Iterative Feedback Tuning Applied to a Two-Link Robot Arm

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Abstract

Iterative feedback tuning (IFT) is a applied to a two-link robot arm. Using a simulation model the robot arm the applicability of IFT is investigated for a nonlinear and multivariable system. Due to the strong cross-coupling between the two links a decoupling controller is needed in order to obtain satisfactory results. An approach for obtaining a decoupling controller is investigated. It is also shown how IFT can be used to tune the controller parameters for a particular trajectory of the arm. The influence of friction is also investigated.

Key words: Iterative Feedback Tuning, Robotics

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1 Introduction

Iterative feedback tuning (IFT), as initially proposed in Hjalmarsson et al. (1994), is a method for tuning of the parameters in feedback control systems without needing an explicit model of the system to be controlled. In the IFT method one iteration in the tuning procedure corresponds to one updating of the controller coefficients. The method has been successfully applied to e.g. mechanical systems in Hjalmarsson et al. (1995), where control of a flexible transmission system is considered, as well as to problems in chemical process control in Hjalmarsson et al. (1998).

In the original paper, Hjalmarsson et al. (1994), control of scalar linear time invariant systems was discussed, but later it has been shown that the method is applicable and useful also for control of systems containing nonlinearities in Hjalmarsson (1998) and multivariable systems in Hjalmarsson and Birkeland (1998). In this paper IFT will be used for tuning of the PD-loops in a two-link robot arm, i.e. a system that is both nonlinear and multivariable. The paper summarizes some of the experiences of the IFT method that are presented in Collignon and Rousseaux (1998) and is an extension of Gunnarsson et al. (1999).

The paper is organized as follows. In Section 2 a description is given of the two-link manipulator that will be studied, and in Section 3 a brief introduction to the iterative feedback tuning method is given. In Section 4 the IFT method is used to tune two separate control loops, while PD-controllers in combination with fixed static decoupling are used in Section 5. In Section 6 both the decoupling controllers and the PD-controllers are tuned using IFT. In Section 7 the method is evaluated when a more demanding reference trajectory is used, and in Section 8 some effects of friction are illustrated. Finally some conclusions are given in Section 9.

2 Robot Model

The system that is considered in the paper is the two-link robot arm with rotational joints shown in Figure 1.

Starting from the general model of the motion of the robot

$$M(q)\ddot{q} + C_c(q,\dot{q})\dot{q} + G(q) + \tau_F(q) = \tau \tag{1}$$

it is assumed that both links have length L, equal mass m and that the center of mass is located at the center of each link. The torque τ itself will be used as input signal, which means that no actuators are included in the model.

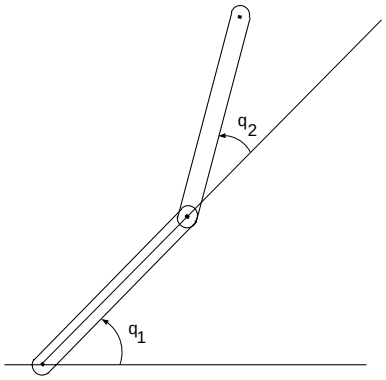

Fig. 1. Two-link arm

Since the mathematical model will contain several trigonometric functions the notations

$$s_1 = \sin q_1 \quad s_2 = \sin q_2 \quad s_2 = \sin q_2 \quad c_2 = \cos q_2 \tag{2}$$

and

$$s_{12} = \sin(q_1 + q_2) \qquad c_{12} = \cos(q_1 + q_2) \tag{3}$$

are used. Using the results presented in Spong and Vidyasagar (1989) (Section 6.4) the following expressions for the inertia matrix, the Coriolis and centrifugal forces, the gravitational, and friction forces are obtained. The inertia matrix is given by

$$M(q) = mL^2 \begin{pmatrix} \frac{5}{3} + c_2 & \frac{1}{3} + \frac{1}{2}c_2 \\ \frac{1}{3} + \frac{1}{2}c_2 & \frac{1}{3} \end{pmatrix} \tag{4}$$

while the Coriolis and centrifugal forces are given by

$$C_c(q, \dot{q})\dot{q} = -\frac{mL^2 s_2}{2} \begin{pmatrix} \dot{q}_2^2 + 2\dot{q}_1\dot{q}_2 \\ -\dot{q}_1^2 \end{pmatrix} \tag{5}$$

and finally the gravitational forces are given by

$$G(q) = \frac{mgL}{2} \begin{pmatrix} 3c_1 + c_{12} \\ c_{12} \end{pmatrix} \tag{6}$$

Finally the term $\tau_F(q)$ represents the friction forces acting on the arm.

The robot model and the control system are implemented in Simulink using the state variables

$$x_1 = q_1 \quad x_2 = \dot{q}_1 \quad x_3 = q_2 \quad x_4 = \dot{q}_2 \tag{7}$$

and the state equations are based on the straightforward formulation

$$\ddot{q} = M^{-1}(q)(\tau - C_c(q,\dot{q})\dot{q} - G(q) - \tau_F(q)) \tag{8}$$

In the simulations the numerical values $m = 10$ and $L = 1$ are used. The controllers are working in discrete time using sampling frequency 100 Hz. In all experiments it is assumed that the motions are carried out in the horizontal plane, which means that no gravitational forces are present. The term $G(q)$ is therefore assumed to be zero. Experiments with gravitation are presented in Collignon and Rousseaux (1998). In the first experiments the manipulator is simulated without friction, and the effects of friction are discussed in Section 8.

3 Iterative Feedback Tuning

Thorough introductions to the iterative feedback tuning (IFT) method can be found in e.g. Hjalmarsson et al. (1994) and Hjalmarsson et al. (1998), and in this section only the main points of the method will be discussed. The aim in IFT is to iteratively update the parameters in a feedback controller

$$\tau = C(\rho)(r - q) \tag{9}$$

where ρ denotes the vector of controller parameters. In this application $C(\rho)$ is a 2×2 discrete time transfer function matrix. The updating is carried out using data collected when the control system is operating in closed loop, and it is based on minimization of a criterion reflecting the control system performance and the control signal magnitude. In this paper the criterion to be minimized is given by

$$J(\rho) = \frac{1}{2}\sum_{t=1}^{N} \tilde{q}_1^2(\rho) + \tilde{q}_2^2(\rho) + \lambda\tau_1^2(\rho) + \lambda\tau_2^2(t) \tag{10}$$

where λ is a positive scalar. The signals $\tilde{q}_i$ denote the difference between the desired and achieved joint angles, i.e.

$$\tilde{q}_i = q_{d,i} - q_i(\rho) \tag{11}$$

The desired output is specified as the output obtained when a reference signal q_{ref} is fed through a reference system T_d, which means

$$q_d = T_d q_{ref} \tag{12}$$

The criterion (10) can be extended by including both both frequency domain and time domain weighting, but these possibilities are omitted here.

Using IFT the regulator parameters are updated as

$$\rho_{i+1} = \rho_i - \gamma_i R_i^{-1} \hat{J}'(\rho_i) \tag{13}$$

where the matrix R_i is used to modify the search direction, the scalar γ_i is used to adjust the step size, and $\hat{J}'(\rho_i)$ is an approximation of the gradient of the criterion J with respect to the regulator parameters. The key idea in IFT is that it is possible to generate (an estimate of) the gradient of the criterion $J(\rho)$ without using any model of the open loop system. The gradient is instead formed purely using input/output-data from the closed loop system in a two-stage procedure: Let $r^{1,i} = q_{ref}$ be the reference signal in the first experiment of iteration i. Apply this signal to the control system and collect N samples of the input signal $\tau^{1,i}$ and output signal $q^{1,i}$ from the closed loop system. The superscripts denote the first stage in the formation of the gradient in iteration i. Then let $r^{2,i} = q_{ref} - q^{1,i}$ be used as reference signal in a second experiment, which yields the output $q^{2,i}$. By appropriate filtering of this signal an approximation of the gradient can be obtained. For a detailed description of the method see e.g. Hjalmarsson et al. (1998).

4 Tuning of Single Loop PD-controllers

Initially the control system will be treated as two separate SISO problems, i.e. the transfer function matrix in equation (9) is assumed to be diagonal. This means that the control system structure in Figure 2 will be considered.

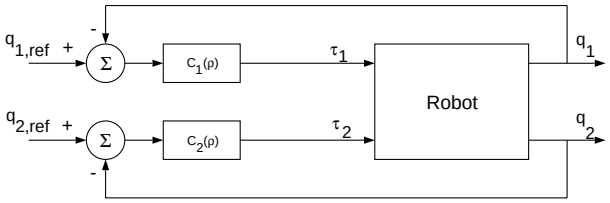

Fig. 2. PD-controller structure

There are a number of choices that have to be made when applying IFT, and these choices include the initial controllers, the reference model, the criterion, and the step size in each iteration. The initial controllers are in this case chosen by discretization of a continuous time PD-controller

$$u(t) = K_P e(t) + K_D \dot{e}(t) \tag{14}$$

which gives

$$u(kT) = (K_P + \frac{K_D}{T}) e(kT) - \frac{K_D}{T} e(kT - T) \tag{15}$$

In this particular application the parameter values $K_P = 100$ and $K_D = 30$ give reasonable initial performance, and the initial controllers are hence chosen as

$$C_1(\rho) = C_2(\rho) = 3100 - 3000z^{-1} \tag{16}$$

The reference signals are generated using the function `jtraj` in the ROBOTICS TOOLBOX Corke (1996), and the reference signals for the two joints are shown in Figure 3.

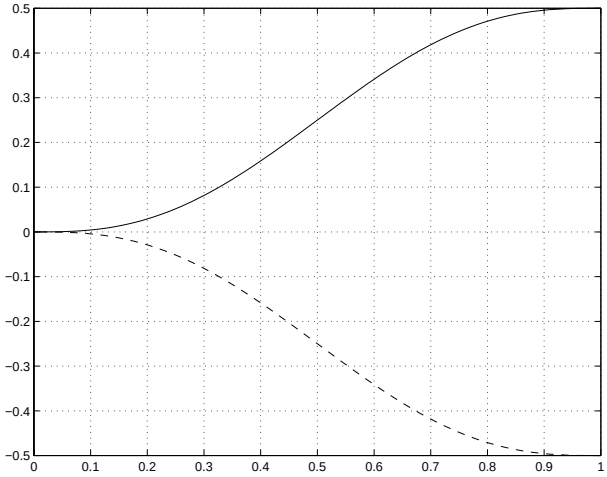

Fig. 3. Reference signals (rad). Solid line: $q_{1,ref}$. Dashed line: $q_{2,ref}$

The reference system is chosen as $T_d(z) = z^{-5}$ for both channels, it is desired that the outputs follow the reference signals with a delay of 0.05 s. The number of delays is also a design variable and it will affect the gain of the controllers. In the first experiment the weights on both output signals are 10^3, while the weights on the control signals are zero. The size of the weight is not important here since no weight is put on the control signal, and 10^3 is chosen just in order to obtain a convenient level of the criterion value. The step size to in general chosen to one. The matrix R_i in the update equation (13) was chosen such that the Gauss-Newton search direction was achieved. See Hjalmarsson et al. (1998) for further information.

Applying IFT under these conditions almost immediately leads to a situation where the zero of the controller moves outside the unit circle. One method to avoid this situation is to add a disturbance signal, of suitable character, to the control signals generated by the PD-regulators. The purpose of the disturbance signals is to obtain sufficiently high low frequency gain of the control loop. Therefore it is suitable to choose a low frequency disturbance signal, and in this application a square wave is used. It is then necessary to choose the period time and amplitude of the disturbance signals. In this case the first disturbance signal had period time 1 s and amplitude 4, and the second disturbance signal had period time 0.5 s and amplitude 2.

During the first two iterations the value of the criterion decreases, as can be seen in Table 1, and the output signals approach the reference signals. After the second iteration, however, high frequency oscillations, with frequency about half of the Nyquist frequency, occur, in particular in the control signals. Further iterations, also with a shorter step length, lead to deteriorated control performance and an unstable closed loop system.

Table 1
Tuning of C_1 and C_2 with $\lambda = 0$.

Iteration	Criterion	Step size
0	4.8	
1	1.8	1.0
2	1.0	1.0

Since single loop control has been applied to a multivariable system it is likely that the oscillations are caused by coupling effects in the system. This can also be found by carrying out an approximate linear analysis where the system is considered around zero angle for both joints and neglecting the Coriolis and centrifugal forces. The system can then approximately be described as

$$M_0 \ddot{q}(t) = \tau(t) \tag{17}$$

where

$$M_0 = M(0) = mL^2 \begin{pmatrix} 8/3 & 5/6 \\ 5/6 & 1/3 \end{pmatrix} \tag{18}$$

Laplace transforms give

$$Q(s) = \frac{M_0^{-1}}{s^2} T(s) \tag{19}$$

where $T(s)$ is the Laplace transform of the torque. Transforming this linear system to discrete time and applying the PD-controllers obtained in the experiment when the oscillations occur it is found that the closed loop system has poles close to the unit circle, in the vicinity of the imaginary axis, i.e. the poles will result in oscillations with around half the Nyquist frequency. This result coincides well with the observations above. The conclusion becomes that it is not possible to treat the problem as two single loop problems and that the coupling has to be taken into consideration.

5 PD-tuning with Fixed Decoupling

In a first attempt to handle the cross-coupling in the system the approximate linear model in equation (19) shall be used. From this model it is clear that, in the linear case, it is possible to obtain exact decoupling by using a static decoupling defined by the inertia matrix itself in zero position, i.e. the matrix M_0. Practically this implies introducing a new input signal $\bar{\tau}(t)$ and letting the actual torque signal be computed as

$$\tau(t) = M_0 \bar{\tau}(t) \tag{20}$$

where the signals in $\bar{\tau}(t)$ are then generated by PD-controllers as in the previous section. The structure of the control system is given in Figure 4. By letting factors from the decoupling controller be included in C_1 and C_2 only two decoupling constants remain. At zero position the theoretical values of the decoupling constants are $K_1 = 5/16 = 0.3125$ and $K_2 = 2.5$ respectively.

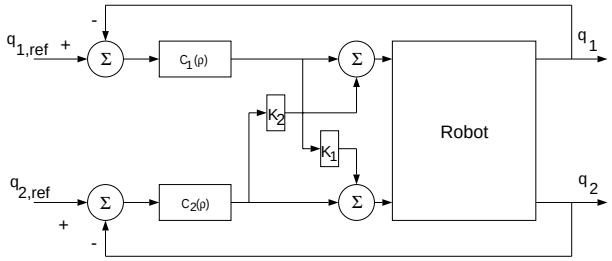

Fig. 4. PD-controller structure with decoupling

In this case the PD-controllers are initiated as

$$C_1(\rho) = 10100 - 10000z^{-1} \qquad C_2(\rho) = 1100 - 1000z^{-1} \tag{21}$$

These choices were made by considering the linear approximation at zero angles in combination with the decoupling. The system can then approximately be seen as two separate SISO systems. In the criterion the weight is 10^3 on the output signals and zero on the control signals. In Table 2 the results from the tuning procedure using the fixed decoupling are shown. A large improvement, measured in terms of the criterion value, is obtained by the introduction of the decoupling.

During the first iterations the criterion value is dominated by the error between desired angle and the actual angle, but as the iterations proceed the influence of the extra disturbance signals increases. Therefore the disturbance signals are removed after the third iteration. This can be done without getting any problems with the controller zeros moving outside the unit circle. During the first iterations it is necessary to include the extra disturbance signals to avoid this phenomenon.

Table 2
PD-tuning using theoretical K_1 and K_2.

Iteration no	Criterion	Step size
0	3.0	
1	0.9	1
2	0.2	1
3	0.07	1
4	0.03	1
5	0.007	1
6	0.005	1

Figure 5 shows the error signals after convergence. The output signals follow the outputs of the reference models almost exactly. Since the reference model consists of five time delays it is natural that there will be a delay between the reference angles and the joint angles.

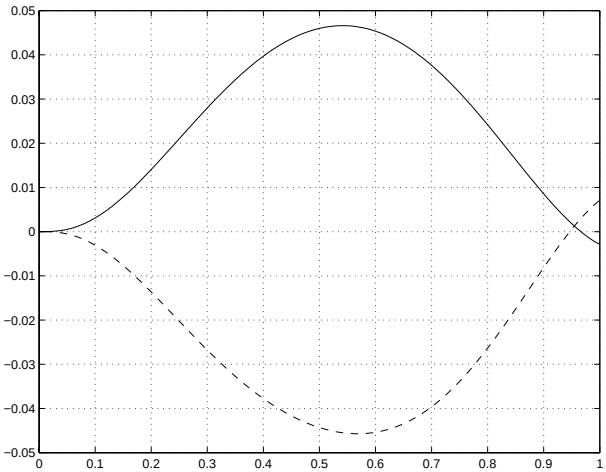

Fig. 5. Error signals (rad) after tuning. Solid line: $q_{1,ref} - q_1$ Dashed line: $q_{2,ref} - q_2$

6 Tuning of Decoupling Controllers

In the previous section the usefulness of the decoupling controllers was shown, but in order to compute the decoupling a model of the robot was used. Since IFT is meant to be a model free method it is not realistic to assume that such a model is available. It is therefore natural to consider the problem of tuning also the decoupling controllers. In the control system there are then four controllers to update, two decoupling controllers and two PD-controllers. There are then several possible ways to organize the tuning of the controllers, and the strategy that has been found useful here is to first tune the decoupling

controllers while keeping C_1 and C_2 fixed and then tune the PD-controllers while keeping K_1 and K_2 fixed.

When tuning the decoupling constant K_1 the aim is that a change in $\bar{\tau}_1(t)$ shall have as small influence as possible on the second joint angle $q_2(t)$. Therefore the reference signal shown in Figure 3 is applied to the first joint while the second reference signal is zero. In the criterion the weight on the second output $q_2(t)$ is set to 10^3 while the weight on $q_1(t)$ is set to zero. The PD-controllers are initiated according to equation (16) while $K_1 = K_2 = 0.01$. In the tuning procedure only K_1 is updated in each iteration. It turns out that the tuning of K_1 is very quick and that the iterations have converged after only two steps. The results are summarized in Table 3. The final value of the decoupling constant is $K_1 = 0.313$ which is the same as obtained using the approximate linear model of the robot arm. With this decoupling the second joint angle satisfies $\mid q_2(t) \mid < 1.5 \cdot 10^{-4}$ during the movement of $q_1(t)$.

Table 3
Tuning of K_1

Iteration no	Criterion	Step size
0	1.4	
1	0.003	1
2	0.000	1

The conditions when tuning the second decoupling constant K_2 were the opposite compared to the tuning of K_1. This means that the reference signal for joint one was zero while a reference signal of the type shown in Figure 3 was applied to the second joint. A weight of 10^3 was put on $q_1(t)$ while the weight on $q_2(t)$ was zero. The same initial values of the controllers as in the case above were used with the exception of K_1, where the result from the tuning was used. Also now the decoupling constant converged quickly and the evolution of the criterion is shown in Table 4. The tuning resulted in the final value $K_2 = 2.436$ which is slightly different from the theoretical value at zero angle. This is however logical since the second joint changes from zero to -0.5 during the movement. In this case the remaining coupling is larger that in the previous case and $\mid q_1(t) \mid < 7 \cdot 10^{-3}$ during the movement of $q_2(t)$. When tuning K_2 it was found necessary to reduce the step size in the first iteration in order to avoid to too large value of K_2, from which it was difficult to recover.

After having tuned the decoupling controllers it is straightforward to tune the PD-controllers keeping the decoupling controllers fixed. The situation is similar to the case when the theoretically computed decoupling coefficients were used, with only slight difference in the numerical values. Since the principle behavior is the same as presented above the iteration results are excluded.

Table 4
Tuning of K_2

Iteration no	Criterion	Step size
0	1.4	
1	0.1	0.5
2	0.006	1
3	0.005	1

7 Tuning for a Faster Movement

In the previous experiments smooth and fairly slow reference trajectories were used. Since the movement of the second joint was rather small, between 0 and 0.5 rad, the nonlinear effects due to the the change of the inertia matrix were rather small. In order to evaluate the IFT method in a more demanding situation the simulations in this section shall be based on a trajectory where $q_1(t)$ goes from 0 to $\pi/2$ rad and $q_2(t)$ goes from 0 to $-\pi$ rad. Furthermore it is required that the movement is done during 0.3 seconds, i.e. more than three times faster than in the previous experiments. The reference trajectories are shown in Figure 6. In these experiments the sampling frequency is increased to 1000 Hz. Since the reference model $T_d = z^{-5}$ is used also in this case and the sampling interval is reduced this also contributes to increased performance requirements.

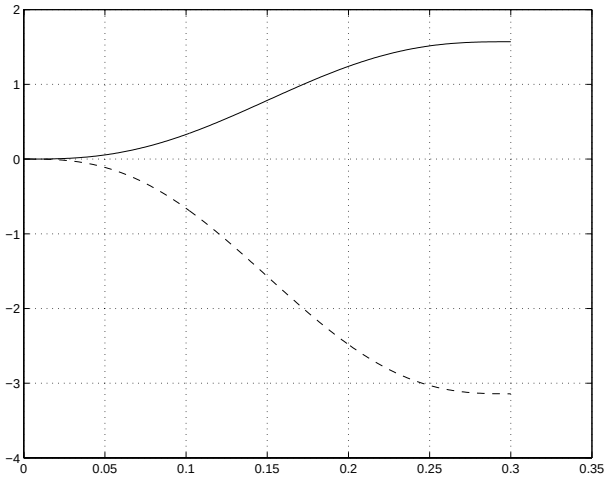

Fig. 6. Reference signals (rad). Solid line: $q_{1,ref}$. Dashed line: $q_{2,ref}$

The procedure will be the same as before, i.e. first the decoupling constants are tuned and then the PD-controllers. The results of the tuning of the decoupling controllers are given in Table 5 and Table 6. The tuning of K_1 converges in two steps giving $K_1 = 0.312$. The second joint angle satisfies $\mid q_1(t) \mid < 1.4 \cdot 10^{-3}$. The tuning of K_2 is somewhat slower, and after the fourth iteration

no improvement of the criterion is achieved. The final value $K_2 = 2.288$ gives the deviation $\mid q_1(t) \mid < 0.08$ during the movement of $q_2(t)$.

Table 5
Tuning of K_1

Iteration no	Criterion	Step size
0	20	
1	0.006	1
2	0.000	1

Table 6
Tuning of K_2

Iteration no	Criterion	Step size
0	36	
1	1.0	1
2	0.3	1
3	0.2	1

Using the tuned values of K_1 and K_2 the next step is to tune the PD-controllers. Using the initial values in equation (16) the results shown in Table 7 are obtained.

Table 7
PD-tuning using tuned K_1 and K_2.

Iteration no	Criterion	Step size
0	1280	
1	850	1
2	370	1
3	140	1
4	46	1
5	14	1
6	4.0	1
7	1.6	1

Further improvements can however be achieved by re-tuning the decoupling coefficients. Using data from normal movements of the robot-arm first K_1 and then K_2 are re-tuned. K_1 is first reduced to 0.272 giving the value 1.1 of the criterion. In the second tuning K_2 is reduced to 1.9 which further reduces the criterion to 1.0.

8 Tuning for a System with Friction

The tuning experiment have so far been carried out without friction in the model of the robot arm. Since all manipulators in reality contain friction it is highly motivated to investigate the effects of friction on the IFT tuning procedure. This section will present results from tuning experiments similar to those presented in Section 6, where first the decoupling controllers and then the PD-controllers are tuned. The manipulator is extended with both viscous and Coulomb friction, which means that the term $\tau_F(q)$ in equation (1) is given by

$$\tau_F = \begin{pmatrix} c_v\dot{q}_1 + c_c sign(\dot{q}_1) \\ c_v\dot{q}_2 + c_c sign(\dot{q}_2) \end{pmatrix} \tag{22}$$

For simplicity the friction coefficients of both joints are the same. In the simulations the friction coefficients are chosen as $c_v = c_c = 5$.

The first stage in the IFT procedure is to tune the two decoupling controller as described in Section 6. The PD-controllers are in this case chosen according to equation 16. Two important observations were made using this tuning. First, it is important to choose the initial values of the decoupling controller large enough. A too small value of e.g. K_1 in Figure 4 implies that the input to the second link will be too small to overcome the friction during the gradient experiment and the link will not move. Therefore the initial values were chosen equal to one when tuning both K_1 and K_2. The second observation is that the tuned values of the decoupling controller differ from the ones obtained in the friction free case. With the particular values of the friction coefficients used here the tuned values are $K_1 = 0.35$ and $K_2 = 0.7$ respectively, i.e. essentially lower for K_2.

When the decoupling controllers have been determined the PD-controllers are initialized according to equation 21, and the tuning of the PD-controllers is carried out. The results are presented in Table 8. Also here disturbance signals are added to the input signals in order to avoid the zero of the controllers to move outside the unit circle. The disturbance signals are removed after the second iteration. Table 8 shows that the convergence is quite rapid. After the fifth iteration no further improvement is obtained. The final value of the criterion is approximately twice the value in the friction free case.

Figure 7 shows the difference between reference and joint angles for the case with friction. For the second joint the error is somewhat larger, in particular during the final part of the movement.

The results obtained here are restricted to one particular trajectory for each

Table 8
PD-tuning with friction.

Iteration no	Criterion	Step size
0	8.8	
1	1.5	1
2	0.15	1
3	0.02	1
4	0.013	1
5	0.012	1

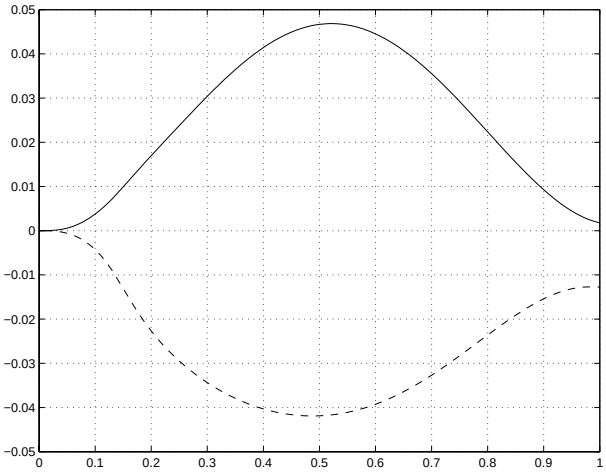

Fig. 7. Error signals (rad) after tuning for the case with friction. Solid line: $q_{1,ref} - q_1$ Dashed line: $q_{2,ref} - q_2$

joint and one particular set of friction coefficient values, and therefore too general conclusions should be avoided. In a more demanding situation a trajectory with several zero crossings for the velocity should be used. Further investigations of the influence of friction in this problem can be found in Collignon and Rousseaux (1998).

9 Conclusions

This paper has illustrated that IFT is applicable to a system that is both nonlinear and multivariable. Due to the strong cross-coupling in the system it was found necessary to use a decoupling controller, and one possible way to determine a suitable cross-coupling has been tested with good results. It has been illustrated that IFT can be used also when the system contains friction. The investigations presented above have some limitations that require further studies, like e.g. the influence of disturbances. One possibility that can be

explored further is the use of time domain and frequency domain weighting in the criterion. Application of the IFT method involves several choices, like the initial controllers, step size, order of tuning of the different controllers, etc, and the results here represent some possible choices. General guidelines for these choices require further work.

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