An Iterative Procedure for Semi-Blind Symbol Estimation in a Multipath SISO Channel Context Exploiting Finite Alphabet Properties[†]

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Abstract— **In this paper, we present a Maximum Likelihood (ML) symbol detector for single-carrier block transmissions in the context of SISO channels with multipath. This ML detector is based on an Iterative Least Squares with Projection (ILSP) algorithm exploiting both finite alphabet properties of the transmitted signal and its cyclic prefixed structure in order to approach ML detection in a cheap way. Since the initial channel estimate is crucial for the convergence of the ILSP algorithm, we propose a computationally cheap stochastic method for computing an initial channel estimate using a variant of Cyclic Prefix Only (CP-Only): the Known Symbol Padding Only (KSP-Only) technique. The resulting channel model is sufficiently accurate to be used as a starting point for the iterations. The final result is a computationally affordable direct symbol estimation method that yields promising results in terms of Bit Error Rate (BER).**

I. Introduction

The constantly increasing need for high data rate transmission systems has driven the research in broadband communications in the last years. Multipath effects, resulting in frequency-selective fading, are a major impediment of these broadband communication systems since they introduce Inter Symbol Interference (ISI), which needs to be tackled by appropriate techniques. However, these are most often computationally demanding. In this context, block transmission techniques (multi-carrier or single-carrier) based on the use of a Cyclic-Prefix (CP) have attracted a lot of attention in the last years for they allow an efficient and computationally cheap ISI cancellation procedure [1], [2]ISI can be suppressed by a single-tap frequency-domain equalisation on blocks of data symbols using FFT and IFFT operations relying on the fact that the blocks of transmitted data are made cyclic by the use of a CP, whose length is at least equal to the channel order. Multi-carrier block transmission techniques based on a CP (i.e., Discrete Multi-Tone (DMT) techniques) perform an IFFT at the transmitter after which the CP is added; the receiver performs an FFT followed by a one-tap frequency domain equalisation. These techniques can be used in combination with carrier loading, which allows the transmitter to optimize the PSD of the transmitted signal as a function of the channel [3], [1, p 7]. If the channel is unknown to the transmitter, however, this loading is not applicable and the performance of the system becomes very sensitive to the frequency-selectivity of the channels. Another drawback of classical DMT is the occurrence of large peaks in the transmitted signal, known as the Peak to Average Power Ratio (PAPR) problem [4]. In this paper, we consider the case where the channel is unknown to the transmitter, making carrier loading unapplicable. We therefore focus on single-carrier block transmission techniques based on a CP (i.e., CP-Only techniques) [5, pp 103-104], [2, p 36], where the transmitter simply adds a CP to every block of data symbols. This solves the PAPR problem since the transmitted data sequences now show finite alphabet properties, while keeping the advantage of computationally cheap ISI mitigation. The sensitivity to highly frequency-selective channels is also tackled by this technique. A variant of CP-Only that has recently been proposed is known as Known Symbol Padding Only (KSP-Only) [6] [7] [8]. In the latter, a sequence of known symbols is padded to every block of transmitted data symbols, which makes the data pseudo-cyclic and allows the same equalisation scheme as in the classical CP-Only context. The known symbols can be further exploited for channel estimation while the percentage of redundant symbols remains roughly the same.

The signals that are transmitted in digital communication systems usually have finite alphabet propreties, which is a very rich data structure that can be exploited for optimal symbol estimation. In [9], iterative methods exploiting finite alphabet properties of the transmitted signals have been proposed in the context of narrowband MIMO systems without multipath effects. One of these methods, known as Iterative Least Squares Projections (ILSP), approaches Maximum Likelihood (ML) detection of the transmitted symbols in a cheap way.

In this paper, we extend this work to a multipath SISO environment maintaining low computational complexity whilst still approaching ML detection. To achieve this we propose a novel approach that combines ILSP with CP-Only transmission techniques. This context is relevant to wideband applications where the Multiple Access protocols are similar to the ones used for GSM systems (TDMA and FDMA).

Experimental results show that the initial channel estimate is critical for the convergence of the ILSP algorithm. In order to compute a sufficiently accurate initial channel estimate, we rely on KSP-Only modulation, which is a special case of CP-Only. It allows us to find an initial estimate in a cheap way based on

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a stochastic method. This initial channel estimate is the only reason for using KSP-Only instead of CP-Only; the ILSP procedure can be used in a classical CP-Only framework provided a sufficiently accurate channel estimate is made available by other means.

This paper is structured as follows. In section II, we present the data model that will be used later on in this paper. For the sake of simplicity, we directly present the model for KSP-Only; the extension to CP-Onlyis straightforward. In section III we present the ML detector in this context. In section IV, we propose a recursive projection algorithm who's performance approaches the ML detector. In section V, we propose a stochastic method that exploits the known symbols and provides a sufficiently accurate channel estimate for initialising the ILSP algorithm. In section VI, we present simulation results and we finally draw some conclusions in section VII.

II. Data Model

We are working in a Known Symbol Padding Context with L^{th} order SISO channels.

Define a transmission channel of order L:

$$\mathbf{h} = [h_0 \dots h_L] \tag{1}$$

Define a training sequence $\mathbf{t}$ of length T:

$$\mathbf{t} = [t_1 \dots t_T] \tag{2}$$

with the condition $T \geqslant L$.

Let P be the number of subcarriers that will be considered later on by the receiver. The sequence of transmitted data symbols $\mathbf{s}$ is organised in blocks of length $B = P - T$. The nth transmitted data block is defined as

$$\mathbf{s}^n = [s(nB+1) \dots s((n+1)B)] \tag{3}$$

The transmitter transmits a burst of $\mathcal{V}$ blocks where every block is padded with the sequence $\mathbf{t}$. The total transmitted sequence is thus $\mathbf{x} = [\ \mathbf{t}\ \ \mathbf{s}^1\ \ \mathbf{t}\ \ \dots\ \ \mathbf{t}\ \ \mathbf{s}^{\mathcal{V}}\ \ \mathbf{t}\ ]$. We consider the channel to be constant across the transmission of an entire burst. Define the matrix of transmitted data, omitting the first transmitted training sequence as

$$\mathbf{X} = \begin{bmatrix} \mathbf{s}^{1T} & \cdots & \mathbf{s}^{\mathcal{V}T} \\ \mathbf{t}^T & \cdots & \mathbf{t}^T \end{bmatrix} \tag{4}$$

The received sequence $\mathbf{y}$ is the convolution of the transmitted sequence with the channel:

$$y(i) = h_0 x(i) + h_1 x(i-1) + \dots + h_L x(i-L) + n(i) \tag{5}$$

where $x(i)$ is the ith element of $\mathbf{x}$ and $n(i)$ is the Additive White Gaussian Noise (AWGN) at the receiver. The received symbols are organised in $\mathcal{V}$ blocks of length P on which Frequency Domain Equalisation will be performed: $\mathbf{y}^n = [y(T+(n-1)P+1) \ \cdots\ y(T+NP)]$ is the nth received block. These blocks are organised in a $(P \times \mathcal{V})$ matrix:

$$\mathbf{Y} = [\ \mathbf{y}^{1T} \quad \cdots \quad \mathbf{y}^{\mathcal{V}T}\ ] \tag{6}$$

Define the $(P \times P)$ circulant channel matrix

$$\mathbf{H}_{circ} = \begin{bmatrix} h_0 & 0 & \dots & 0 & h_L & \dots & h_2 & h_1 \\ h_1 & h_0 & 0 & \dots & 0 & h_L & \dots & h_2 \\ & \ddots & \ddots & \ddots & & \ddots & \ddots & \\ h_L & \dots & h_0 & 0 & \dots & & 0 & 0 \\ 0 & h_L & \dots & h_0 & 0 & \dots & & 0 \\ 0 & 0 & \ddots & & \ddots & \ddots & & \\ \vdots & & \ddots & \ddots & & \ddots & & \vdots \\ 0 & \dots & & 0 & h_L & \dots & h_0 & 0 \\ 0 & & \dots & & 0 & h_L & \dots & h_0 \end{bmatrix} \tag{7}$$

Exploiting the cyclic structure of the transmitted symbols, the transmission scheme reads:

$$\mathbf{Y} = \mathbf{H}_{circ}\mathbf{X} + \mathbf{N} \tag{8}$$

where $\mathbf{N}$ is the AWGN matrix. Define the frequency-domain equivalent of the channel:

$$\mathbf{h}_f = \mathcal{F}_P \begin{bmatrix} \mathbf{h}^T \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{9}$$

where $\mathcal{F}_P$ is the P-point FFT matrix. The circulant structure of the channel matrix $\mathbf{H}_{circ}$ allows us to describe the transmission scheme in a simplified way using FFT and IFFT operations [2]:

$$\mathbf{Y} = \mathcal{I}_P \mathbf{H}_f \mathcal{F}_P \mathbf{X} + \mathbf{N} \tag{10}$$

where $\mathcal{I}_P$ is the P-point IFFT matrix and $\mathbf{H}_f = diag(\mathbf{h}_f)$ is the diagonal matrix containing the frequency-domain description of the channel.

Note that the percentage of training symbols to the total number of transmitted symbols for this KSP-Only context is equal to L/P when $T = L$. In a classical CP-Only context, the percentage of redundant symbols is $(L/P)(1+L/P)^{-1}$ which tends to L/P as the number of subcarrier increases. This shows that for a sufficiently large number of subcarriers P, CP-Only and KSP-Only are equivalent as far as the percentage of redundant symbols is concerned. Hence we do note waste extra bandwith using KSP-Only instead of CP-Only [8].

III. Maximum Likelihood Estimation

The data model shows us that the received signals can be modelled as deterministic sequences corrupted by AWGN. The log likelihood function of the transmitted data is

$$\mathcal{L} = -\alpha - \beta \ln(\sigma^2) - \frac{1}{\sigma^2} \sum_{n=1}^{\mathcal{V}} \|\mathbf{y}^{nT} - \mathcal{I}_P \mathbf{H}_f \mathcal{F}_P \mathbf{x}^{nT}\|^2 \tag{11}$$

where α and β are constants. The ML detector maximises $\mathcal{L}$ with respect to the unknown parameters $\mathbf{H}_f$ and $\mathbf{x}^n$, $n = 1 \cdots \mathcal{V}$, which is equivalent to the following minimisation problem:

$$\min_{\mathbf{H}_f, \mathbf{X}} \|\mathbf{Y} - \mathcal{I}_P \mathbf{H}_f \mathcal{F}_P \mathbf{X}\|^2 \tag{12}$$

where the elements of $\mathbf{X}$ are constrained to a finite alphabet and $\mathbf{H}_f$ has to be a diagonal matrix with unconstrained elements on the diagonal. Multiplying both terms by $\mathcal{F}_P$ and using the notations $\mathbf{X}_f = \mathcal{F}_P\mathbf{X}$ and $\mathbf{Y}_f = \mathcal{F}_P\mathbf{Y}$ yields the following equivalent problem:

$$\min_{\mathbf{H}_f,\mathbf{X}_f} \|\mathbf{Y}_f - \mathbf{H}_f\mathbf{X}_f\|^2 \quad (13)$$

with the same diagonal structure constraint on $\mathbf{H}_f$. $\mathbf{X}_f$ is constrained by the finite alphabet structure of $\mathbf{X}$. This is a nonlinear separable optimisation problem with mixed discrete and continuous variables. The optimisation can be carried out in two steps [9]. We first minimise this expression with respect to $\mathbf{H}_f$. To force $\mathbf{H}_f$ to have the desired diagonal structure, we separate the problem in P independant problems:

$$\min_{\mathbf{H}_f(j,j)} \|\mathbf{Y}_f(j,:) - \mathbf{H}_f(j,j)\mathbf{X}_f(j,:)\|^2, \;\; j = 1 \cdots P \quad (14)$$

whose solution is

$$\hat{\mathbf{H}}_f(j,j) = \mathbf{Y}_f(j,:)\mathbf{X}_f(j,:)^H \left(\mathbf{X}_f(j,:)\mathbf{X}_f(j,:)^H\right)^{-1} \quad (15)$$

where ^{H} denotes the complex conjugate transpose of a matrix and $\mathbf{A}(j,:)$ is the jth row of a matrix $\mathbf{A}$. Inserting these results that only depend on $\mathbf{X}_f$ and $\mathbf{Y}_f$ into (13) allows us to find the ML sequence by enumarating over all the possible values for the matrix $\mathbf{X}$, computing the value of the minimisation term, and choosing the matrix that minimises that cost function. This includes the computation of $\mathbf{X}_f$ and $\hat{\mathbf{H}}_f$ for every possible combination of the inputs. The computational cost of this method, which is exponential in P, $\mathcal{V}$, and the alphabet size, severely limits its practical interest.

IV. Iterative Least Squares with Projection

In the previous section, we derived an expression for the ML detector which is separable in its continuous and discrete variables. In this section, we apply an Iterative Least Squares with Projections algorithm inspired by [9] that uses this property to approach the ML detector by iteratively minimising the cost function (12) for one variable and then for the other.

Assume we have an initial channel estimate $\hat{\mathbf{H}}_f$. We first minimise the cost function (12) with respect to $\mathbf{X}$ with a fixed $\hat{\mathbf{H}}_f$:

$$\min_{\mathbf{X}} \|\mathbf{Y} - \mathcal{I}_P\hat{\mathbf{H}}_f\mathcal{F}_P\mathbf{X}\|^2 \quad (16)$$

This is equivalent to a classical frequency-domain equalisation on the received symbols to compute a soft estimate of the transmitted symbols:

$$\hat{\mathbf{X}} = \mathcal{I}_P\hat{\mathbf{H}}_f^{-1}\mathcal{F}_P\mathbf{Y} \quad (17)$$

Note that $\hat{\mathbf{H}}_f^{-1}$ is a diagonal matrix with $\hat{\mathbf{H}}_f^{-1}(j,j) = \hat{\mathbf{H}}_f(j,j)^{-1}$. This step is followed by a finite alphabet projection of the soft estimates: $\hat{\mathbf{X}} = FAP(\hat{\mathbf{X}})$, where FAP denotes the finite alphabet projection operation. Note that in the special case of KSP-Only, the last T symbols of every column are known in advance and are thus used instead of the estimates provided at each iteration.

We then use this estimate to minimise the cost function in its alternative formulation (13) with respect to $\mathbf{H}_f$, where $\mathbf{X}$ is given by the previous step:

$$\min_{\mathbf{H}_f} \|\mathbf{Y}_f - \mathbf{H}_f\hat{\mathbf{X}}_f\|^2 \quad (18)$$

Again we split this into P parallel independent problems to force $\mathbf{H}_f$ to have a diagonal structure:

$$\min_{\mathbf{H}_f(j,j)} \|\mathbf{Y}_f(j,:) - \mathbf{H}_f(j,j)\hat{\mathbf{X}}_f(j,:)\|^2, \;\; j = 1 \cdots P \quad (19)$$

whose solution is

$$\hat{\mathbf{H}}_f(j,j) = \mathbf{Y}_f(j,:)\hat{\mathbf{X}}_f(j,:)^H \left(\hat{\mathbf{X}}_f(j,:)\hat{\mathbf{X}}_f(j,:)^H\right)^{-1} \quad (20)$$

This new channel estimate is used to compute a new $\hat{\mathbf{X}}$ and we proceed iteratively. The iterations are stopped when two consecutive finite alphabet estimates of the transmitted sequences are the same. The convergence point is a maximum of the log likelihood function $\mathcal{L}$. It should be noted that a fairly good initial channel estimate is needed to keep the algorithm from getting stuck in irrelevant local minima. In the following section we therefore provide a cheap channel estimation technique for initialising the algorithm, which allows this iterative procedure to converge to minima that are close or equal to the ML sequence. It should be noted that an important advantage of this ILSP method is its low computational complexity

The use of an Iterative Least Squares with Enumerations (ILSE) algorithm is an alternative solution to ILSP that always converges. However, in our context, this ILSE procedure has a complexity $\mathcal{O}\left(2^P\right)$ which makes it totally unaffordable for most practical systems where the number of subcarriers is large. For a practical system with 64 carriers using BPSK modulation, there are in total $1.84\ 10^{19}$ different combinations that have to be enumerated. This approach will therefore not be developed here.

V. Initial Channel Estimate

Define $\mathbf{y}_T^n$ as the $T + L$ -long vector of received symbols capturing all the energy of the nth transmitted training sequence:

$$\mathbf{y}_T^n = [y((n-1)P+1) \cdots y((n-1)P+T+L+1)] \quad (21)$$

where $n = 1 \cdots \mathcal{V}+1$. There are $\mathcal{V}+1$ such vectors. Define $\mathbf{T}^n$ as

$$\mathbf{T}^n = \begin{bmatrix} t_1 & x_B^{n-1} & \cdots & x_{B-L+2}^{n-1} & x_{B-L+1}^{n-1} \\ \vdots & t_1 & & \vdots & \vdots \\ t_T & \vdots & \ddots & x_B^{n-1} & \vdots \\ x_1^n & t_T & & t_1 & x_B^{n-1} \\ \vdots & x_1^n & \ddots & \vdots & t_1 \\ \vdots & \vdots & & t_T & \vdots \\ x_L^n & x_{L-1}^n & \cdots & x_1^n & t_T \end{bmatrix} \quad (22)$$

where x_i^n is used as an equivalent notation for $x^n(i)$. Using (5) allows us to express $\mathbf{y}_T^n$ as :

$$\mathbf{y}_T^{n\,T} = \mathbf{T}^n \begin{bmatrix} h_0 \\ \vdots \\ h_L \end{bmatrix} + \mathbf{n}^{n\,T} \quad (23)$$

This expression contains both deterministic and random variables. If one looks at the expected value of ${\mathbf{y}_T^n}^T$, assuming $\mathbf{E}\{x^i(j)\} = \mathbf{E}\{n(k)\} = 0 \quad \forall i,j,k$, which is the case if the noise is zero-mean and if the data symbols are equiprobable, (23) reads.

$$\mathbf{E}\{{\mathbf{y}_T^n}^T\} = \mathcal{T} \begin{bmatrix} h_0 \\ \vdots \\ h_L \end{bmatrix} \tag{24}$$

where

$$\mathcal{T} = \mathbf{E}\{\mathbf{T}^n\} = \begin{bmatrix} t_1 & 0 & \cdots & 0 & 0 \\ \vdots & t_1 & \ddots & \vdots & \vdots \\ t_T & \vdots & \ddots & 0 & \vdots \\ 0 & t_T & & t_1 & 0 \\ \vdots & 0 & \ddots & \vdots & t_1 \\ \vdots & \vdots & \ddots & t_T & \vdots \\ 0 & 0 & \cdots & 0 & t_T \end{bmatrix} \tag{25}$$

Define the average value of $\mathbf{y}_T^n$ as:

$$\overline{\mathbf{y}_T} = \frac{1}{\mathcal{V}+1} \sum_{i=1}^{\mathcal{V}+1} \mathbf{y}_T^i \tag{26}$$

It is clear that $\lim_{\mathcal{V}\to\infty} \overline{\mathbf{y}_T} = \mathbf{E}\{\mathbf{y}_T^n\}$ and we will therefore use it as an approximation of $\mathbf{E}\{\mathbf{y}_T^n\}$ even with a finite $\mathcal{V}$. We can thus write

$$\overline{\mathbf{y}_T}^T \simeq \mathcal{T} \ \mathbf{h}^T \tag{27}$$

from which we derive a channel estimate based on the knowledge of the training symbols:

$$\hat{\mathbf{h}}^T = \left(\mathcal{T}^H \mathcal{T}\right)^{-1} \mathcal{T}^H \overline{\mathbf{y}_T}^T \tag{28}$$

This estimate is used to initialise $\mathbf{H}_f$ in the ILSP procedure. Experimental results show that this initial estimate yields good performance of the algorithm. Any other initial channel guess can be used as a starting point for the iterations, the advantage of this one being its very low computational complexity.

VI. Simulation Results

In this section, we present simulation results obtained with the proposed method for Rayleigh-fading channels with $L = 5$, $T = 5$ and $P = 64$. The simulations were performed using BPSK symbols for transmission. The simulations were performed on a large number of randomly generated Rayleigh-fading channels with approximately 10^7 data symbols estimated for each point of the graph.

Fig. 1 shows the Bit Error Rate (BER) for the proposed method as a function of the SNR expressed in dB. The different curves show the results for different values of the burstlength $\mathcal{V}$. The lower curve shows the theoretical BER floor for L^{th} order Rayleigh-fading SISO channels using BPSK signals [10, p 955], the upper curve shows the BER obtained with classical ZF equalisation in the frequency domain assuming perfect channel knowledge.

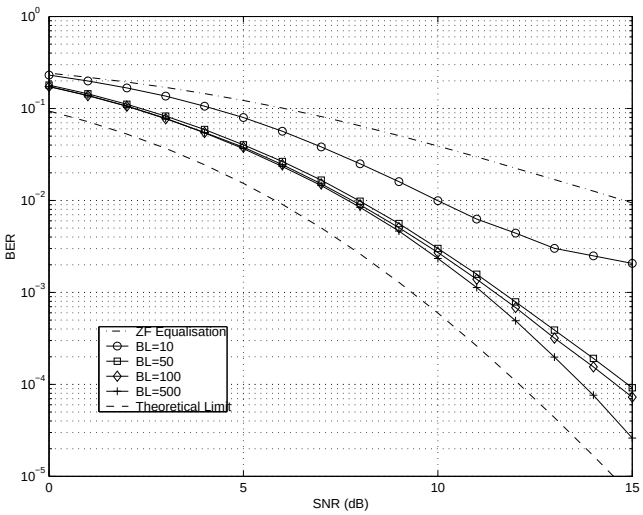

Fig. 1. BER vs. SNR for different Burst Lengths (BLs).

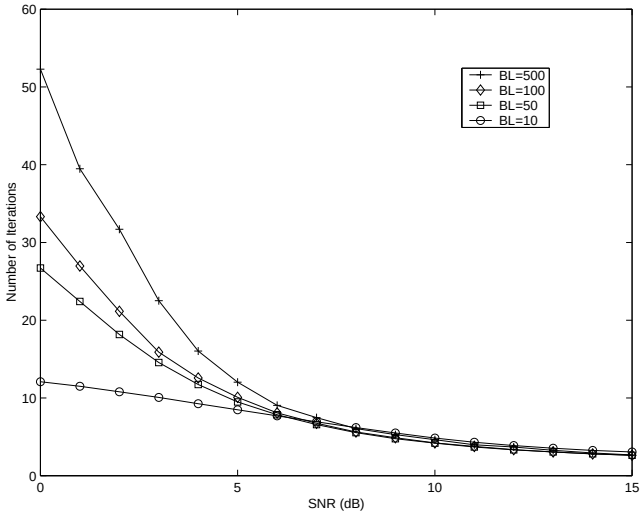

Fig. 2. Num. of It. vs. SNR for different Burst Lengths (BLs).

The achieved BER approaches the theoretically achievable limit as $\mathcal{V}$ increases. This might be because the curve of the cost function gets smoother as the number of data symbols increases, which keeps the algorithm from getting stuck in local minima, or because the initial estimate becomes more accurate.

Fig. 2 shows the average number of iterations that were required for the algorithm to converge. This number is maintained at a quite low level thanks to the good initial channel estimates. This is especially true for high SNRs. It should be noted that large $\mathcal{V}$'s, which yield better BER performance, require more iterations, especially for low SNRs.

VII. Conclusions

In this paper, we have proposed an iterative method for semi-blind direct symbol estimation in a SISO context with multipath. A considerable advantage of this method is that it achieves a very low Bit Error Rate, approaching the ML sequence estimate, while maintaining the computational complexity at reasonable levels. Future work on this topic includes the extension of this

method to time-varying channels and to MIMO systems. The optimisation of the training sequences used in the KSP-Only context is another topic that will be investigated.

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