Iterative Feedback Tuning Applied to Robot Joint Controllers

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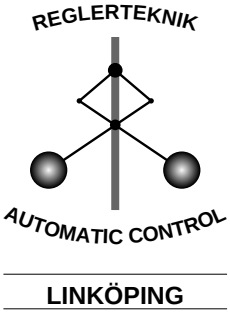

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ITERATIVE FEEDBACK TUNING APPLIED TO ROBOT JOINT CONTROLLERS

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Abstract: Tuning of robot joint controllers using iterative feedback tuning (IFT) is considered. Using a simulation model of a two-link robot arm the applicability of IFT is investigated for a nonlinear and multivariable system. Due to the strong cross-coupling between the two links a decoupling controller is needed in order to obtain satisfactory results. An approach for obtaining a decoupling controller is investigated. It is also shown how IFT can be used to tune the controller parameters for a particular trajectory of the arm.

Keywords: Iterative methods, robotic manipulators, decoupling, multivariable, nonlinear.

1. INTRODUCTION

Iterative feedback tuning (IFT), as initially proposed in Hjalmarsson *et al.* (1994*a*), is a method for tuning of the parameters in feedback control systems without needing an explicit model of the system to be controlled. In the IFT method one iteration in the tuning procedure corresponds to one updating of the controller coefficients. The method has been successfully applied to e.g. mechanical systems Hjalmarsson *et al.* (1995), where control of a flexible transmission system is considered, as well as to problems in chemical process control Hjalmarsson *et al.* (1998).

In the original paper Hjalmarsson *et al.* (1994*a*) control of scalar linear time invariant systems was discussed, but later it has been shown that the method is applicable and useful also for control of systems containing nonlinearities Hjalmarsson (1998) and multivariable systems Hjalmarsson and Birkeland (1998). In this paper IFT will be used for tuning of the PD-loops in a two-link robot arm, i.e. a system that is both nonlinear and multivariable. The paper summarizes some of the experiences of the IFT method that are presented in Collignon and Rousseaux (1998).

The paper is organized as follows. In Section 2 a brief introduction to the method of iterative feedback tuning is given, and in Section 3 a description is given of the two-link manipulator that will be studied. In Section 4 the IFT method is used to tune two separate control loops, while PD-controllers in combination with fixed static decoupling are used in Section 5. In Section 6 both the decoupling controllers and the PD-controllers are tuned using IFT. In Section 7 the method is tested in a situation where a more demanding trajectory is applied to the robot. Finally some conclusions are given in Section 8.

2. ITERATIVE FEEDBACK TUNING

In this section a brief introduction to the IFT method for a linear SISO case is given. A thorough introduction can be found in Hjalmarsson *et al.* (1994*a*) and Hjalmarsson *et al.* (1994*b*). Even though the system to be studied here is nonlinear and multivariable the presentation of the method is restricted to the linear SISO case. The different aspects of the application of the method to nonlinear and multivariable systems are discussed in Hjalmarsson (1998) and Hjalmarsson and Birkeland (1998)

Consider therefore the discrete time system

$$y = G_0 u \tag{1}$$

which is controlled using the feedback controller

$$u = C(\rho)(r - y) \tag{2}$$

where ρ denotes the vector of controller parameters. For simplicity a disturbance free deterministic case is considered here. Omitting the time argument the control objective is expressed as minimization of the criterion

$$J(\rho) = \frac{1}{2}\sum_{t=1}^{N}(\tilde{y}^2(\rho) + \lambda u^2(\rho)) \tag{3}$$

where the signal $\tilde{y}(\rho)$ denotes the deviation of the actual output $y(\rho)$ from a desired output y_d, i.e.

$$\tilde{y}(\rho) = y(\rho) - y_d \tag{4}$$

The desired output is specified as the output obtained when r is fed through a reference system T_d which means

$$y_d = T_d r \tag{5}$$

In the criterion (3) both frequency domain and time domain weighting can be used but these possibilities are omitted here.

The key idea in IFT is that it is possible to generate (an estimate of) the gradient of the criterion $J(\rho)$ without using any model of the open loop system. The gradient is instead formed purely using input/output-data from the closed loop system. Differentiating $J(\rho)$ in equation (3) gives

$$J^{'}(\rho) = \sum_{t=1}^{N}(\tilde{y}(\rho)y^{'}(\rho) + \lambda u(\rho)u^{'}(\rho)) \tag{6}$$

Straightforward differentiation of $\tilde{y}(\rho)$ gives

$$\tilde{y}^{'}(\rho) = y^{'}(\rho) = \frac{C^{'}}{C}T_0(r - T_0 r) \tag{7}$$

where T_0 denotes the transfer operator of the closed loop system, i.e.

$$T_0 = \frac{CG_0}{1 + CG_0} \tag{8}$$

Differentiation of u gives

$$u^{'}(\rho) = \frac{C^{'}}{C}G_u(r - T_0 r) \tag{9}$$

where G_u is the transfer operator from reference signal to control signal

$$G_u = \frac{C}{1 + CG_0} \tag{10}$$

Furthermore $C^{'}$ denotes the vector obtained when the controller is differentiated with respect to the controller coefficients in ρ.

This leads to the formulation of the following two-stage procedure for generating $\tilde{y}^{'}(\rho)$, which is the key idea in IFT: Let $r_1^i = r$ be the reference signal in the first experiment of iteration i. Apply this signal to the control system and collect N samples of the input signal u_1^i and output signal y_1^i from the closed loop system. Then let $r_2^i = r - y_1^i$ be used as reference signal in a second experiment, which yields the output y_2^i. Filtering of y_2^i through the filter $C^{'}/C$ gives the derivative $\tilde{y}^{'}(\rho)$. Denoting the experimentally computed derivatives $\hat{y}^{'}(\rho)$ and $\hat{u}^{'}(\rho)$ respectively the approximate gradient can be expressed

$$\hat{J}^{'}(\rho) = \sum_{t=1}^{N}(\tilde{y}(\rho)\hat{y}^{'}(\rho) + \lambda u(\rho)\hat{u}^{'}(\rho)) \tag{11}$$

With the gradient available it is possible to form an iterative procedure for updating the controller parameters

$$\rho_{i+1} = \rho_i - \gamma_i R_i^{-1}\hat{J}^{'}(\rho_i) \tag{12}$$

where the matrix R_i is used to modify the search direction and the scalar γ_i is used to adjust the step size.

3. ROBOT MODEL

The system that shall be considered in the paper is the two-link robot arm with revolute joints shown in Figure 1.

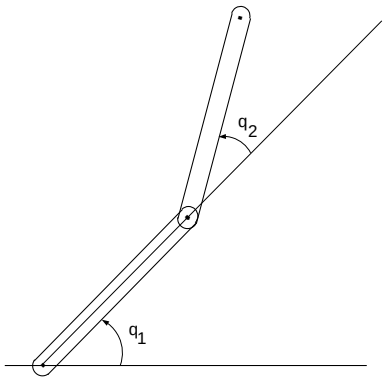

Fig. 1. Two-link arm

Starting from the general model of the motion of the robot

$$M(q)\ddot{q} + C_c(q,\dot{q})\dot{q} + G(q) + \tau_F(q) = \tau \qquad (13)$$

it shall be assumed that both links have equal length L, equal mass m and that the center of mass is located at the center of each link. The torque τ itself will be used as input signal, which means that no actuators are included in the model.

Since the mathematical model will contain several trigonometric functions the notations

$$s_1 = \sin q_1 \quad s_2 = \sin q_2 \quad s_2 = \sin q_2 \quad c_2 = \cos q_2 \qquad (14)$$

and

$$s_{12} = \sin(q_1 + q2) \qquad c_{12} = \cos(q_1 + q_2) \qquad (15)$$

are used. Using the results presented in Spong and Vidyasagar (1989) (Section 6.4) the following expressions for the inertia matrix, the Coriolis and centrifugal forces, the gravitational, and friction forces are obtained. The inertia matrix is given by

$$M(q) = mL^2 \begin{pmatrix} \frac{5}{3} + c_2 & \frac{1}{3} + \frac{1}{2}c_2 \\ \frac{1}{3} + \frac{1}{2}c_2 & \frac{1}{3} \end{pmatrix} \qquad (16)$$

while the Coriolis and centrifugal forces are given by

$$C_c(q,\dot{q})\dot{q} = -\frac{mL^2 s_2}{2} \begin{pmatrix} \dot{q}_2^2 + 2\dot{q}_1\dot{q}_2 \\ -\dot{q}_1^2 \end{pmatrix} \qquad (17)$$

and finally the gravitational forces are given by

$$G(q) = \frac{mgL}{2} \begin{pmatrix} 3c_1 + c_{12} \\ c_{12} \end{pmatrix} \qquad (18)$$

Finally the term $\tau_F(q)$ represents the friction forces acting on the arm.

The robot model and the control system are implemented in Simulink using the state variables

$$x_1 = q_1 \quad x_2 = \dot{q}_1 \quad x_3 = q_2 \quad x_4 = \dot{q}_2 \qquad (19)$$

and the state equations are based on the straightforward formulation

$$\ddot{q} = M^{-1}(q)(\tau - C_c(q,\dot{q})\dot{q} - G(q) - \tau_F(q)) \qquad (20)$$

In the simulations the numerical values $m = 10$ and $L = 1$ are used. The controllers are working in discrete time using sampling frequency 100 Hz.

4. TUNING OF SINGLE LOOP PD-CONTROLLERS

Initially the control system will be treated as two separate SISO problems, which means that the control system structure in Figure 2 will be considered.

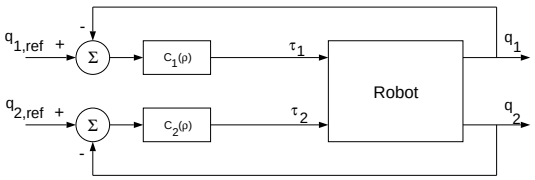

Fig. 2. PD-controller structure

The initial controllers are chosen as

$$C_1(\rho) = C_2(\rho) = 1100 - 1000z^{-1} \qquad (21)$$

and the reference signals are generated using the function `jtraj` in the ROBOTICS TOOLBOX Corke (1996). The reference signals for the two joints are shown in Figure 3.

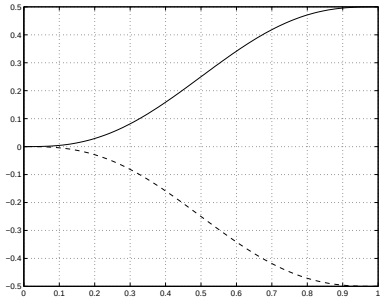

Fig. 3. Reference signals (radians). Solid line: $q_{1,ref}$. Dashed line: $q_{2,ref}$

The reference system is chosen as $T_d(z) = z^{-1}$ for both channels, and in the first experiment the weight on both output signals are 10^3 while the weight on the control signals are zero. The size of the weight is not important here since no weight is put on the control signal, and 10^3 is chosen just in order to obtain a convenient level of the criterion value.

During the first four iterations the value of the criterion decreases, as can be seen in Table 1, and the output signals approach the reference signals. After the fourth iteration, however, high frequency oscillations, with frequency about half of the Nyquist frequency, occur, in particular in the control signals.

Table 1. Tuning of C_1 and C_2 with $\lambda = 0$.

Iteration	Criterion	Step size
0	10.1	
1	7.4	0.2
2	5.3	0.2
3	4.0	0.2
4	3.2	0.2

On obvious way to handle this problem is to introduce weights on the control signals, and in the second experiment the control signal weights are 10^{-2} on both control signals. Starting from the same initial controllers as in the previous case the value of the criterion continues to decrease during the iterations, as can be seen in Table 2, and no high frequency oscillations occur. The

resulting controllers that are obtained after the iterations have converged however give too poor performance to be really useful.

Table 2. Tuning of C_1 and C_2 with $\lambda = 0.01$

Iteration	Criterion	Step size
0	11.8	
1	6.4	1.0
2	5.7	1.0
3	5.6	1.0

Since single loop control has been applied to a multivariable system it is likely that the oscillations are caused by coupling effects in the system. This can also be found by carrying out an approximate linear analysis where the system is considered around zero angle for both joints and neglect the Coriolis, centrifugal, and friction forces. The system can then approximately be described as

$$M_0 \ddot{q}(t) = \tau(t) \tag{22}$$

where

$$M_0 = M(0) = mL^2 \begin{pmatrix} 8/3 & 5/6 \\ 5/6 & 1/3 \end{pmatrix} \tag{23}$$

Laplace transforms give

$$Q(s) = \frac{M_0^{-1}}{s^2} T(s) \tag{24}$$

where $T(s)$ is the Laplace transform of the torque. Transforming this linear system to discrete time and applying the PD-controllers obtained in the experiment when the oscillations occur it is found that the closed loop system has poles close to the unit circle, in the vicinity of the imaginary axis, i.e. the poles will result in oscillations with around half the Nyquist frequency. This result coincides well with the observations above. The conclusion becomes that it is not possible to treat the problem as two single loop problems and that the coupling has to be taken into consideration.

5. PD-TUNING WITH FIXED DECOUPLING

In a first attempt to handle the cross-coupling in the system the approximate linear model in equation (24) shall be used. From this model it is clear that, in the linear case, it is possible to obtain exact decoupling by using a static decoupling defined by the inertia matrix itself in zero position, i.e. the matrix M_0. Practically this implies introducing a new input signal $\bar{\tau}(t)$ and letting the actual torque signal be computed as

$$\tau(t) = M_0 \bar{\tau}(t) \tag{25}$$

where the signals in $\bar{\tau}(t)$ are then generated by PD-controllers as in the previous section. The structure of the control system is given in Figure 4. By letting factors from the decoupling controller be included in C_1 and C_2 only two decoupling constants remain. At zero position the theoretical values of the decoupling constants are $K_1 = 5/16 = 0.3125$ and $K_2 = 2.5$ respectively.

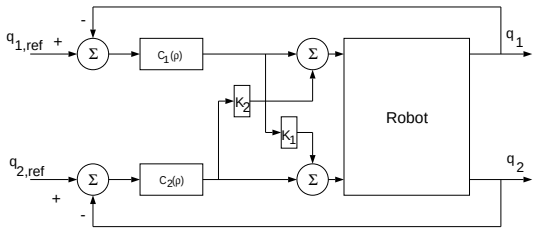

Fig. 4. PD-controller structure with decoupling

In Table 3 the results from the tuning procedure using the fixed decoupling are shown. The PD-controllers are initiated according to equation (21) and the weight is 10^3 on the output signals and zero on the control signals. A large improvement is obtained by the introduction of the decoupling.

Table 3. PD-tuning using theoretical K_1 and K_2.

Iteration no	Criterion	Step size
0	25.3	
1	6.7	1
2	1.9	1
3	0.5	1
4	0.1	1
5	0.02	1
6	0.002	1
7	0.000	1

6. TUNING OF PD- AND DECOUPLING CONTROLLERS

In the previous section the usefulness of the decoupling controllers was shown, but in order to compute the decoupling a model of the robot was used. Since IFT is meant to be a model free method it is not realistic to assume that such a model is available. It is therefore natural to consider the problem of tuning also the decoupling controllers. In the control system there are then four controllers to update, two decoupling controllers and two PD-controllers. There are then several possible ways to organize the tuning of the controllers, and the strategy that has been found useful here is to first tune the decoupling controllers while keeping C_1 and C_2 fixed and then tune the PD-controllers while keeping K_1 and K_2 fixed.

When tuning the decoupling constant K_1 the aim is that a change in $\bar{\tau}_1(t)$ shall have as small influence as possible on the second joint angle $q_2(t)$. Therefore the reference signal shown in Figure 3 is applied to the first joint while the second reference signal is zero. In the criterion the weight on the second output $q_2(t)$ is set to 10^3 while the weight on $q_1(t)$ is set to zero. The PD-controllers are initiated according to equation (21) while $K_1 = K_2 = 0.01$. In the tuning

procedure only K_1 is updated in each iteration. It turns out that the tuning of K_1 is very quick and that the iterations have converged after only two steps. The results are summarized in Table 4. The final value of the decoupling constant is $K_1 = 0.313$ which is the same as obtained using the approximate linear model of the robot arm. With this decoupling the second joint angle satisfies $\mid q_2(t) \mid < 1.8 \cdot 10^{-4}$ during the movement of $q_1(t)$.

Table 4. Tuning of K_1

Iteration no	Criterion	Step size
0	2.1	
1	0.009	1
2	0.000	1

The conditions when tuning the second decoupling constant K_2 were the opposite compared to the tuning of K_1. This means that the reference signal for joint one was zero while a reference signal of the type shown in Figure 3 was applied to the second joint. A weight of 10^3 was put on $q_1(t)$ while the weight on $q_2(t)$ was zero. The same initial values of the controllers as in the case above were used with the exception of K_1, where the result from the tuning was used. Also now the decoupling constant converged quickly and the evolution of the criterion is shown in Table 5. It is however noticeable that the step size had to be reduced in the first step in order to achieve good behavior. The tuning resulted in the final value $K_2 = 2.416$ which is slightly different from the theoretical value at zero angle. This is however logical since the second joint changes from zero to -0.5 during the movement. In this case the remaining coupling is larger that in the previous case and $\mid q_1(t) \mid < 0.01$ during the movement of $q_2(t)$.

Table 5. Tuning of K_2

Iteration no	Criterion	Step size
0	2.5	
1	0.011	0.2
2	0.010	1

After having tuned the decoupling controllers it is straightforward to tune the PD-controllers keeping the decoupling controllers fixed. The situation is similar to the case when the theoretically computed decoupling coefficients were used, with only slight difference in the numerical values. For completeness the results are shown in Table 6.

7. A MORE DEMANDING TRAJECTORY

In the previous experiments smooth and fairly slow reference trajectories were used. Since the movement of the second joint was rather small, between 0 and 0.5 radians, the nonlinear effects

Table 6. PD-tuning using tuned K_1 and K_2.

Iteration no	Criterion	Step size
0	23.3	
1	6.2	1
2	1.7	1
3	0.42	1
4	0.089	1
5	0.015	1
6	0.001	1
7	0.000	1

due to the the change of the inertia matrix were not that big. In order to evaluate the IFT method in a more demanding situation the simulations in this section shall be based on a trajectory where $q_1(t)$ goes from 0 to $\pi/2$ radians and $q_2(t)$ goes from 0 to $-\pi$ radians. Furthermore it is required that the movement is done during 0.3 seconds, i.e. more than three times faster than in the previous experiments. The reference trajectories are shown in Figure 5. In these experiments the sampling frequency is increased to 1000 Hz.

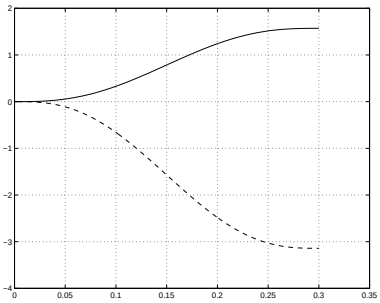

Fig. 5. Reference signals (radians). Solid line: $q_{1,ref}$. Dashed line: $q_{2,ref}$

The procedure will be the same as before, i.e. first the decoupling constants are tuned and then the PD-controllers. The results of the tuning of the decoupling controllers are given in Table 7 and Table 8. The tuning of K_1 converges in one step giving $K_1 = 0.313$. The second joint angle satisfies $\mid q_1(t) \mid < 9 \cdot 10^{-4}$. The tuning of K_2 is somewhat slower, and after the third iteration no improvement of the criterion is achieved. The final value $K_2 = 2.343$ gives the deviation $\mid q_1(t) \mid < 0.06$ during the movement of $q_2(t)$.

Table 7. Tuning of K_1

Iteration no	Criterion	Step size
0	20.1	
1	0.000	1

Table 8. Tuning of K_2

Iteration no	Criterion	Step size
0	33,4	
1	2.7	1
2	0.18	1
3	0.12	1

Using the values of K_1 and K_2 the next step is to tune the PD-controllers. Using the initial

values in equation (21) the results shown in Table 9 are obtained. Using the controllers obtained after iteration 6 the control signal exhibits high frequency oscillations during the last part of the movement.

Table 9. PD-tuning using tuned K_1 and K_2.

Iteration no	Criterion	Step size
0	1679	
1	488	1
2	182	1
3	61	1
4	18	1
5	4.0	1
6	2.5	0.2

Further improvements can however be achieved by re-tuning the decoupling coefficients. Using data from normal movements of the robot-arm first K_2 and then K_1 are re-tuned. K_2 is first reduced to 1.601 giving the value 0.54 of the criterion. In the second tuning K_1 is reduced to 0.197 which further reduces the criterion to 0.10. The error signals obtained after the final tuning are shown i Figure 6. Noticeable is that the trajectory used here resulted in different controller coefficients that for the previous trajectory, and this is of course a consequence of the nonlinear character of the system. An interpretation of this behavior is that the IFT method produces values of the controller parameters that are optimized for a particular trajectory. This is a feature that might be useful in some cases.

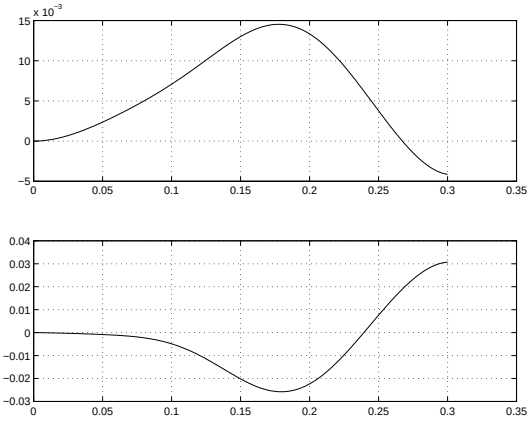

Fig. 6. Error signals (radians) after tuning. Upper figure: $q_{1,ref} - q_1$ Lower figure: $q_{2,ref} - q_2$

8. CONCLUSIONS

This paper has illustrated that IFT is applicable to systems that are both nonlinear and multivariable. Due to the strong cross-coupling in the system it was found necessary to use a decoupling controller, and one possible way to determine a suitable cross-coupling has been tested with good results. The investigations presented above however have several limitations that require further studies. Effects of friction, gravitation, and disturbances have not been considered. The controllers have furthermore been of low order, and the possibilities in IFT to use both time domain and frequency domain weighting have not been utilized. All these factors are subjects for future studies.

ACKNOWLEDGMENTS

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