it is typical to have only a fraction of the end-customer on the signal coordination group. Lastly, we simulate a full MAC, i.e. with full receiver coordination. In this case, there is one group on the receiver side, i.e. $\mathcal{R}_1 = \{1, 2, 3, 4, 5\}$. In Fig. 7, we depict the three respective rate regions. It should come

as no surprise that more coordination translates into larger data rates. We remark that signal coordination over all the lines provides a substantial improvement compared to partial signal coordination.

V. Conclusion

The resource allocation problem in a multi-user environment has been a popular research topic in the past decade. We notice, however, that the majority of the papers on the subject make strong assumptions about the network infrastructure. For example, many papers assume that coordination is only available on the service provider side or that all users have only one transceiver. In this paper, we propose a general framework and algorithm for the weighted rate maximization problem in a multi-user system. Our framework includes systems with any number of users, any number of transceivers, any number of tones, any kind of coordination on both the transmitter and on the receiver sides, and synchronous or asynchronous transmission. In what concerns the coordination, we construct channel matrices from all transmitters to all receivers that have as building blocks the channel matrices relating to the MIMO IC and that depend on the kind of coordination on the transmitter and receiver sides. The algorithm divides the problem into two parts, one dealing with the spectrum coordination (i.e. power allocation) and the other dealing with signal coordination (i.e. the spatial separation typical of MIMO systems).

Some extensions that would be interesting to explore would be per-transceiver power constraints and non-linear receivers. Per-transceiver power constraints are more realistic than the per-user power constraints used in this paper, but they also lead to more complex problems. A solution to this problem is presented for the MIMO IC scenario in [28].

Appendix
Derivation of the ICI matrices

To evaluate the impact of the asynchronous transmission, we focus on one victim and one interferer user, as depicted in Fig. 3. The victim user, without loss of generality denoted as user 1, has its DMT symbol for its ith transceiver denoted by $\mathbf{x}_i \in \mathbb{C}^K$ (unlike the remaining of this paper the subscript does not denote user). The interferer user, without loss of generality denoted as user 2, has its DMT symbol denoted by $\mathbf{u}_i$. There are a total of $U+V$ transceivers in the network, where U is the number of transceivers for user 1 and V for user 2. We number them sequentially. For example, if each of the users has two transceivers, we refer to the DMT symbols as $\mathbf{x}_1$,$\mathbf{x}_2$, $\mathbf{u}_3$ and $\mathbf{u}_4$. Symbols $\mathbf{u}_{i,(1)}$ and $\mathbf{u}_{i,(2)}$, $i = U+1, \ldots, U+V$, interfere with the reception of user 1, where the bracketed subscripts denote time. Without loss of generality, we consider time instants (1) and (2).

Before proceeding, we define some variables. We assume that the CP, whose length is given by $L_{\rm cp}$, is longer than both the direct and crosstalk channel impulse response. The delay between the transmission of the two users is given by η, $0 \leq \eta \leq 1$. We define $\mathbf{F}$ and $\mathbf{F}^{\rm H} \in \mathbb{C}^{K\times K}$ as the DFT and IDFT matrices, respectively. The matrices

$$\widetilde{\mathbf{C}} = \left[\begin{array}{c:c} \mathbf{0}_{K\times L_{\rm cp}} & \mathbf{I}_K \end{array}\right] \tag{27}$$

and

$$\mathbf{C} = \left[\begin{array}{c:c} \mathbf{0}_{L_{\rm cp}\times(K-L_{\rm cp})} & \mathbf{I}_{L_{\rm cp}} \\ \hdashline \multicolumn{2}{c}{\mathbf{I}_K} \end{array}\right],$$

where $\widetilde{\mathbf{C}} \in \mathbb{R}^{K\times(K+L_{\rm cp})}$ and $\mathbf{C} \in \mathbb{R}^{(K+L_{\rm cp})\times K}$, respectively remove and insert the CP. The matrices $\mathbf{S}_{(1)}$ and $\mathbf{S}_{(2)}$ capture the effect of the time offset. Define $\omega = \lfloor \eta(K+L_{\rm cp}) \rfloor$ as the number of samples in delay. These matrices are given by

$$\mathbf{S}_{(1)} = \left[\begin{array}{c:c} \mathbf{0}_{(K+L_{\rm cp}-\omega)\times\omega} & \mathbf{I}_{(K+L_{\rm cp}-\omega)} \\ \hdashline \multicolumn{2}{c}{\mathbf{0}_{\omega\times(K+L_{\rm cp})}} \end{array}\right] \tag{28}$$

and

$$\mathbf{S}_{(2)} = \left[\begin{array}{c:c} \mathbf{0}_{(K+L_{\mathrm{cp}}-\omega)\times(K+L_{\mathrm{cp}})} & \\ \hdashline \mathbf{I}_{\omega} & \mathbf{0}_{\omega\times(K+L_{\mathrm{cp}}-\omega)} \end{array}\right]. \tag{29}$$

Here $\mathbf{S}_{(1)},\ \mathbf{S}_{(2)} \in \mathbb{R}^{(K+L_{\mathrm{cp}})\times(K+L_{\mathrm{cp}})}$. If η is equal to zero or one, then the system is synchronized and $\mathbf{S}_{(1)} = \mathbf{I}_{(K+L_{\mathrm{cp}})}$ and $\mathbf{S}_{(2)} = \mathbf{0}_{(K+L_{\mathrm{cp}})\times(K+L_{\mathrm{cp}})}$ or vice-versa.

The signal on the receiver side for user 1 is given by

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{A}\mathbf{u}_{(1)} + \mathbf{B}\mathbf{u}_{(2)} + \mathbf{z}. \tag{30}$$

Here $\mathbf{r}, \mathbf{z} \in \mathbb{C}^{UK}$ are respectively the received signal and background noise for user 1; the vector $\mathbf{x} = [\mathbf{x}_1^{\mathrm{T}} \dots \mathbf{x}_U^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{C}^{UK}$ is the concatenation of the DMT symbols from all transceivers from user 1—the vectors $\mathbf{u}_{(1)}, \mathbf{u}_{(2)} \in \mathbb{C}^{VK}$ are defined similarly; the matrix $\mathbf{H} \in \mathbb{C}^{KU\times KU}$ has a block structure where the (i,j)-th block, $i,j = 1,\dots,U$, is given by $\mathbf{F}\widetilde{\mathbf{C}}\mathbf{G}_{i,j}\mathbf{C}\mathbf{F}^{\mathrm{H}}$; $\mathbf{G}_{i,j} \in \mathbb{C}^{(K+L_{\mathrm{cp}})\times(K+L_{\mathrm{cp}})}$ is a Toeplitz matrix with first column $\left[\mathbf{g}_{i,j}^{\mathrm{T}}\ \mathbf{0}_{1\times(K+L_{\mathrm{cp}}-L)}\right]^{\mathrm{T}}$ and first row $\left[g_{i,j}(1)\ \mathbf{0}_{1\times(K+L_{\mathrm{cp}}-1)}\right]$, where $\mathbf{g}_{i,j} \in \mathbb{C}^{L}$ is the L-tap channel impulse response from transmitter j to receiver i of user n and is considered constant in time; $\mathbf{A}, \mathbf{B} \in \mathbb{C}^{KU\times KV}$ have a block structure, where the $(i, j-U)$-th block, $i = 1,\dots,U$ and $j = U+1,\dots,U+V$, are given respectively as $\mathbf{F}\widetilde{\mathbf{C}}\mathbf{G}_{i,j}\mathbf{S}_{(1)}\mathbf{C}\mathbf{F}^{\mathrm{H}}$ and $\mathbf{F}\widetilde{\mathbf{C}}\mathbf{G}_{i,j}\mathbf{S}_{(2)}\mathbf{C}\mathbf{F}^{\mathrm{H}}$.

Because $L \leq L_{\mathrm{cp}}$, the process of insertion and removal of the CP creates a square circulant matrix, which is in turn diagonalized by the DFT and IDFT operations. Hence the (i,j)th block of $\mathbf{H}$, where $i,j = 1,\dots,U$, is given by $\mathbf{F}\widetilde{\mathbf{C}}\mathbf{G}_{i,j}\mathbf{C}\mathbf{F}^{\mathrm{H}} = \mathrm{diag}\{\mathbf{h}_{i,j}\}$, where $\mathbf{h}_{i,j} \in \mathbb{C}^{K}$ is the corresponding channel frequency response. Because of the offset represented by the matrices $\mathbf{S}_{(1)}$ and $\mathbf{S}_{(2)}$, the operation $\widetilde{\mathbf{C}}\mathbf{G}_{i,j}\mathbf{S}_{(1)}\mathbf{C}$ and $\widetilde{\mathbf{C}}\mathbf{G}_{i,j}\mathbf{S}_{(2)}\mathbf{C}$ fail to produce a circulant matrix, and therein lies the effect of the asynchronicity.

The next step is to write a per-tone version of (30):

$$\mathbf{r}^k = \mathbf{H}^k\mathbf{x}^k + \sum_{q\in\mathcal{K}} \mathbf{A}^{k,q}\mathbf{u}_{(1)}^q + \mathbf{B}^{k,q}\mathbf{u}_{(2)}^q + \mathbf{z}^k. \tag{31}$$

Here $\mathbf{x}^k,\ \mathbf{z}^k \in \mathbb{C}^U$, $\mathbf{u}_{(1)},\ \mathbf{u}_{(2)} \in \mathbb{C}^V$. We define $\mathbf{x}^k = [x_1^k\ \ \cdots\ \ x_U^k]^{\mathrm{T}}$. Other vectors are defined similarly. We also have $\mathbf{H}^k \in \mathbb{C}^{U\times U}$ and $\mathbf{A}^{k,q}, \mathbf{B}^{k,q} \in \mathbb{C}^{U\times V}$. The matrix $\mathbf{H}^k$ is obtained from $\mathbf{H}$ by substituting

the block (i,j) by the (k,k) element of block (i,j), with $i,j=1,\ldots,U$. The matrices $\mathbf{A}^{k,q}$ and $\mathbf{B}^{k,q}$ are obtained by substituting the block $(i,j-U)$ by the (k,q) element of block $(i,j-U)$, with $i=1,\ldots,U$ and $j=U+1,\ldots,U+V$.

Substituting $\mathbf{u}^q_{(1)}=\mathbf{T}^q_u\mathbf{x}^q_u$ and $\mathbf{u}^q_{(2)}=\mathbf{T}^q_u\bar{\mathbf{x}}^q_u$, we arrive at (23).

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