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Transportation Planning and Traffic Flow Models

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Abstract

In this paper, we focus on the different traffic flow models that exist in literature. Due to our frequently encountered confusion among traffic engineers and policy makers, this paper goes into more detail about transportation planning models on the one hand, and traffic flow models on the other hand. The former deal with households that make certain decisions which lead to transportation and the use of infrastructure, as opposed to the latter which explicitly describe the physical propagation of traffic flows in a road network. Our goal is not to give a full account (as that would be a dissertation of its own, given the broadness of the field), but rather to impose upon the reader a thorough feeling for the differences between transportation planning and traffic flow models. Because of the high course of progress over the last decade (or even during the last five years), this paper tries to chronicle both past models, as well as some of the latest developments in this area.

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Due to our frequently encountered confusion among traffic engineers and policy makers when it comes to transportation planning models and the role that traffic flow models play therein, this paper strives to alleviate that bewilderment. The material elaborated upon in this paper, spans a broad range going from transportation planning models that operate on a high level and deal with households that make certain decisions which lead to transportation and the use of infrastructure, to traffic flow models that explicitly describe the physical propagation of traffic flows in a road network.

1 Transportation planning models

Before going into detail about the possible mathematical models that describe the physical propagation of traffic flows, it is worthwhile to cast a glance at a higher level, where transportation planning models operate. The main rationale behind transportation planning systems, is that travellers within these systems are motivated by making certain decisions about their wishes to participate in social, economical, and cultural activities. The ensemble of these activities is called the *activity system*. Because these activities are spatially separated (e.g., a person's living versus work area), the need for transportation arises. In such a system, the so-called *household activity patterns* form the main explanation for what is seen in the transportation network.

These models have as their primary intent the performing of impact and evaluation studies, and conducting 'before and after' analyses. The fact that such transportation studies are necessary, follows from a counter-intuitive example whereby improving the transportation system (e.g., by making extra infrastructure available), can result in an *increase* of the travel times. This phenomenon, i.e., allowing more flexible routing that results in more congestion, is known as Braeß' paradox, after Dietrich Braeß [33]. The underlying reason for this counter-intuitive behaviour, is that people generally only *selfishly* try to minimise their own travel times, instead of considering the effects they have on other people's travel times as well [228].

As transportation is inherently a temporal and spatial phenomenon, we first take a look at the concept of land-use models and their relation to the socio-economical behaviour of individual people. In the two subsequent sections, we consider two types of transportation planning models, i.e., the classic trip-based models, and the class of activity-based models, respectively. The section concludes with a brief reflection on the economist's view on transportation systems.

1.1 Land use and socio-economical behaviour

As already stated, transportation demand arises because of the desire to participate in a set of activities (e.g., social, economical, cultural, ...). In order to deduce this *derived* transportation demand, it is necessary to map the activity system and its spatial separations. This process is commonly referred to as *land use*, mainly playing the role of forging a relation between economical and geographical sciences. In general, land-use models seek to explain the growth and layout of urban areas (which is not strictly determined by economical activities alone, i.e., ethnic considerations et cetera can be taken into account),

Because transportation has spatial interactions with land use and vice versa, it can lead to a kind of chicken-and-egg problem [252]. For example, building a new road will attract some economical activity (e.g., shopping malls et cetera), which can lead to a possible increase of the travel demand. This in turn, can lead to an increase of extra economical activity (because of the well-suited location), and so on, resulting in a local reorganisation of the spatial structure. Resolving this chicken-and-egg paradox, is typically done by means of feedback and iterations between land-use and transportation models, whereby the former provide the basic starting conditions for the latter models (with sometimes a reversal of the models' roles).

In the following two sections, we first shed some light on several of the archetypical land-use models, after which we take a look at some of the more modern models for land use in the context of geosimulation.

1.1.1 Classic land-use models

The discussion given in this section, talks about several kinds of land-use models that — at their time — were considered as landmark studies. That said, the models presented here should be judged as being general in that they deal with (pre-)industrial American societies in the first part of the 20^{th} century. They are devised to gain insight into the general patterns that govern the growth and evolution of a city. As such, they almost never 'fit' perfectly, leading to the obvious criticism that they are more applicable to American cities than elsewhere. Notwithstanding these objections, the models remain very useful as explanations for the mechanisms underpinning the socio-economical development of cities.

One of the oldest known models describing the relation between economic markets and spatial distances, is that of Johann Heinrich von Thünen [283]. As the model was published in 1826, it presents a rather 'pre-industrial' approach: the main economical ingredients are based on agricultural goods (e.g., tomatoes, apples, wheat, ...), whereas the transportation system is composed of roads on which carts pulled by horses, mules, or oxen ride. The spatial layout of the model, assumes an *isolated state* (self-sustaining and free of external influences), in which

a central city location is surrounded by concentric regions of respectively farmers, wilderness, field crops, and meadows for grazing animals. All farmers aim for maximum profits, with transportation costs proportionally with distance, thus determining the land use around the city centre.

Some 100 years later, inspired by von Thünen's simple and elegant model, Ernest W. Burgess developed what is known as the *concentric zone model* [40]. It was based on observations of the city of Chicago at the beginning of the 20th century. As can be seen in the left part of Fig. 1, Burgess considered the city as growing around a central business district (CBD), with concentric zones of respectively the industrial factories and the low-, middle-, and high-class residents. The outermost ring denotes the commuter zone, connecting the CBD with other cities. As time progresses, the city develops and the radii of these concentric zones would grow by processes of 'invasion' and 'succession': an inner ring will expand, invading an outer ring that in turn has to grow, in order to make space.

Fifteen years after Burgess' theory, Homer Hoyt introduced refinements, resulting in the *sector model* [139]. One of the main incentives, was the observation that low-income residents were typically located in the vicinity of railroads. His model accommodates this kind of observation, in that it assumes that a city expands around major transportation lines, resulting in wedge-shaped patterns (i.e., sectors), stretching outward from the CBD. A typical example of this development, can be seen in the right part of Fig. 1.

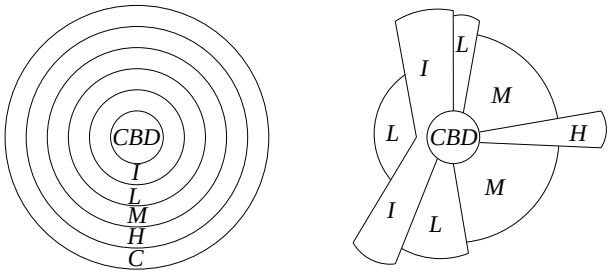


Figure 1: Typical examples of two models relaying the evolution of land use. *Left:* the concentric zone model of Burgess. *Right:* the sector model of Hoyt. In both figures, *CBD* corresponds to the central business district, *I* to the industrial factories, *L*, *M*, and *H* to the low-, middle-, and high-class residents respectively. In the Burgess model at the left, *C* denotes the commuter zone.

Halfway the previous century, Chauncy D. Harris and Edward L. Ullman were convinced that the previous types of models did not correspond to many of the encountered cities. The main reason for this discrepancy, was to be found in the stringent condition of a central area being surrounded by different zones. As a solution to this shortcoming, Harris and Ullman presented their *multiple nuclei model* [108]. Their theory assumed that in larger cities, small suburban areas could develop into fully fledged business districts. And although Harris and Ullman did not dispose of the CBD as the most important city centre, their smaller 'nuclei' would take on roles of being areas for specialised socio-economical activities.

To end our discussion of classic land-use models, we highlight the work of Peter Mann in 1965 [187], who considered a *hybrid model* for land-use representation. He combined both Burgess's and Hoyt's models, when deriving a model that described a typical British city. In his model that studied the cities of Huddersfield , Nottingham, and Sheffield, the CBD still remained the central location, surrounded by zones of pre- and post 1918 housing respectively. Dispersed around the outer concentric zone, the low-, middle-, and high-class residents would live. A most notable feature of Mann's model, is the fact that he considered the industrial factories to be on one side of the city, with the high-class residents diametrically opposed (the rationale being that high-class residents would prefer to stay upwind of the factories' smoke plumes).

1.1.2 The modern approach to land-use models

In the current time of living, most modern citizens have a different behaviour than their former counterparts at the beginning of the 20th century. It seems there is an increased trend towards expansion, as people are feeling more comfortable about covering larger distances, e.g., working in a busy city centre or at a remote industrial facility, coupled with living on the countryside). The activities related to working, living, and recreation appear to occur at substantially different spatial locations. Furthermore, several urban regions are composed of unique ethnic concentrations, among other things leading to the conclusion that the emphasis on the geographical aspect of a city gets less important during its evolution.

Recognising these radical changes in the development, modern land-use models approach the integration of an activity system from a completely different perspective. The growth of a city is represented as the evolution of a *multi-agent system*, in which a whole population of individual households is simulated. Due to the tremendous increase in computational power over the last two decades, these large-scale simulations are now possible. As an example, it is feasible to consider residential segregation in urban environments: within these environments (e.g., the city and housing market), individual agents (i.e., households) interact locally in a well-defined manner, leading to emergent structures, i.e., the evolving city. Besides data surveys that try to capture the households' behaviour, the basic landscape and mapping data is fed into *geographical information systems* (GIS) that is coupled with a *computer aided design* (CAD) representational model of the real world (although the difference between the traditional GIS and CAD concepts is slowly fading away) [285]. A recent example of such an all-encompassing approach, is the work related to the Urban-

Sim project, where researchers try to interface existing travel models with new land use forecasting and analysis capabilities [284]. It is being developed and improved by the Center for Urban Simulation and Policy Analysis at the University of Washington.

To conclude this section, we refer to the work of Benenson and Torrens, who adopted the terminology of *geosimulation* [25]. Their methodology is based on what they call the 'collective dynamics of interacting objects'. As such, geosimulation hinges on the representation of what we would call a *socio-economy* that is simulated, taking into account hitherto neglected dynamic effects (e.g., demographic changes, shifts of the economic activities, ...).

1.2 Trip-based transportation models

The relation between activity patterns and the transportation system has a long history, starting around 1954 with the seminal work of Robert B. Mitchell and Chester Rapkin [196]. They provided the first integrated study, establishing a link that introduced a framework for transportation analysis, primarily intended for studying large scale infrastructure projects [192]. Their methodology was based on four consecutive steps (i.e., submodels), collectively called the *four step model* (4SM). In 1979, Manheim casted the model's structure into a larger framework of transportation systems analysis, encapsulating both activity and transportation systems [186]. Central to this framework, was the notion of 'demand and performance procedures', which we can validly call *demand* and *supply procedures*. In a typical setup, they respectively represent the traffic that wants to use this infrastructure and the road infrastructure. For a more historically tinted recollection of the trip-based approach, we refer the reader to the outstanding overview of Boyce [30].

> With respect the 4SM's history, a subtle — almost forgotten — fact is that the classic four step model was actually conceived independently from the integrated network equilibrium model proposed by Beckmann, McGuire, and Winsten in the mid-fifties; the 4SM can actually be perceived as a trimmed-down version of this latter model [30]. Intriguingly, over the years, the work of the 'BMW trio' has had profound impacts on the mathematical aspects of determining network equilibria, optimal toll policies, algorithms for variational inequalities, stability analyses, supply chains, ... [5, 207, 30, 31].

In the next four sections, we consider the basic entities and assumptions of the four step model, followed by a brief overview of the four individual submodels with some more detail on the fourth step (traffic assignment), concluding with some remarks on the criticisms often expressed against the four step model. For a more extensive survey of the four step model, we refer the reader to the books of Sheffi [255] and Ortuzar and Willumsen [226].

1.2.1 Basic entities and assumptions

The basic ingredients on which the four step model is rooted, are the *trips*. These trips are typically considered at the household level, and relate to aggregate information (individuals are no longer explicitly considered). This level of detail, essentially collapses the whole tempo-spatial structure of transportation planning based on individual travellers into bundles of trips, going from one point in the transportation network to another.

In the four step model, one of the most rigid assumptions is that all trips describe departure and arrival within the planning period (e.g., the morning commute). Furthermore, the usage of the model's structure is intended for large-scale planning purposes, excluding small infrastructural studies at e.g., a single intersection of urban roads. Another assumption is based on the fact that an entity within the four step model has to make certain decisions, e.g., what is the departure time, which destination is picked, what kind of transportation (private or public) will be used, which route will be followed, ... In many cases, these decisions are considered concurrently, but the four step model assumes they are made independently of each other. And finally, as each submodel needs input, most of the data is aggregated into *spatial zones* (often presumed to be distinguished by socio-economic characteristics) in order to make the model computationally feasible. These zones are typically represented by their centrally located points, called *centroids*.

1.2.2 The four steps

Within the four step model, the first three steps (I) – (III) can collectively be seen as a methodology for setting up the travel demand, based e.g., on land use and other socio-economical activities. This travel demand is expressed as *origin-destination* (OD) pairs (by some respectively called 'sources' and 'sinks'), reflecting the amount of traffic that *wants* to travel from a certain origin to a certain destination (these are typically the zones mentioned in the previous section). The last step (IV) then consists of loading this travel demand onto the network, thereby assigning the *routes* that correspond to the trips.

(I) Trip generation
In an essential first step, transportation engineers look at all the trips that on the one hand originate in certain zones, and on the other hand arrive in these zones. As such, the first step comprises what are called the *productions* and *attractions*. Central to the notion of a trip, is the *motive* that instigated the trip. An example of such a motive is a home-based work trip, i.e., a trip that originates in a household's residential area, and arrives in that household's work area. Other examples include recreational and social motives, shopping, ... and the *chaining of activities*. Based on these intentions, productions and attractions consist of absolute counts,

denoting the number of trips that depart from and arrive in each zone. Because of this, productions and attractions are in fact *trip ends*. Both of them are derived using techniques based on regression analysis, category analysis, or even logit models. As different models can be used for the derivations of the number of productions and attractions, an a posteriori balancing is performed that equalises both results. In the end, step (I) gives the magnitude of the total travel demand on the network. Note that all activities (i.e., the original motives) are at this point in effect transformed and *aggregated* into trips. More importantly, these trips are *only considered for a specific time* period (e.g., the morning rush hour).

(II) Trip distribution

Once the total number of productions and attractions for all zones in the transportation network is known, the next step then consists of deriving how many trips, originating in a certain zone, arrive at another zone. In other words, step (II) *connects* trip *origins* to their *destinations* by distributing the trips. The result of step (II) is then the construction of a complete *origin-destination* table (*OD table*). In such an OD table (or *OD matrix* as some people say), an element at a row i and a column j denotes the total number of trips departing from origin zone O_i and arriving in destination zone D_j. Diagonal elements denote intra-zonal trips. Note that step (II) does not state anything about the different *routes* that can be taken between two such zones; this is something that is derived in the final step (IV). Because of the implicit assumption in step (I), namely that all trips are considered for a specific time period, the same premise holds for all the derived OD tables. Consequently, the four step model is applied for different time periods, e.g., during rush hours or off-peak periods. In this context, we advise to use the nomenclature of *time-dependent* or *dynamic* OD tables, denoting OD tables that are specified for a certain period, e.g., from 07:00 until 08:00 (or even tables given for consecutive quarter-hours).

Considering the fact that an OD table contains a large amount of unknown variables (it is a considerably under determined system of equations), several techniques have been introduced to deal with this problem by introducing additional constraints. If an OD table for a previous period (called a *base table*) is known, then a new OD table can be derived by using a so-called *growth factor model*. Another method is by using *gravity models* (also known as *entropy models*, see e.g., the discussion in by Helbing and Nagel [122]), which are based on *travel impedance functions*. These functions reflect the relative attractiveness of a certain trip e.g., based on information retrieved from household travel surveys. In most cases, they are calibrated as power or exponential functions. One of the harder problems that still remains to be solved, is how to deal with so-called *through trips*, i.e., trips that originate or end outside of the study area. Horowitz and Patel for example, directly incorporate rudimentary geographical information and measured link flows into a model that allows to derive through-trip tables, using a notion of external stations located in an external territory. Application of his methodology to regions in Wisonsin and Florida, result in reasonable estimates of link flows that are comparable with empirically obtained data [137].

Besides using results from productions and attractions, gathering the necessary information for construction of OD tables can also be done using other techniques. An equivalent methodology is based on the consideration of *turning fractions* at intersections. The process can be largely automated when using video cameras coupled with image recognition software. Furthermore, there literally exist thousands of papers devoted to the estimation of origin-destination matrices, mostly applicable to small-scale vehicular transportation networks and local road intersections. Some past methodologies used are the work of Nihan and Davis who developed a recursive estimation scheme [223], the review Cascetta and Nguyen who casted most earlier methods into a unified framework [44], and Bell who estimated OD tables based on constrained generalised least squares [21]. An example of a more recent technique is the work of Li and De Moor who deal with incomplete observations [167].

(III) Mode choice / modal split

Once the origin-destination table for the given network and time period is available, the next step deals with the different *modes of transportation* that people choose between. Typical examples are the distinction between private and public transportation (both vehicular and railroad traffic). The 'split' in this step, refers to the fact that the OD table obtained from step (II), is now divided over the supported transportation modes. To this end, *discrete choice theory* is a popular tool that allows a disaggregation based on the choice of individual travellers, e.g., by using utility theory based on a nested logit model [23]. A modern trend in this context is to work with fully *multimodal transportation networks*; these multi-layered networks provide access points for changing from one layer (i.e., mode of transportation) to another [273].

Historically, steps (I) — production and attraction — and (III) were executed simultaneously, but nowadays they are considered separate from step (I): the main reason is the fact that the modal choice is not only dependent on e.g., a household's income, but also on the type of trip to be undertaken, as well as the trip's destination. As a result, the modal split can be intertwined with step (II), trip distribution, or it can be executed subsequently after step (II). In the former case, the same kind of travel impedance functions are used in combination with an adjusted gravity model, whereas in the latter case, a hierarchic logit model can be used.

(IV) Traffic assignment

At this point in the four step model, the total amount of trips undertaken by the travellers is known. The fourth and final step then consists of finding out *which routes* these travellers follow when going from their origins to

their destinations, i.e., which sequence of consecutive links they will follow ? In a more general setting, this process is known as *traffic assignment*, because now the total travel demand (i.e., the trips) are assigned to routes in the transportation network. Note that in some approaches, an iteration is done between the four steps, e.g, using the traffic assignment procedure to calculate link travel times that are fed back as input to steps (II) and (III).

It stands to reason that all travellers will endeavour to take the *shortest route* between their respective origins and destinations. To this end, a suitable measure of distance should be defined, after which a shortest path algorithm, e.g., Dijkstra's algorithm [81], can calculate the possible routes. Such a notion of distance typically includes both spatial and temporal components, e.g., the physical length of an individual link and the travel time on this link, respectively. The use of the travel time is one of the most essential and tangible components in travellers' route choice behaviour. Note that in a more general setting, the distance can be considered as a *cost*, whereby travellers then choose the *cheapest route* (i.e., the quickest route when time is interpreted as a cost). Daganzo calls these formulations the *forward shortest path problem*, as opposed to the *backward shortest path problem* that tries to find the cheapest route for a given arrival time [73].

The basic principles that underlay route choice behaviour of individual travellers, were developed by Wardrop in 1952, and are still used today. In his famous paper, relating space- to time-mean speed, Wardrop also stated two possible criteria governing the distribution of traffic over alternative routes [289]:

> **User equilibrium (W1):** *"The journey times on all the routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route."*

> **System optimum (W2):** *"The average journey time is a minimum."*

The above two criteria are based on what is called the *Nash equilibrium* in game theory [209], albeit that now a very large number of individuals are considered[1]. In the first criterion (W1), it is assumed that all individuals' decisions have a negligible effect on the performance of others. Two, more important, fundamental principles here are the fact that in the equilibrium situation, there is *no cooperation between individuals* assumed, and that all individuals make their decisions in an *egoistic and rational* way [107]. In real-life traffic, everybody is expected to follow the first criterion (W1), such that the whole system can settle in an equilibrium in which no one is better off by choosing an alternative route. In this respect, the work of Roughgarden is interesting because it provides a mathematical basis for the quantification of the worst loss of social welfare due to *selfish routing*, and the management of networks that limit these effects in order to obtain a socially desirable outcome [254]. In contrast to this user equilibrium situation, the second criterion (W2) is unlikely to occur spontaneously. However, when the perceived cost of a route by a traveller is changed to a *generalised* or *marginal cost* (i.e., including the costs of the effects brought on by adding an extra vehicle to the travel demand), then a system optimum is achieved with respect to these latter costs. In any case, as some people will be better off, others will be worse off, but the transportation system as a whole will be best off.

The above two principles, are a bit idealistic, in the sense that there are many exceptions to these behavioural guidelines. For example, in urban city centres, a significant part of the congestion can be brought on by vehicles looking for parking space. Furthermore, many drivers just follow their *usual route*, because this is the route they know best, and they know what to expect with respect to travel time. In a broader setting, this make these 'standard' routes more *appealing* to road users than other unfamiliar alternative routes. In some cases however, travellers will opt for these less known routes, thereby possibly entering the *risk* of experiencing a higher travel time as has been concluded in the work of Chen and Recker [50]. Another fact that we expect to have a non-negligible effect on the distribution of traffic flows, is that nowadays more and more people use *intelligent route planners* to reach their destinations. These planners take into account congestion effects, as the trip gets planned both spatially and temporally. This will result in a certain percentage of the population that is informed either *pre-route* or *en-route*, and these people can consequently change their departure time or actual route (e.g., through route guidance), respectively. Another interesting research problem arises because transportation infrastructure managers should then be able to adapt their policies to the changing travel patterns. For example, how should a policy maker optimally control the traffic when only 20% of the population will follow the proposed route guidance ?

Due to the importance of the subject, we have devoted two separate sections in this dissertation to the concept of traffic assignment. In these sections, we discuss the traffic assignment procedure in a bit more detail, considering two prominent methodologies from a historic perspective, namely *static* versus *dynamic* traffic assignment.

[1] The difference between a Wardrop and a Nash equilibrium is a subtle but important one. In the Wardrop case, an infinite number of individuals is considered, each seeking their own optimum. Note that the concept 'infinite number of individuals' can practically be approximated by 'a large amount of individuals'. The Nash case also considers an infinite number of individuals, but they are now grouped into a finite number of classes, with each class seeking its own optimum. If in this latter case the number of classes goes to infinity, then the Nash equilibrium converges to a Wardrop equilibrium [112].

1.2.3 Static traffic assignment

The classic approach for assigning traffic to a transportation network, assumes that all traffic flows on the network are *in equilibrium*. In this context, the *static traffic assignment* (STA) procedure can be more correctly considered as dealing with *stationary* of *steady-state* flows: the travel demand and road infrastructure (i.e., the supply) are supposed to be time-independent, meaning that the calculated link flows are the result of a constant demand. In a typical setup, this entails the assignment of an hourly (or even daily) OD table to the network (e.g., during on- and off-peak periods), resulting in *average* flows for the specified observation period. Because the STA methodology neglects time varying congestion effects (it assumes constant link flows and travel times), various important phenomena such as queue spill back effects are not taken into account.

In general, several possible techniques exist for achieving an STA. The first one assumes (i) that all drivers will choose the same cheapest route between a pair of origins and destinations, (ii) that they all have the same *perfect information* about the links' impedances, and (iii) that these impedances are considered to be constant, i.e., independent of a link's traffic load (so no congestion build-up is taken into account). As the methodology implies, this is called an *all-or-nothing assignment* (AON). A second technique refines this notion, whereby differences among drivers are introduced (i.e., giving rise to imperfect information), resulting in a *stochastic assignment*. In this methodology, the link travel impedances are assumed to be probabilistically distributed: for each link in the network, an impedance is drawn from the distribution after which an AON assignment is performed on the resulting network. This Monte Carlo process is repeated until a certain termination criterion is met.

Both previous methods carry a significant drawback with respect to link capacities, that is to say, no effects are taken into account due to the fact that an increased flow on a link will generally result in an increase of the travel time (i.e., the link's impedance). To this end, a third method introduces *capacity restraints* such that an increase of the travel demand on a link, will result in a higher cost (thereby possibly changing the route with the cheapest cost). This method is called an *equilibrium assignment*, and just like as in the second method, a *stochastic equilibrium assignment* version can be derived, taking into account travellers' imperfect knowledge. The underlying assumption is that all travellers behave according to Wardrop's user equilibrium (W1). Furthermore, the capacity restraints are included in the travel impedance functions, as they are now synonymously called *travel time (loss) functions*, *congestion functions*, *volume delay functions*, *link impedance functions*, or even *link performance functions*. A popular form of these functions that express the travel time T in function of the flow q on a link, is the *Bureau of Public Roads* (BPR) power function [38]:

$$T = T_{\text{ff}} \left(1 + \alpha \left(\frac{q}{q_{\text{pc}}} \right)^{\beta} \right). \qquad (1)$$

In this BPR relation, the coefficients α and β determine the shape of the function. An example of such a function is depicted in Fig. 2. For low flows, the BPR function is rather flat and the travel time corresponds to the travel time T_{ff} under free-flow conditions. When higher flows occur on the link, the coefficient β determines the threshold at which the BPR function rises significantly (in some formulations it *asymptotically* approaches the capacity flow). The travel time will increase with the ratio of the flow q and the so-called *practical capacity* q_{pc}. This latter characteristic is derived from the value of the travel time under congested conditions. As a result, the practical capacity is different from the maximum capacity of a link as defined by a fundamental diagram. Finally, note that a serious disadvantage associated with these BPR functions in combination with static traffic assignment, is the fact that the travel demand on the network at a certain time does not always correspond to the actual physical flows that can be sustained. Under congested conditions, this implies that the flows in the STA approach can be higher than the physically possible link capacities (which are different from the previously mentioned practical capacities), leading to an incorrect assignment with faulty oversaturated links.

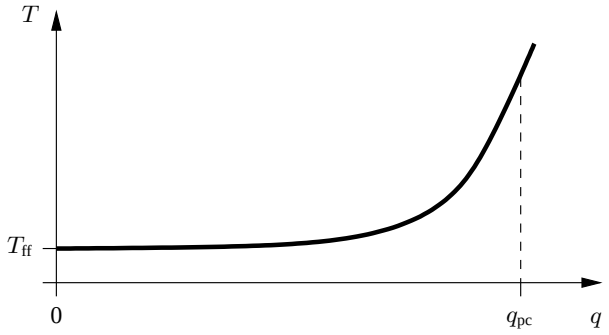


Figure 2: The Bureau of Public Roads (BPR) function, relating the travel time to the flow. It is based on the travel time T_{ff} under free-flow conditions and the practical capacity q_{pc} of the link under consideration.

Once the travel time of a link can be related to its current flow using e.g., a BPR function, an iterative scheme is adopted to calculate the equilibrium traffic assignment. Popular implementations are the *Frank-Wolfe algorithm* [91], and the *method of successive averages* (MSA). The former method is based on principles of optimisation theory, as demonstrated by Beckmann et al. [20, 31] who reformulated the Wardrop equilibrium as a convex optimisation problem, consisting of a single objective function with linear inequality constraints based on the Karush-Kuhn-Tucker (KKT) conditions, thereby resulting in a global minimum. Because travellers do not have perfect information, Daganzo and Sheffi formulated a variation

on Wardrop's first criterion (W1), whereby all traffic distributes itself over the network with respect to a *perceived travel time* of the individual drivers [77]. The resulting state of flows on the network is called a *stochastic user equilibrium* (SUE), as opposed to the *deterministic user equilibrium* (DUE). Note that a further discrimination is possible, as proposed by the work of Chen and Recker, who also make a distinction between travellers' perception errors on the one hand, and network uncertainty (i.e., stochasticity of the travel times) on the other hand [50]. For a thorough overview of the STA approach, we refer the reader to the work of Patriksson [229].

Although, as mentioned earlier, time varying congestion effects are not taken into account, the STA approach does fit nicely into the concept of long-term transportation planning. For short-term analyses however, these effects can have a significant impact on the end results, thus requiring a more detailed approach to traffic assignment.

1.2.4 Dynamic traffic assignment

As explained in the previous paragraphs, the static traffic assignment heavily relies on simple travel time functions (e.g., BPR). An associated problem with these is the difficulty in capturing the concept of 'capacity of a road'. In reality, congestion is a dynamic phenomenon, whereby its temporal character is not to be neglected. To tackle theses problems inherent to the STA approach, a more dynamic treatment of traffic assignment is necessary [180]. A fundamentally important aspect in this *dynamic traffic assignment* (DTA) procedure, is the fact that congestion has a temporal character, meaning that its buildup and dissolution play an important role: the *history of the transportation system* should be taken into account (e.g., congestion that occurs due to queue spill back) [66]. Neglecting this time dependency by assuming that the entry of a vehicle to a link instantaneously changes the flow on that link, results in what is called *Smeed's paradox*. This leads to incorrect behaviour as a result of a violation of the so-called 'first-in, first-out' (FIFO) property, because now a vehicle can leave link earlier then a vehicle that enters it later (i.e., arriving earlier by departing later) [260]. The methodology of dynamic traffic assignment was now designed to deal with all these particular aspects. The DTA technique is composed of two fundamental components:

Route choice
Just as in the STA approach, each traveller in the transportation network follows a certain route based on an instinctive criterion such as e.g., Wardrop's (W1). The associated component that takes care of travellers' route choices, can be complemented to allow for imperfect information. Another, more important, aspect related to the route choice, is a traveller's *choice of departure time*. An STA approach assumes that all traffic of a given OD table is simultaneously assigned to the network, whereas DTA coupled with departure time choice can spread the departures in time (leading to e.g., realistically spreading of the morning peak's rush hour).

Dynamic network loading (DNL)
Instead of using simple travel time functions, a DTA approach typically has a component that loads the traffic onto the network. In fact, this step resembles the *physical propagation* of all traffic in the network. In order to achieve reliable and credible results, a good description of the network's links is necessary, as well as the behaviour of traffic at the nodes connecting the links within this network (i.e., this is a mandatory requirement to achieve correct modelling of queue spill back). The DNL component in the DTA approach has been an active field of research during the last decade, and it still continues to improve the state-of-the-art. Testimonies include the use of *analytic models* that give correct representations of queueing behaviour, as well as detailed simulations that describe the propagation of individual vehicles in the transportation network. Note that in the case of *simulation-based* (also called *heuristic*) traffic assignment, the route choice and DNL components can be iteratively executed, whereby the former establishes a set of routes to follow, and the latter step feedbacks information to the route choice model until a certain termination criterion is met (e.g., a relaxation procedure). Furthermore, using simulation-based traffic assignment with very large road networks is not always computationally feasible to calculate all shortest paths. As a result, it might be beneficial to resort to simplifications of either the simulation model (e.g., using faster queueing models), or the number of paths to consider (e.g., based on the hierarchy inherently present in the road network) [253]. Finally, we mention the work of Astarita who provides an interesting classification of DNL models, based on the discretisation with respect to the spatial and temporal dimensions, as well as with respect to the modelling of the traffic demand [10].

Despite the appealing nature of simulation-based DTA, there is in contrast to the STA approach, no unified framework that deals with the convergence and stability issues [94, 93, 233].

Some examples of these DTA mechanisms are: Gawron who uses a queueing model to develop a simulation-based assignment technique that is able to deal with large-scale networks and is proven to be empirically stable [94, 93], Bliemer who developed an analytical DTA approach (with different user-classes) based on a variational inequality approach [26], Bliemer's work furthermore culminated in the development of *INteractive DYnamic traffic assignment* (INDY) [185] which — in combination with the OmniTRANS[2] commercial transportation planning software — can be used as a fully fledged DTA analysis tool [280], Lo and Szeto who developed a DTA formulation based on a variational inequality approach leading to a dynamic user equilibrium [174], the group

[2] `http://www.omnitrans-international.com`

of Mahmassani who is actively engaged in the DTA scene with the development of the DYNASMART (*DYnamic Network Assignment-Simulation Model for Advanced Roadway Telematics*) simulation suite [189], ... An excellent comprehensive overview of several traditional DTA techniques is given by Peeta and Ziliaskopoulos [234].

Another important field of research, is how individual road travellers react to the route guidance they are given. In his research, Bottom considered this type of *dynamic traffic management* (DTM), providing route guidance to travellers whilst taking into account their *anticipated behaviour* during e.g., incidents [28]. Taking this idea one step further, it is possible to study the interactions between the behaviour of travellers in a road network, and the management of all the traffic controls (e.g., traffic signal lights) within this network. An example of such a *dynamic traffic control* (DTC) and DTA framework, is the work of Chen who considers the management from a theoretic perspective based on a non-cooperative game between road users and the traffic authority [51].

1.2.5 Critique on trip-based approaches

Considering its obvious track record of the past several decades, the *conventional* use of the trip-based approach is — to our feeling — running on its last legs. By 'conventional' we denote here the fact that the current *state-of-the-practice* is still firmly based on the paradigm of static traffic assignment, despite the recent (academic) progress on the front of dynamic traffic assignment techniques. The four step model still largely dominates the commercial business of transportation planning, although its structure remained largely unchanged since its original inception. As mentioned earlier, in the case of STA, all trips are assumed to depart and arrive within the specified planning period. This leads to an unnatural discrepancy between models and reality in congested areas during e.g., a morning rush hour: some travellers *want* to make a trip and, in the former case, are perfectly allowed to achieve this trip, whereas in the latter case they are in fact physically *unable* to make the trip due to dynamical congestion effects.

In order to facilitate this disagreement between the balancing of travel demand versus supply (i.e., the transportation infrastructure), the DTA approach is gaining importance as more features are provided. An example of such a feature includes the framework of congestion pricing, where we have an incorporation of departure time choice models coupled with the derivation of optimal road tolls. Some noteworthy studies that have been carried out in this respect, are the work of de Palma and Marchal who present the METROPOLIS toolbox, allowing the simulation of large-scale transportation networks [78], the work of Lago and Daganzo who combined a departure time equilibrium theory with a fluid-dynamic model in order to assess congestion policy measures [159], the work of Szeto and Lo who coupled route choice and departure time choice with the goal of numerically handling large-scale transportation networks [261, 175]. Closely related to Lago's and Daganzo's work is that of Yperman et al., who determined an optimal pricing policy, describing the demand side with a bottleneck model and an analytical fluid-dynamic model as the DNL component [298].

At this point, we should mention some of the complications associated with the traditional method of modelling traffic flow propagation using queue-based analogies. Historically, there have been two different queueing techniques with FIFO discipline that describe this aspect in a trip-based assignment procedure:

- Point queues (also called *vertical queues*): this type of queue has no spatial extent. Because vehicles can *always* enter the queue, and leave it after a certain delay time, congestion is incorrectly modelled. A well-known example of a model based on this queueing policy is Vickrey's bottleneck model [281].
- Spatial/physical queues (also called *horizontal queues*): a queue of this type has an associated *finite capacity*, i.e., a buffer storage. Vehicles can only enter the queue when there is enough space for them available.

The correct modelling of congestion effects such as queue spill back, is of fundamental importance when assessing certain policy measures like e.g., road pricing schemes. To this end, the use of vertical queues should be abandoned, in favour of horizontal queues. However, even horizontal queues have problems associated with them: the buildup and dissolution of congestion in a transportation network are flawed, e.g., vehicles that are leaving the front of a queue *instantly open up a space at the back of this queue*, thus allowing an upstream vehicle to enter. This leads to shorter queue lengths, because the physical queueing effect of individual vehicles (i.e., the upstream propagation of the empty spot) is absent [93, 256, 47, 159]. In order to alleviate this latter issue, a more realistic velocity should be adopted for the backward propagating kinematic wave, thus calling for more advanced modelling techniques that *explicitly* describe the propagation of traffic (e.g., fluid-dynamic approaches, models with dynamical vehicle interactions, ...).

As often is the case, a model's criticisms can be found in its underlying assumptions. In the case of the four step approach, it is obvious that all information regarding individual households is lost because of its aggregation to a trip level. As was already recognised from the start, the individual itself loses value during this conversion process. This opened the door towards another approach to transportation planning, more precisely *activity-based modelling* (ABM) which is discussed in the next section.

A final complaint that is more common around many of these grotesque models, is their requirement of a vast amount of specific data. In many cases, a diverse range of national studies are carried out, having the goal of

gathering as much data as possible. Regardless of this optimism, some of the key problems remain, e.g., it is still not always straightforward to properly interpret and adapt this data so it can be used as input to a transportation planning tool.

1.3 Activity-based transportation models

As it was widely accepted that the rationale for travel demand can be found in people's motives for participating in social, economical, and cultural activities, the classic trip-based approach nevertheless kept a strong foothold in the transportation planning community. Instead of focussing attention elsewhere, the typical institutional policy was to ameliorate the existing four step models [191]. However, some problems persistently evaded a solution with the trip-based approach, e.g., shops that remain open late, employers who introduce flexible working hours, the consideration of joint activities by members of a household, ...

In the next few sections, we illustrate how all this changed with the upcoming field of activity-based transportation planning. We first describe its historic origins, after which we move on to several of the approaches taken in activity-based modelling. The concluding section gives a concise overview of some of the next-generation modelling techniques, i.e., large-scale agent-based simulations.

1.3.1 Historic origins

The historic roots of the activity-based approach can probably be traced back to 1970, with the querulous work of Torsten Hägerstrand [106]. He asserted that researchers in regional sciences should focus more on the intertwining of both disaggregate spatial and temporal aspects of human activities, as opposed to the more aggregate models in which the temporal dimension was neglected. This scientific field got commonly termed as *time geography*; it encompasses all time scales (i.e., from daily operations to lifetime goals), and focusses on the constraints that individuals face rather than predicting their choices [194].

Central to Hägerstrand's work was the notion of so-called *space-time paths* of individuals' activity and travel behaviour. In a three-dimensional space-time volume, two spatial dimensions make up the physical world plane, with the temporal dimension as the vertical axis. The journey of an individual is now the path traced out in this space-time volume: consecutive visits to certain locations are joined by a curve, with vertical segments denoting places where the individual remained stationary during a certain time period. The complete chain of activities (called a *tour*) is thus joined by individual *trip legs*. In this respect, the space-time path represents the *revealed outcome* of an unrevealed behavioural process [191]. An example of such a path can be seen in Fig. 3: we can see a woman going from her home in Boulder (Colorado, USA), to the university's campus, followed by a visit to the post office and grocery store, and finally returning home [80]. Note that Hägerstrand extended his notion in the space-time volume to include *space-time prisms* that encapsulate and effectively *constrain* all of a person's reachable points (i.e., all his/her possible space-time paths), given a certain maximum travel speed as well as both starting and ending points within the volume [61]. This environment is sometimes also referred to as a person's *action space*, enveloping that person's *time budget*.

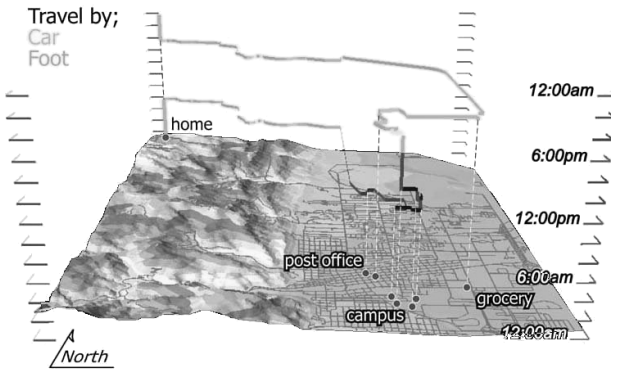


Figure 3: An example of a space-time path showing an individuals' activity and travel behaviour in the space-time volume: the two spatial dimensions make up the physical world plane, with the vertical axis denoting the temporal dimension. In this case, we can see a woman going from her home in Boulder (Colorado, USA), to the university's campus, followed by a visit to the post office and grocery store, and finally returning home (image reproduced after [80]).

Contrary to the belief that the field of activity-based transportation planning found its crux with the dissatisfaction of trip-based modelling, it grew and emerged spontaneously as a separate research study into human behaviour [191]. The underlying idea however remained the same as in the trip-based approach, namely that travel decisions arise from a need to participate in social, economical, and cultural activities. But as opposed to the more aggregated trip-based view, the basic units here are individual activity patterns, commonly referred to as *household activity patterns*. In this context, the activity-based approach then studies the interactions between members of a household, and the relation to their induced travel behaviour [12].

1.3.2 Approaches to activity-based modelling

Departing from Hägerstrand's initial comments, activity-based research progress has been slowly but steadily. In contrast with the development of the trip-based approach that culminated in the four step model, *there is no explicit general framework that encapsulates the activity-based modelling scheme*. There were however early compre-

hensive studies into human activities and their related travel behaviour, e.g., the synopsis provided by Jones et al. [145]. As the field began to mature, certain ingredients could be recognised, e.g. [12]:

- the *generation of activities*, which can be regarded as the equivalent of the production/attraction step in trip-based modelling,
- the modelling of *household choices*, i.e., with respect to their activity chains; this includes choosing starting time and duration of the activity, its location as well as a modal choice,
- the *scheduling of activities*, outlining how a household plans and executes the tasks of its members for long-, middle-, and short-term activities, going from year- and lifetime-long commitments, to daily operations.

During the last three decades, many research models that encompass activity scheduling behaviour have been developed. An excellent overview is given by Timmermans, who makes a distinction between *simultaneous* and *sequential models* [265]. The former class is based on full activity patterns (e.g., for one whole day), whereas the latter is based on an explicit modelling of the activity scheduling process. Simultaneous models comprise utility-maximisation models and mathematical programming models (e.g., Recker's household activity pattern problem – HAPP, [247]). Sequential models are frequently implemented as so-called computational process models (CPM), acknowledging the belief that individuals do not arrive at optimal choices, but rather employ context-dependent heuristics.

As an example, we illustrate the seminal STARCHILD model, which was originally a simultaneous model based on the maximisation of individuals' utilities. Based on Hägerstrand's notion and derivatives thereof, i.e., the central idea that an individual's travel behaviour is constrained by its space-time prism, Recker et al developed the STARCHILD research tool addressing activity-based modelling [248, 249]. The model hinges on three interdependent consecutive steps: (i) the generation of household activities, (ii) constructing choice sets for these activities, as well as scheduling them, and (iii) constraining these choices within the boundaries of the space-time prism [191]. Note that the principal critique to the model's operation, was — and today still is — its need for an extensive amount of specifically tailored data that encompasses Hägerstrand's concepts. Just as with the four step model, these data are arduous to come by. In short, most of the data are based on and transformed from e.g., conventional trip-based surveys, *travel diaries* (e.g., the MOBEL (Belgium) and MOBIDRIVE (Germany) surveys of Cirillo and Axhausen [59]) and the like, although more recently passive GPS based information is collected [12, 191, 194].

In the future, a complete integration of activity generation, scheduling, and route choice (DTA) is expected to take place, on the condition that suitable empirical data will become available. We must however be careful not to be too optimistic, as e.g., Axhausen states that depending on the 'research-political' adoption of the activity-based approach, *"both a virtuous circle of progress or a vicious circle of stagnation are a possibility for the future"* [12]. An even more harsh argumentation was voiced by Timmermans, who looked back at the development of the integration between land-use models and transportation planning [266]. In his overview, he identified three waves, i.e., (i) aggregate spatial interaction-based models, (ii) utility-maximising multinomial logit-based models, and (iii) activity-based detailed microsimulation models. His final conclusion states that, despite the advances in finer levels of spatial detail, the scientific field has not undergone any significant theoretical progress. And although there exists a pronounced need for better behavioural models, the critique remains that this implies a tremendous complexity, hence the insinuation that many of the approaches are in fact based on black-box models.

1.3.3 Towards elaborate agent-based simulations

One of the most notable critiques often expressed against classic trip-based approaches such as the four step model, is the fact that all eye for detail at the level of the individual traveller is lost in the trip aggregation process. Activity-based modelling schemes try to circumvent this disadvantage by starting from a fundamentally different basis, namely individual household activity patterns. To this end, it is necessary to retain all information regarding these individual households during the planning process.

As hinted at earlier, an upcoming technique that fits nicely in this concept, is the methodology of *multi-agent simulations*. In such models, the individual households are represented as *agents*; the models then allow these agents to make independent decisions about their actions. These actions span from long-term lifetime residential housing decisions, the mid-term planning of daily activities, to even short-term decisions about an individual's driving behaviour in traffic. The following description of such a simulation system is based on the work of the group of Nagel et al. [199, 13, 14, 15, 202, 122]:

- As a first step, a *synthetic population* of agents is generated. There is a close relation with the common land-use models, as these agents come from populations that should be correctly seeded, i.e., they should entail a correct demographic representation of the real world. Once the synthetic population is available to the model, the next step is to *generate activity patterns* (i.e., activity chains), generate these activities' *locations*, and finally the *scheduling* of the activities, as described in the previous section. Finally, *mode* and *route choice* form the bridge between the activity-based model and the transportation layer. As a consequence, it is beneficial to deal

with agents' plans directly, rather than to rely on the information contained in OD tables [13].

- The component that represents the *physical propagation of agents* throughout e.g., the road network, sits at the lowest level of the model. In this case, the necessary ingredients constitute the physical propagation of individual vehicles in the traffic streams. Popular models are *traffic cellular automata* and/or *queueing models*, allowing a fast and efficient simulation of individual agents in a network. Higher level models such as e.g., pure fluid-dynamic models are inherently not suitable because they operate on a more aggregated basis and consequently ignore the individuality of each agent in the system. Note that this latter type of model can be deemed appropriate, on the condition that they can incorporate the tracking of individual particles by e.g., a smoothed particle hydrodynamics method [14].

- An important issue that revolves around the two previous aspects, is the *clear absence of a rigidly defined direction of causality*, i.e., when exactly do people choose their travel mode, is it before the planning of activities, or is it rather a result from the planning process ? This problem can be dealt with in a broader context, wherein agents make certain plans about their activities, and *iteratively learn* by replanning and rescheduling (either on a *day-to-day* or *within-day* basis). This process of *systematic relaxation* continues until e.g., a Wardrop equilibrium (W1) is reached (see step (IV) in section 1.2.2 for more details). However, note that the question of whether or not people in reality strive to reach such an equilibrium, and whether or not such an equilibrium is even reached, remains an open debate. At this stage in the model, we are clearly dealing with aspects from evolutionary game theory, be it cooperative or non-cooperative. In this context, the concept of within-day replanning by agents is getting more attention, as it constitutes a necessary prerequisite for intelligent transportation systems, i.e., when and how do travellers react (e.g., *en-route choice*) to changes (e.g., control signals, incidents, ...) in their environment [14] ?

The above description of a multi-agent activity-based simulation system may seem straightforward, nevertheless, no complete practical implementation exists today. The model suite that comes the closest to reaching the previously stated goals, is the *TRansportation ANalysis and SIMulation System* (TRANSIMS[3]) project. This project is one part of the multi-track Travel Model Improvement Program of the U.S. Department of Transportation, the Environmental Protection Agency, and the Department of Energy in the context of the Intermodal Surface Transportation Efficiency Act and the Clean Air Act Amendments of 1990. Its original development started at Los Alamos National Laboratory, but a commercial implementation was provided by IBM Business Consulting.

Since its original inception, TRANSIMS has been applied to a various range of case studies. One of the most notable examples, is the truly country-wide agent-based detailed microsimulation of travel behaviour in Switzerland (see also Fig. 4) [282, 245]. A similar study encompassing the iteration and feedback between a simulation model and a route planner, was carried out for the region of Dallas [198, 204]. In the context of large-scale agent-based simulations, queueing models were employed as a TRANSIMS component by the work of Simon et al. for the city of Portland [256], as well as the work of Gawron [93, 94] and Cetin et al. [47]. Because of the computational complexity involved in dealing with the enormous amount of agents in real-world scenarios, a popular approach is the use of parallel computations, as described in the work of Nagel and Rickert [203]. Another example of this last type of simulations, is the work of Chopard and Dupuis who applied the methodology of large-scale simulations to the city of Geneva [53, 52, 83].

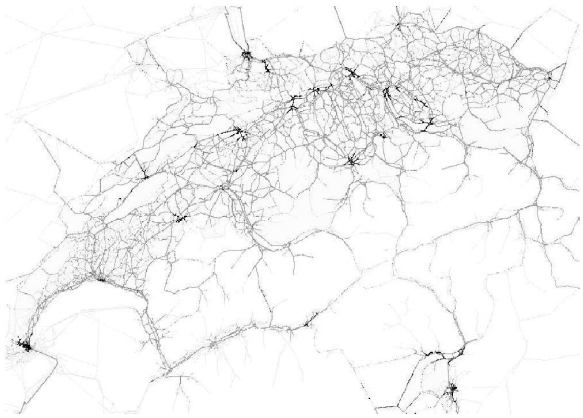


Figure 4: An example of a multi-agent simulation of the road network of Switzerland, around 08:00 in the morning: each vehicle is indicated by a single grey pixel, with low-speed vehicles coloured black. The image clearly reveals more vehicular activity (and congestion) in the city centres than elsewhere in the country (image reproduced after [282]).

As a final remark, we would like to draw attention to some more *control-oriented aspects of multi-agent simulations*. In this respect, the transportation system is considered as a whole, whereby the agents are now the local controllers within the system (e.g., traffic lights, variable message signs, ...), instead of individual households as was previously assumed. Using a coordinated control approach, is it then possible to achieve a system optimum. An example of such a control system is the *Advanced Multi-agent Information and Control for Integrated multi-class traffic networks* (AMICI[4]) project from The Netherlands. As one of its goals, it strives to provide routing information to different classes of road users, as

[3] `http://www.transims.net`

[4] `http://www.amici.tudelft.nl`

well as controlling them by means of computer simulated agents who operate locally but can be steered hierarchically.

1.4 Transportation economics

Most of the work related to traffic flow theory has been considered by researchers with roots in engineering, physics, mathematics et cetera. With respect to transportation planning, the scene has shifted somewhat during the last couple of decades towards policy makers who test and implement certain strategies, based on e.g., the four step modelling approach. Around 1960 however, another branch of scientists entered the field of transportation planning: economists developed standard models that viewed transportation as a market exchange between demand and supply.

Generally stated, the economics of transportation does not exclusively focus on traffic as a purely physical phenomenon, but also takes into account the fact that *transportation incurs certain costs*, both to the individual as well as to the society as a whole [172].

In the following sections, we describe the setting in which economists view transportation, after which we discuss the concept of road pricing.

1.4.1 The economical setting

In the context of economic theory, a transportation system can be seen as an interaction between *demand* (profits) and *supply* (costs). In a static setting, both demand and supply are frequently described by means of functions: they are expressed as the *price* for a good associated with the *quantity* of that good. In transportation economics, quantity is frequently identified as the *number of trips made* (e.g., by the macroscopic characteristic of traffic flow) [20]. In the remainder of this section, we use the term *travel demand* to denote the demand side, as opposed to the supply side which is composed of the transportation infrastructure (including changes due to incidents, ...). In a more broader context, travel demand is typically described as the amount of traffic volume that *wants* to use a certain infrastructure (i.e., the supply): when demand thus exceeds the infrastructure's capacity under congested conditions (implying queueing), this supply effectively acts as a constraint to the present volume of traffic flow.

According to the aforementioned conventions, a demand side function is expressed as a certain cost associated with a level of flow (i.e., number of trips). We call such a curve a *travel demand function* (TDF), and it is typically decreasing with increasing flow; an example of such a function can be seen as the dotted curve in Fig. 5. Note that, to be correct, the depicted curve actually represents a *marginal travel demand*: it gives the additional profit that is received with the obtaining of one extra unit (the total amount of profit is just the area under (i.e., the integral of) the demand curve). Translated to a transportation system, this means that the benefits of a traveller tend to decrease with increasing travel demand (i.e., congestion).

In similar spirit, we can consider a supply side curve, i.e., price (costs) versus quantity (flow). One of the most used approaches for describing traffic flow operations at the supply side from an economical point of view, is the use of an *average cost function* (ACF) [277] as proposed by A.A. Walters in 1961 [288]. The theory was based on the description of traffic flow by means of fundamental diagrams. Consider for example Greenshields' simple linear relation between density k and space-mean speed $\overline{v}_s$ [105, 182]. The corresponding $\overline{v}_{s_e}(q)$ fundamental diagram, consists of a tilted parabola, lying on its side [182]. As the travel time T is inversely proportional to the space-mean speed $\overline{v}_s$, Walters' idea was to assume certain *costs* related to the travel demand. Some examples of these costs are those associated with [27, 110, 111]:

- (i) the construction of the transportation infrastructure,
- (ii) vehicle ownership and use,
- (iii) taxes,
- (iv) travel time, i.e., *value of time* (VOT),
- and (v) environmental and social costs.

Based on these costs, and using the relation between travel time and travel demand, Walters derived a functional relationship for the economical cost C associated with the travel demand q. This relationship (i.e., the ACF) denotes the supply side of transportation economics; an example is the thick solid curve in Fig. 5.

Once both travel demand and average cost functions are known, they can be used to determine the *equilibrium points* that occur at their intersection(s): given both curves, the transportation system is assumed to settle itself at these equilibria, with a certain travel cost associated with the equilibrium traffic demand. Note that because of the nature of the analysis procedure, i.e., using static (stationary) curves, the resulting travel costs are *average* costs, hence the name of the average cost function.

There are some distinct features noticeable in the relation described by the average cost function. For starters, the curve does not go through the origin, i.e., at low travel demands there is already a *fixed cost* incurred. The depicted cost then typically increases with increasing travel demand, mainly due to the contribution of the value of time associated with the travel time. The most striking feature however, is the fact that the curve contains a *backward-bending* upper branch [86]. This peculiar branch has an asymptotic behaviour, i.e.,

$$\lim_{q \to q_{\text{cap}} \to 0} C(q) = +\infty, \tag{2}$$

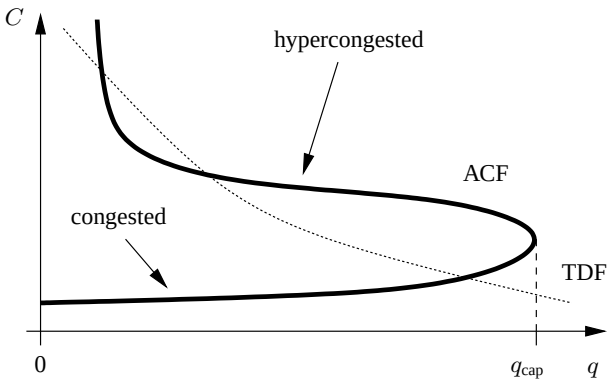


Figure 5: A graphical illustration of the economics of transportation operations: the dotted curve represents the demand side, i.e., the travel demand function (TDF), whereas the thick solid curve represents the supply side, i.e., the average cost function (ACF). Both curves express the cost C associated with the number of trips made (e.g., level of traffic flow q). The latter curve is said to have two states, namely the congested and the hypercongested area (identified as the backward-bending part of the curve). Points where both demand and supply curves intersect each other denote equilibrium points: given both curves, the transportation system as assumed to settle itself at their intersection(s), with a certain travel cost associated with the equilibrium traffic demand.

where we have denoted the path taken by the limit, i.e., passing through the capacity flow q_{cap} towards the upper branch, which in fact corresponds to an increase towards the jam density k_{jam}. Also note the presence of an inflection point (for concave $q_e(k)$ fundamental diagrams), which can be located analytically by differentiating the functional relation twice, and solving it with a right hand side equal to zero.

In contrast to the nomenclature adopted by the traffic engineering community and in this dissertation, economists typically refer to the lower branch of Fig. 5 as the *congested* state, and to the upper branch as the *hypercongested* state. Their line of reasoning being the conviction that in a certain sense, congestion also occurs when the speed drops below the free-flow speed on the free-flow branch [277, 172, 259].

With respect to the relevance of this hypercongested state, there has been some debate in literature. Among most economists there seems to be a consensus, in the sense that the hypercongested branch is actually a transient phenomenon [296, 172]. Walters thought of the branch as just a collection of inefficient equilibria, but it was shown by Verhoef that all equilibria obtained on the hypercongested branch are inherently unstable [277, 259]. Another argument, that discards the use of the branch, goes as explained by Yang and Huang [296]: many traditional economical models of transportation assume a static (stationary) model of congestion, similarly as in classic static traffic assignment described in section 1.2.3. Under this premise, the relations as described by the fundamental diagrams, should be considered for complete links, and not only — as is usual in traffic flow theory — at local points in space and time. Therefore, a link may contain two different states: a free-flow state and a congested state. Hence, the average cost function should only describe the properties that are satisfied on the whole link, and as a result this excludes the global hypercongested regime.

Several ad hoc solutions exist for dealing with this problem, which is a consequence of using cost functions based on stationary equilibria: some of these solutions typically entail the use of vertical segments near the capacity flow in Fig. 5, resulting in finite queueing delays on heavily congested links [277, 296, 259, 278]. Another much used solution that ignores the backward-bending branch, is to directly specify the average cost function based on a link's observed capacity, instead of deriving it through the fundamental diagram of space-mean speed versus flow [172]. Note that in most cases, *the economic interpretation of capacity is different from the one in traffic flow theory*: the capacity considered by economists has a lower value as opposed to its engineering counterpart. A similar example that specifies the relation between travel demand and travel time (e.g., VOT), is the BPR travel impedance function as described in section 1.2.3. Notwithstanding these proposed specific solutions, the mainstream tendency nowadays seems to imply the use of traffic flow models that explicitly describe the dynamics of congestion, either by using queueing models, or more elaborate models based on fluid dynamics or detailed simulations of individual vehicles [296].

1.4.2 Towards road pricing policies

In an economical treatment of transportation, road users in general only take into account their own *private costs*, such as (ii) vehicle ownership and use, (iii) taxes, and (iv) costs related to the travel time. Note that because, as mentioned earlier, we are working with *marginal cost functions*, the cost (i) related to the transportation infrastructure is not taken into account (as this is only a one-time initial cost).

To this end, we consider the average cost function from two different points of view: on the one hand, we have the *private costs* borne by an individual traveller, and on the other hand, we have the *external costs* that the traveller bears to the rest of society. In accordance with economic literature, we call the former associated cost function the *marginal private cost function* (MPCF), and the latter the *marginal social cost function* (MSCF). The extra costs to society brought on by individual travellers, are called *negative externalities*.

In Fig. 6, we have depicted the resulting equilibria that arise from the intersections of the travel demand function with both marginal private and social cost functions (note that we disregard the upper backward-bending branch of the average cost function as was shown in Fig. 5). In an unmanaged society, i.e., where no measures are taken

to change individual travellers' behaviour, the resulting equilibrium will be found at q_{ue}, which is in fact a *user equilibrium* corresponding to a cost as dictated by the marginal private cost function (MPCF) [9]. As travellers handle selfishly, not considering the costs inflicted upon other travellers (e.g., more road users imply more congestion *for everybody*), this pricing method is termed *average cost pricing*. At this equilibrium, the unpaid external cost to society equals the difference between the MSCF and MPCF curves at a demand level of q_{ue}.

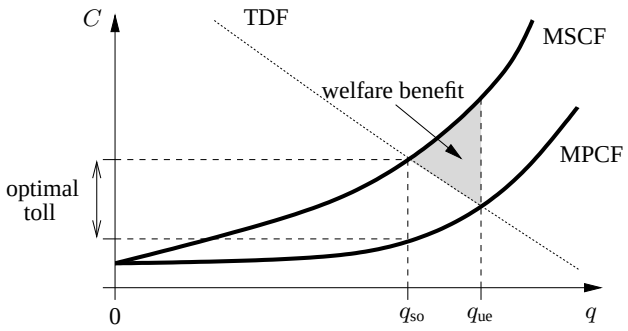


Figure 6: An economical equilibrium analysis based on a travel demand function (TDF) represented by the dotted curve, and marginal private and social cost functions (MPCF and MSCF) represented by the thick solid curves. The user equilibrium is located at a demand of q_{ue}, whereas the system optimum is located at a lower demand of q_{so}. The welfare benefit (indicated as the grey triangular region) can be gained by levying a congestion toll equal to the difference between the marginal social and private cost function defined at the system optimum demand level q_{so}.

As early as in 1920, Arthur Cecil Pigou noted that road users do not take into account the costs they inflict upon other travellers. In order to rectify this situation, he proposed to levy governmental taxes on road use. Pigou actually discussed his idea in a broader economic setting, by making a distinction between the private and the social costs. Charging of a suitable governmental tax could change the balance so the negative externalities would be included, resulting in a new equilibrium [237]. This process is called *internalising the external costs*.

Some years later in 1924, Frank Hyneman Knight further explored Pigou's ideas[5]: Knight fully acknowledged the fact that congestion justified the levying of taxes. In contrast to Pigou however, Knight raised some criticism in the sense that not governmental taxes were necessary, but instead private ownership of the roads would take care of tax levying and consequently resulting in a reduction of congestion [150].

In 1927, Frank Plumpton Ramsey cast this methodology — called *marginal cost pricing* — in the light of *social welfare economics*. This latter type employs techniques from a branch that is called micro-economics, which is an economical treatment of society based on the behaviour of individual producers and consumers. Welfare economics embraces two important concepts:

Efficiency: a measure for assessing how much benefit society gains from a certain policy rule. It can be considered with the *(strict) Pareto criterion* (invented by Vilfredo Pareto), which states that efficiency improves if a policy rule implies an increase of welfare for at least one individual, but no other individual of society is worse off. Nicholas Kaldor and John Hicks restated Pareto's criterion, but this time from the point of view of those who gain and those who lose, respectively. Their *Kaldor-Hicks criterion* states that society gains welfare, but not everybody receives personal gain, i.e., there will be winners and losers. The crucial assumption on the Kaldor-Hicks criterion is that the winners could fully compensate the losers, in theory; whether or not this happens at all, is not the issue.

Equity: if society benefits from a certain policy rule, then its efficiency can be measured using e.g., the Pareto criterion as stated earlier. However, nothing is said about the size of the benefit each individuals of society receives. This is were the concept of equity enters the picture: it refers to a fair distribution of the total benefits over all individuals in society (note that in this case, there typically is a strong correlation with the *income distribution*).

In this context, Ramsey thus stated a policy rule, implying a *maximisation of the social welfare* [244]. In the field of transportation, this can be done by marginal cost pricing, also called *road pricing, congestion tolls, ...* The nature of the measure is that it signifies a demand-side strategy, with the goal of inducing a change in travellers' behaviour. Road pricing typically entails a shift from on-peak to off-peak periods, switching mode (e.g., from private to public transportation), car pooling, route change, ... Considering again Fig. 6, we can see that if users were to consider the marginal social cost function, instead of only their marginal private cost function, this would shift the resulting equilibrium from q_{ue} to q_{so}, which is a *social optimum*. As said at the beginning of this section, travellers do not take into account the negative externalities they cause to the rest of society, and as such, they can be charged with an *optimal toll* that is defined as the difference between both marginal social and private cost functions. Levying the correct congestion toll, would remove the original market failure, resulting in a net social welfare benefit that is visualised as the grey triangular region in Fig. 6. Note that in an ideal world, congestion tolls exactly match the caused negative externalities. In practice however, this can not be accomplished, resulting in so-called *second-best pricing* schemes. Practical real-life examples of this are tolling the beltway around a city

[5] Note that Knight apparently was clued in his research by an error made on Pigou's behalf in his study of a two-route road network [30]. Even more intriguing, is the fact that this type of problem was already considered as far back as 1841, with the work of the German economist Johann Georg Kohl [151].

upon entering it (e.g., London's recent congestion charge), using step-tolls, tolling at fixed time periods instead of based on traffic conditions, ... [173]

Reconsidering the work of Wardrop with respect to the criteria (W1) and (W2) highlighted in section 1.2.2, Beckmann, McGuire, and Winsten found that the system optimum q_{so} can be reached if the standard cost (i.e., journey time) is replaced with a *generalised cost*, which is just the marginal social cost function as described earlier [20]. Consequently, the total travel time in the system can be minimised (from an engineering perspective), by levying a so-called efficiency toll, which corresponds to Ramsey's optimal toll.

One of the most notable extensions in the economic treatment of transportation and congestion tolls, is the seminal work of the late Nobel prize winner William Vickrey [281]. As already stated in section 1.2.5, correct modelling of e.g., queue spill back, is of fundamental importance when assessing the effectiveness of road pricing schemes for example. Vickrey's *bottleneck model* is one step in this direction: it is based on the behaviour of morning commuters, whereby the model takes into account the departure times of all travellers. As everybody's desire is to arrive at work at the same time, some will arrive earlier, others later. Besides the traditional travel time costs, travellers therefore also experience so-called *schedule delay costs*. Levying suitable tolls that depend on the travellers' arrival times, allows to reach a system optimum [8, 172]. It is important to realise here that the levied toll should vary over time and space, in order to correspond to the governing traffic conditions.

To most people in society road pricing is a highly unpopular measure, as well as a controversial political issue, whereby public acceptance is everything [188, 109, 110, 111]. Alternatives to road pricing can include upgrading existing roads and/or public transportation services, strict control-oriented regulation by means of *advanced traffic management systems* (ATMS), issuing elaborate parking systems, fuel taxes, et cetera [7]. In spirit of second-best pricing schemes, it was a wise thing in the UK to connect London's congestion charge to the simultaneous improvement of public transportation [258]. Similarly, the cordon toll system in the city of Oslo, Norway, quickly found acceptance among the population [109]. Nevertheless, road pricing is considered an *unfair* policy measure to most people: households (and firms) with higher incomes, can more easily afford to pay the charge, hence they will keep the luxury of travelling at their own discretion, whilst others might not be able to pay the required toll. As a consequence, an inconsiderate internalisation of the external costs, does not lead to an equitable Pareto optimal scenario. Despite this resistance, there does seem to be a general consensus among members of society that congestion caused by traffic induced by recreational activities, is not tolerated during peak periods; congestion tolling for these travellers is deemed appropriate.

Despite the advances in the methodology underlying road pricing, there is still one major gap that has yet to be filled in, i.e., the equity of the principle, or otherwise stated: where do the gained social welfare benefits (i.e., tax revenues) go in the redistribution ? As Small states, road pricing is more acceptable to the broad public, if it is presented as a complete financial package [257]. As welfare economists debate on whether or not the revenues should go back to the transportation sector or rather elsewhere, Small asserts that inclusion of the former is mandatory for achieving substantial support from both the political side and the investors. Complementary, in order to satisfy the global population, it is advisory to use the collected charges in order to diminish e.g., labour taxes, as they are perceived as being too high [103, 110, 111].

In the end, we should note that both economists and traffic engineers are essentially talking about the same subject, although by approaching it from different angles. In the field of economics, road pricing policies are introduced based on average cost functions, allowing an optimisation of the social welfare. This effectively corresponds to the engineers' idea of static traffic assignment, based on a system optimum using travel impedance functions (see e.g., section 1.2.3). The validity of using these average cost functions (with or without their backward-bending parts as explained in section 1.4.1), has instigated several debates in road pricing literature, most notably between Else and Nash [87, 208], Evans and Hills [88, 89, 130], and Ohta and Verhoef [224, 225, 279].

In continuation, the approach followed by Vickrey's bottleneck model provides a nice, first alternative, using schedule delay costs (see sections 1.2.5 and 1.4.2). Although Vickrey's idea introduces a hitherto absent time dependence, it has the disadvantage that spatial extents are neglected through the assumption of point-queues (see section 1.2.5). Lo and Szeto have rigourously shown that hypercongestion is essentially a spatial phenomenon, and that by neglecting this facet, a road pricing policy might actually worsen traffic conditions [176]. The correct way out of this problem, is by explicitly taking the tempo-spatial characteristics of traffic flows into account. As an engineering analogy, this can be accomplished by introducing dynamic traffic assignment (see section 1.2.4) which uses physical propagation models to describe the buildup and dissolution of congestion (see also some of the models presented in section 1.2.5, e.g., the work of Lago and Daganzo [159]).

2 Traffic flow propagation models

In contrast to the previous section, which dealt with high level transportation planning models, the current section considers traffic flow models that explicitly describe the

physical propagation of traffic flows. In a sense, these models can be seen as being directly applicable for the physical description of *traffic streams*. There exist several methods for discriminating between the families of models that describe traffic flow propagation, i.e., based on whether they operate in continuous or discrete time (or even event-based), whether they are purely deterministic or stochastic, or depending on the *level of detail* (LOD) that is assumed, ... More detailed explanations can found in the overview of Hoogendoorn and Bovy [134]. In this dissertation, we present an overview that is based on the latter method of discriminating between the level of detail. We believe that this classification most satisfactorily describes the discrepancies between the different traffic flow models. Thus, depending on the level of aggregation, we can classify the propagation models into the following four categories:

- **macroscopic** (highest level of aggregation, lowest level of detail, based on continuum mechanics, typically entailing fluid-dynamic models),
- **mesoscopic** (high level of aggregation, low level of detail, typically based on a gas-kinetic analogy in which driver behaviour is explicitly considered),
- **microscopic** (low level of aggregation, high level of detail, typically based on models that describe the detailed interactions between vehicles in a traffic stream),
- and **submicroscopic** (lowest level of aggregation, highest level of detail, like microscopic models but extended with detailed descriptions of a vehicles' inner workings).

Note that some people regard macroscopic models more from the angle of *network models*. In this case, the focus lies on *performance characteristics* such as total travel times (a measure for the quality of service), number of trips, ... [92] To this end, several quantitative models were introduced, such as Zahavi's so-called α relation between traffic flow, road density, and space-mean speed [299], and Prigogine and Herman's *two-fluid theory* of town traffic [128].

2.1 Macroscopic traffic flow models

In this section, we take a look at the class of traffic flow models that describe traffic streams at an aggregated level. We first introduce the concept behind the models (i.e., the continuum approach), after which we discuss the classic first-order LWR model. Because of its historical importance, we devote several sections to the model's analytical and numerical solutions, as well as to some extensions that have been proposed by researchers. We conclude our discussion of macroscopic models with a description of several higher-order models, and shed some light on the problems associated with both first-order and higher-order models.

2.1.1 The continuum approach

Among the physics disciplines, exists the field of *continuum mechanics* that is concerned with the behaviour of solids and fluids (both liquids and gasses). Considering the class of *fluid dynamics*, it has spawned a rich variety of branches such as aerodynamics, hydrodynamics, hydraulics, ...

Underlying these scientific fields, is the *continuity assumption* (also called the *continuum hypothesis*) that they all have in common. In a nutshell, this assumption states that fluids are to be treated as continuous media (in contrast to e.g., molecular gasses, which consist of distinct particles). Stated more rigourously, the macroscopic spatial (i.e., the length) and temporal scales are considerably larger than the largest molecular corresponding scales [62]. As a consequence, all quantities can be treated as being continuous (in the infinitesimal limit). The decision to use either a liquid-like or a gas-like treatment, is based on the *Knudsen number* of the fluid: a low value (i.e., smaller than unity) indicates a fluid-dynamic treatment, whereas a high value is indicative of a more granular medium. In this section, we consider the former approach. In the latter case, we enter the realm of statistical mechanics that deals with e.g., kinetic gasses, requiring the use of the Boltzmann equation (as will be explained in section 2.2 on mesoscopic traffic flow models).

Historically, the fluid-dynamic approach found its roots in the seminal work of Claude Louis Navier (1822), Adhémar de Saint-Venant (1843), and George Gabriel Stokes (1845) [101]. This gave rise to what we know as the *Navier-Stokes equations* (NSE), formulated as a set of *non-linear partial differential equations* (PDEs). For our overview, the most relevant equation is actually the local *conservation law*, stating that the net flux is accompanied by an increase or decrease of material (i.e., fluid). In general, we can discern two subtypes: *compressible* or *incompressible* fluids, and *viscous* or *inviscid* fluids. Incompressibility assumes a constant density (and a high *Mach number*), whereas inviscid fluids have a zero viscosity (with a corresponding high *Reynolds number*) and are typically represented as the *Euler equations*.

Note that the NSE are still not fully understood. The fact of the matter is that the *Clay Mathematics Institute* has devised a list of *Millennium Problems*[6], among which a deeper fundamental understanding of the NSE holds a reward of one million dollar. Because the original Navier-Stokes equations are too complex to solve, scientists developed solutions to specific subproblems, e.g., Euler's version of inviscid fluids. As an example, we give the *Burgers equation*, as derived by Johannes Martinus Burgers [39], for a one-dimensional fluid in the form of a *hyperbolic conservation law*:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2}, \tag{3}$$

[6] `http://www.claymath.org/millennium`

in which the $u \in \mathbb{R}$ typically represents the velocity, and ν is the viscosity coefficient. For inviscid fluids, $\nu = 0$, such that equation (3) corresponds to a *first-order* partial differential equation. This type of *hyperbolic PDE* is very important, as its solution can develop discontinuities, or more clearly stated, it can contain *shock waves* which are of course directly relevant to the modelling of traffic flows. The inviscid Burgers PDE can be solved using the standard *method of characteristics*, as will be explained in further detail in the next three sections.

2.1.2 The first-order LWR model

Continuing the previous train of thought, we can consider traffic as an inviscid but compressible fluid. From this assumption, it follows that densities, mean speeds, and flows are defined as continuous variables, in each point in time and space, hence leading to the names of *continuum models*, *fluid-dynamic models*, or *macroscopic models*.

The first aspect of such a fluid-dynamic description of traffic flow, consists of a *scalar conservation law* ('scalar' because it is a first-order PDE). A typical derivation can be found in [92] and [147]: the derivation is based on considering a road segment with a finite length on which no vehicles appear or disappear other than the ones that enter and exit it. After taking the infinitesimal limit (i.e., the continuum hypothesis), this will result in an equation that expresses the interplay between continuous densities and flows on a local scale. Another way of deriving the conservation law, is based on the use of a differentiable cumulative count function $\widetilde{N}(t,x)$ that represents the number of vehicles that have passed a certain location [215]:

$$\begin{aligned} k(t,x) = -\partial\widetilde{N}(t,x)/\partial x \quad &\text{and} \quad q(t,x) = \partial\widetilde{N}(t,x)/\partial t, \\ &\Downarrow \\ \frac{\partial k(t,x)}{\partial t} = -\frac{\partial^2\widetilde{N}(t,x)}{\partial t\,\partial x} \quad &\text{and} \quad \frac{\partial q(t,x)}{\partial x} = \frac{\partial^2\widetilde{N}(t,x)}{\partial t\,\partial x}, \\ &\Downarrow \\ \frac{\partial k(t,x)}{\partial t} \quad &+ \quad \frac{\partial q(t,x)}{\partial x} = 0, \end{aligned} \tag{4}$$

with the density k and flow q dynamically (i.e., time varying) defined over a single spatial dimension. Lighthill and Whitham were among the first to develop such a traffic flow model [169] (note that in the same year, Newell had constructed a theory of traffic flow at low densities [218]). Crucial to their approach, was the so-called fundamental hypothesis, essentially stating that flow is a function of density, i.e., there exists a $q_e(k(t,x))$ equilibrium relationship [182]. Essentially to their theory, Lighthill and Whitham assumed that the fundamental hypothesis holds at all traffic densities, not just for light-density traffic but also for congested traffic conditions. Using this trick with the fundamental diagram, relates the two dependent variables in equation (4) to each other, thereby making it possible to solve the partial differential equation.

One year later, in 1956, Richards independently derived the same fluid-dynamic model [251], albeit in a slightly different form. The key difference, is that Richards focusses on the derivation of shock waves with respect to the density, whereas Lighthill and Whitham consider this more from the perspective of disruptions of the flow [239]. Another difference between both derivations, is that Richards fixed the equilibrium relation, whereas Lighthill and Whitham did not restrict themselves to an a priori definition; in Richards' paper, we can find the equation $V = a(b - D)$, with V the space-mean speed, D the density, and a and b fitting parameters [251]. Note that all three authors share the following same comment: because of the continuity assumption, the theory only holds for a large number of vehicles, hence the description of "*long crowded roads*" in Lighthill and Whitham's original article.

> Because of the nearly simultaneous and independent development of the theory, the model has become known as the *LWR model*, after the initials of its inventors who receive the credit. In some texts, the model is also referred to as the *hydrodynamic model*, or the *kinematic wave model* (KWM), attributed to the fact that the model's solution is based on characteristics, which are called kinematic waves (e.g., shock waves).

2.1.3 Analytical solutions of the LWR model

Reconsidering equation (4), taking into account the fundamental diagram, the conservation law is now expressed as:

$$k_t + q_e(k)_x = 0, \tag{5}$$

in which we introduced the standard differential calculus notation for PDEs. Recognising the fundamental relation of traffic flow theory, i.e., $q = k\,\overline{v}_s$, equation (5) [182] then becomes:

$$k_t + (k\,\overline{v}_{s_e}(k))_x = k_t + \left(\overline{v}_{s_e}(k) + k\frac{d\overline{v}_{s_e}(k)}{dk}\right)k_x = 0. \tag{6}$$

The above hyperbolic PDE, can be translated into the Burgers equation (3), using a suitable transformation to a dimensionless form as explained in the rigourous mathematical treatment provided by Jüngel [147]. The conservation law (5) can also be cast in a non-linear wave equation, using the chain rule for differentiation [92]:

$$k_t + \frac{dq_e(k)}{dk}k_x = 0. \tag{7}$$

Analytically solving the previous equation using the method of characteristics, results in shock waves that travel with speeds equal to:

$$w = \frac{dq_e(k)}{dk}, \tag{8}$$

i.e., the tangent to the $q_e(k)$ fundamental diagram. This tangent corresponds to the speed w of the backward propagating *kinematic wave* [182]. As a consequence, solutions, being the characteristics, of equation (7) have the following form:

$$k(t, x) = k(x - wt), \tag{9}$$

with the observation that the density is constant along such a characteristic. Note that in order to obtain shock waves that are only decelerating, the used $q_e(k)$ fundamental diagram should be concave (a property that is often neglected) [79]:

$$\frac{d^2 q_e(k)}{dk^2} \leq 0. \tag{10}$$

Starting from an initial condition, the problem of finding the solution to the conservation PDE, is also called an *initial value problem* (IVP), whereby the solution describes how the density evolves with increasing time. The problem is called a *generalised Riemann problem* (GRP) when we consider an infinitely long road with given constant initial densities up- and downstream of a discontinuity.

Because the method of characteristics can result in non-unique solutions, a trick is used to select the correct, i.e., physically relevant, one. Recall from equation (3) that the right-hand side of the Burgers PDE contained a viscosity term ν. The general principle that is adopted for selection of the correct solution, is based on the *Oleinik entropy condition*, which regards the conservation law as a diffuse equation. In this context, the viscosity coefficient is multiplied with a small diffusion constant ϵ. In the *vanishing viscosity limit* $\epsilon \to 0$, the method returns a unique, smooth, and physically relevant solution instead of infinitely many (weak) solutions [164, 201]. For more details with respect to the application of traffic flows, we refer to the excellent treatment given by Jüngel [147].

Ansorge, Bui et al., Velan and Florian later reinterpreted this entropy condition, stating that it is equivalent to a *driver's ride impulse* [6, 37, 276]. Drivers going from free-flow to congested traffic encounter a sharp shock wave, whereas drivers going in the reverse direction essentially pass through all intermediate points on the fundamental diagrams, i.e., the solution generates a fan of waves. It is for this latter case that the 'ride impulse' is relevant: it denotes the fact that stopped drivers prefer to start riding again, resulting in a fan of waves.

A more intuitive explanation can also be given based on the *anticipation* that drivers adopt when they approach a shock wave: their equilibrium speed $\overline{v}_{s_e}(k)$ is also a function of the change in density, e.g.:

$$\overline{v}_{s_e}(k) \doteq \overline{v}_{s_e}(k) - \frac{\nu}{k}\frac{\partial k}{\partial x}. \tag{11}$$

Substituting this new equilibrium relation in equation (6), results in a right-hand side equal to $\nu \frac{\partial^2 k}{\partial x^2}$. Applying the same methodology based on the vanishing viscosity limit of the entropy solution, results in the same unique solution. Because the shock waves are in fact mathematical discontinuities, and as such, infinitesimally small, they are typically 'smeared out' by numerical schemes. In fact, this is just the equivalent of introducing an artificial viscosity (as explained earlier), which allows diffusion (i.e., the combined effect of dissipation and dispersion) of the shock waves. Note that this diffusion is a consequence of the numerical solution, and not necessarily corresponding to the real diffusion processes in a viscous fluid. This numerical smoothing helps to retain numerical stability of the final solution.

Whenever in the solution of the conservation equation, two of its characteristics intersect, the density takes on two different values (each one belonging to a single characteristic). As this mathematical quirk is physically impossible, the entropy solution states that both characteristics terminate and breed a *shock wave*; as such, these shock waves form boundaries that discontinuously separate densities, flows, and space-mean speeds [92]. The speed of such a shock wave is related to the following ratio [239]:

$$w_{\text{shock}} = \frac{\Delta q}{\Delta k}, \tag{12}$$

with $\Delta q = q_u - q_d$ and $\Delta k = k_u - k_d$ the relative difference in flows, respectively densities, up- and downstream of the shock wave.

Note that going from a low to a high density regime typically results in a shock wave, whereas the reverse transition is accompanied by an emanation of a *fan of characteristics* (also called *expansion*, *acceleration*, or *rarefaction waves*). In shock wave theory, the densities on either side of a shock are well defined (i.e., unique solutions exist); along the shock wave however, the density jumps discontinuously from one value to another. In this latter respect, equation (12) is said to satisfy the *Rankine-Hugoniot jump condition*.

The previous remarks with respect to the entropy condition, are closely related to the concavity of the $q_e(k)$ fundamental diagram, as defined by equation (10): for concave $q_e(k)$ fundamental diagrams, all shock waves are *compression waves* going from lower to higher densities. However, for $q_e(k)$ fundamental diagrams that contain convex regions, application of the entropy condition can return the wrong solution [165]. Although the mathematics of using these kinds of fundamental diagrams has

been worked out, see for example the work of Li [168], a unified physical interpretation is still lacking [181, 201]: instead of only deceleration shock waves and acceleration fans, we now also have acceleration shock waves and deceleration fans. Finally, it is important to realise that for non-smooth $q_e(k)$ fundamental diagrams, the entropy condition is not applicable and no fans occur because the correct unique solution is automatically obtained [276].

In Fig. 7, we have depicted a classic example that is often used when illustrating the tempo-spatial evolution of a traffic flow at a traffic light (left part), based on the LWR first-order macroscopic traffic flow model with a triangular $q_e(k)$ fundamental diagram (right part). The application of the traffic flow model is visible in the time-space diagram to the left. A traffic light is located at position x_{light}; it is initially green, and at t_{red} it turns red until t_{green} when it switches back to green. The initial conditions at the road segment are located at point ① on the fundamental diagram. Because all characteristics of the solution are tangential to the fundamental diagram, the results can be elegantly visualised when using a triangular diagram: except for the fan of rarefaction waves (we approximate the non-differentiable tip of the triangle with a smooth one, such that we can show the fan ④ for all didactical intents and purposes), only two kinematic wave speeds are possible. When the traffic light turns red, a queue of stopped vehicles develops. Inside this queue, the jam density state k_j holds, corresponding to point ② on the fundamental diagram. The upstream boundary of the queue is demarcated by the shock wave ③ that is formed by the intersections of the characteristics ① and ②. Downstream of the jam, there are no vehicles: because we are working with a triangular fundamental diagram, the characteristics are parallel to the vehicle trajectories (their speeds are equal to the slopes of points on the free-flow branch). The initial regime at state ① and the 'empty' regime downstream of the queue are separated from each other by a *contact discontinuity* or *slip*. When the traffic light turns green again, the queue starts to dissipate, whereby the solution of characteristics becomes a fan of rarefaction waves ④, taking on all speeds between states ② and ① on the fundamental diagram. A final important aspect that can be seen from Fig. 7, is the fact that in the LWR model the outflow from a jam, i.e., going from a high to a low density regime, always proceeds via the capacity-flow regime at q_{cap}: so *there is no capacity drop in the LWR model* because the outflow is always optimal.

To conclude our summary of analytical derivations, we point the reader to the significant work of Newell, who in 1993 cast the LWR theory in an elegant form. The key ideas he introduced were on the one hand the use of cumulative curves for deriving the conservation law, and on the other hand the use of a triangular $q_e(k)$ fundamental diagram [215]. Due to Newell's work, traffic flow analysis in this respect gets very simplified, as it is now possible to give an exact *graphical* solution to the LWR model for both free-flow and congested conditions [216]. To complete his theory, Newell also provided us with a means to include multi-destination flows, i.e., specifications of which off-ramp vehicles will use to exit the motorway [217]. Note that for the LWR model with a parabolic $q_e(k)$ fundamental diagram and piece-wise linear and piece-wise constant space and time boundaries, respectively, Wong and Wong recently devised an exact analytical solution scheme. Their method is based on the efficient tracking and fitting of generated and dispersed shock waves within a time-space diagram [295].

2.1.4 Numerical solutions of the LWR model

Besides the previous *analytic derivation* of a solution to the conservation law expressed as a PDE, it is also possible to treat the problem *numerically*. By trying to find a numerical solution to the PDEs, we enter the field of *computational fluid dynamics* (CFD). In a typical setup, the 'fluid domain' is first discretised into adjacent cells (called a one-dimensional *mesh*) of size ΔX (note that all cells need not to be equal in size), after which an *iterative scheme* is used to update the cells' states (e.g., the density k in each fluid cell) at discrete time steps $m\ \Delta T$ with $m \in \mathbb{N}_0$. Typically, this entails *finite difference schemes* (or in a broader context, *finite element methods* or *finite volume methods*), which replace the continuous partial derivative with a *difference operator*, thereby transforming the conservation equation into a *finite difference equation* (FDE). Examples of these difference operators are the *forward difference operator* $\Delta f(x) = f(x+1) - f(x)$ and the *backward difference operator* $\nabla f(x) = f(x) - f(x-1)$, which is not to be confused with the gradient vector of $f(x)$. Examples of finite difference schemes are the *central scheme*, the *Lax-Friedrichs scheme*, the *downwind scheme*, the *upwind scheme*, the *MacCormack scheme*, the *Lax-Wendroff scheme*, the *Steger-Warming Flux Splitter scheme*, the *Rieman-based Harten-van Leer-Lax and Einfeldt scheme*, ... For a more complete overview of these schemes, we refer the reader to the work of Helbing and Treiber [124], Jüngel [147], and Ngoduy et al. [222]. A practical software implementation of a *moving-mesh finite-volume solver* for the previously mentioned hyperbolic PDEs, can be found in van Dam's TraFlowPACK software [272]. LeVeque also developed a numerical solver, called CLAWPACK, that is designed to compute numerical solutions to hyperbolic partial differential equations using a wave propagation approach [166]. A central precaution for all these schemes, is the so-called *Courant-Friedrichs-Lewy (CFL) condition* which guarantees numerical stability of the algorithms; for traffic flows, it has the physical interpretation that no vehicles are allowed to 'skip' cells between consecutive time steps (i.e., all physical information that has an influence on the system's behaviour should be included):

$$\Delta T \leq \frac{\Delta X}{\overline{v}_{\text{ff}}}. \tag{13}$$

Just over a decade ago, Daganzo constructed a numerical

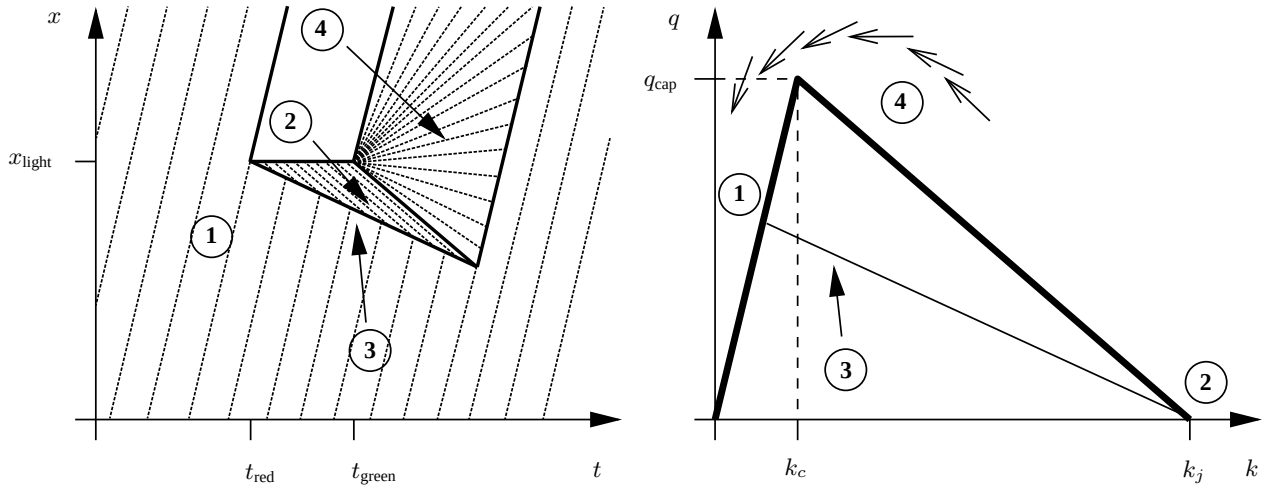


Figure 7: Example of an analytical solution based on the LWR first-order macroscopic traffic flow model with a triangular $q_e(k)$ fundamental diagram. *Left:* a time-space diagram with a traffic light located at position x_{light}. It is green, except during the period from t_{red} until t_{green}. The solution is visually sketched by means of the characteristics that evolve during the tempo-spatial evolution of the traffic flow. *Right:* a triangular fundamental diagram, with the initial conditions at state ①. When the traffic light is red, a queue develops in which the jam density state at point ② holds. Its upstream boundary is demarcated by the shock wave ③. When the queue starts to dissipate, the solution of characteristics generally becomes a fan of rarefaction waves ④.

scheme based on finite difference equations. It is known as the *cell transmission model* (CTM), which solves the LWR model using a trapezoidal $q_e(k)$ fundamental diagram [63]. At the heart of his model lies a discretisation of the road into finite cells of width ΔX, each containing a certain number of vehicles (i.e., an average cell density). When time advances, these vehicles are transmitted from upstream to downstream cells, taking into account the capacity constraints imposed by the downstream cells. The CTM converges to the LWR model in the limit when $\Delta X \to 0$. In 1995, Daganzo extended the model to include network traffic, i.e., two-way merges and diverges, thereby allowing for the correct modelling of dynamic queue spill backs [64]. He also cast the model in the context of *Godunov FDE methods*[7], allowing for arbitrary $q_e(k)$ fundamental diagrams. The exchange of vehicles between neighbouring cells is then governed by so-called *sending* and *receiving functions* [65]. Lebacque derived a similar numerical scheme that performed the same functions as Daganzo's CTM, at approximately the same time (the debate on whomever was first is still not resolved) [161, 162]. In his derivation, he employed the terms *demand* and *supply functions* to denote the exchange of vehicles between cells. He also provided the means to handle general (i.e., multi-way) merges and diverges. Both the original cell transmission model and an implementation of the Godunov scheme for the LWR model with an arbitrary $q_e(k)$ fundamental diagram, were provided by Daganzo et al. in the form of a software package called NETCELL [46]. Note that, as mentioned earlier, numerical methods tend to smear out the shock waves; this diffusion is therefore a consequence of the solution methodology and not of the LWR model itself [177].

Daganzo also developed another methodology for numerically solving the LWR equations, based on a *variational formulation*. Rather than extending the existing concept of a conservation equation coupled with a vanishing viscosity limit, he derived a solution based on the principles behind cumulative curves. The initial value problem becomes well-posed, and the methodology is able to handle complex boundary conditions. In short, the problem is transformed into finding shortest paths in a network of arcs that comprise the kinematic waves; as a surplus, the method is computationally more efficient than traditional solutions based on conservation laws [74, 75].

Traditional cell-based numerical methods are fairly computationally intensive, because they have to discretise the road entirely (even in regions where there is no variation in density), resulting in a solution that is composed of linear shock waves and continuous fans (i.e., the rarefaction waves). In order to derive a solution that is computationally more efficient, Henn proposed to replace the continuous fans of rarefaction waves with a discrete set of angular sectors (i.e., the density now varies with discrete steps). The efficiency now stems from the fact that, instead of a whole array of cells, only list structures need to be maintained [126].

Only recently, a combination of Daganzo's CTM with a triangular $q_e(k)$ fundamental diagram and Newell's cumulative curves was constructed by Yperman et al, resulting in the *link transmission model* (LTM). Because whole links can be treated at once, the LTM's computational efficiency is much higher than that of classic numerical

[7] Sergei K. Godunov's numerical solution of PDEs is considered as a breakthrough in computational fluid dynamics: it provides a unique solution based on a stable Riemann problem [102, 162, 70].

solution schemes for the LWR model, whilst retaining the same accuracy [**?**].

With respect to the applicability of the LWR model to real-life traffic flows, we refer the reader to two studies: the first was done by Lin and Ahanotu in the course of the *California Partners for Advanced Transit and Highways* (PATH) programme (formerly known as the *Program on Advanced Technology for the Highway*). In their work, they performed a validation for the CTM with respect to the formation and dissipation of queues, concluding that the most important first-order characteristics (correlations in measurements of free-flow traffic at successive detector stations, and the speed of the backward propagating wave under congested conditions) perform reasonable well when comparing them to field data [171].

A second, more thorough and critical study was done by Nagel and Nelson. In it, they scrutinise the LWR model, both with concave $q_e(k)$ fundamental diagrams and those with convex regions. Their main conclusion states that it remains difficult to judge the model's capabilities on a fair basis, largely due to the fact that there do not exist many real-world data sets which also contain a geometrical description of the local infrastructural road layout. This latter ingredient is a requirement for assessing whether or not an observed traffic breakdown is either spontaneously induced or due to the presence of an active bottleneck (because the LWR model constitutes a strictly deterministic model) [201].

2.1.5 Flavours of the LWR model

Considering this elegant first-order traffic flow model, its main advantages are that it is simple, and in a sense reproduces the most important features of traffic flows (i.e., shock waves and rarefaction waves). However, because of its restriction to a first-order partial differential equation, certain other effects, such as stop-and-go traffic waves, capacity drop and hysteresis, traffic flow instabilities, finite acceleration capabilities, ... can not be represented [169]. In many cases, these 'deficiencies' can be tackled by switching to higher-order models, as will be elaborated upon in section 2.1.6. Interestingly, in their original paper, Lighthill and Whitham recognised the fact that drivers tend to anticipate on downstream conditions, changing their speed gradually when crossing shock waves. This in fact necessitates a diffusion term in the conservation equation that captures a density gradient.

Instead of using a higher-order model, traffic flow engineers can also resort to more sophisticated approaches, such as extensions of the first-order model. To conclude this section, let us give a concise overview of some of the model flavours that have been proposed as straightforward extensions to the seminal LWR model.

An interesting set of extensions launched, was created by Daganzo, dealing with two classes of vehicles, of which one class can use all lanes of a motorway, whereas the other class is restricted to a right-hand subset of these lanes. When the capacity of the latter vehicles in regular lanes is exceeded, a queue will develop in those lanes, but the former vehicles will still be able to use the other lanes; this is called a *2-pipe* regime. Similarly, if the capacity of the yet freely flowing vehicles is exceeded, all lanes enter a queued state, which is called a *1-pipe* regime. In short, interactions between vehicles in this and the following models are nearly always considered from a user equilibrium perspective [68]. Daganzo et al. applied the theory to a case where there is a set of special lanes on which only priority vehicles can drive. The theory was also suited to describe congestion on a motorway diverge, such that the motorway itself can still be in the free-flow regime [69]. For the special case of queue spill back at a motorway's off-ramp, Newell also provided a graphical solution that is based on the use of cumulative curves [220].

Continuing the previous train of thought, Daganzo provided a logical extension: he again considered different lanes, but now introduced two different types of drivers: aggressive ones (called *rabbits*) and timid ones (called *slugs*). Daganzo himself states that a correct traffic flow theory should involve both human psychology and lane-changing aspects, leading him to such a *behavioural* description [71]. The theory was also used to explain the phenomenon of a capacity funnel [182]: according to the theory, once a capacity drop occurs, the recovery to the capacity-flow regime can not occur spontaneously, thereby requiring an exogeneous mechanism. Daganzo provides an explanation, called the *pumping phenomenon*: drivers temporarily accept shorter time headways downstream of an on-ramp, leading to a 'pumped state' of high-density and high-speed traffic, or in other words, a capacity-flow regime [72]. Chung and Cassidy later provided a validation of the theory, by applying it to describe merge bottlenecks on multi-lane motorways in Toronto (Canada) and Berkeley (California). In their study, they introduced the concept of *semi-congestion*, denoting a regime in which on vehicle class enters a state with a reduced mean speed, whereas the other vehicle class can still travel unimpeded. Their findings indicated an agreement between both shock wave speeds empirically observed and predicted by the model [58].

An interesting case to which the LWR theory can be applied, is the problem of *moving bottlenecks* as stated by Gazis and Herman [96]. Examples of such bottlenecks are slower trucks on the right shoulder lanes, which can impede upstream traffic. Newell was among the first to try to give a satisfactory consistent treatment of this type of bottlenecks. The trick he used was to translate the problem into a moving coordinate system that is travelling at the bottleneck's velocity. This resulted in a description of a stationary bottleneck, after which the classic LWR theory can be applied [219]. Although the theory is sound, there exist some serious drawbacks, mainly due to its underlying assumptions. For example, the moving bottlenecks are assumed to be long convoys, and other drivers'

behaviours are not affected by the bottlenecks' speeds; even more serious is the fact that the theory is not valid for very light traffic conditions, and that several strange effects are predicted by the theory (e.g., a bottleneck with increasing speed can result in a lower upstream capacity). To this end, Muñoz and Daganzo applied the previously mentioned behavioural model with rabbits and slugs to the problem of a moving bottleneck. Their theory performs satisfactorily and agrees well with empirically observed motorway features. However, because of the fact that it relies on the LWR model, it is not entirely valid for bottlenecks that travel at high speeds under light traffic conditions. In this latter case, they state that driver differences are much more important than the dynamics dictated by the kinematic model [197].

Another theory that deals with the problem of moving bottlenecks, is the one proposed by Daganzo and Laval: they treat moving bottlenecks as a sequence of consecutive *fixed obstructions* that have the same capacity restraining effects. Despite the fact that the previous theory of Muñoz and Daganzo has a good performance, it does not easily lend itself to discretisation schemes that allow numerical solutions. In contrast to this, the hybrid theory (fixed obstructions coupled with the LWR model dynamics) of Daganzo and Laval holds high promise as they have shown that it can be discretised in a numerically stable fashion [76]. As a continuation of this work, Lavel furthermore investigated the power of these fixed obstructions, allowing him to capture lane-changes as random events modelled by moving bottlenecks in a LWR 1-pipe regime. It is suggested that (disruptive) lane changes form the main cause for instabilities in a traffic stream. This leads the 'Berkeley school' to the statement that incorporating lane-change capabilities into multi-lane macroscopic models seems a prerequisite for observing effects such as capacity drops, kinematic waves of fast vehicles, ... [160] In this respect, Jin provides a theory that explicitly takes into account to effects of lane changes [144]. The starting point in this model, is the presence of certain road areas in which traffic streams mix. The underlying assumption here is that all lane changes lead to the same traffic conditions in each lane: the crucial element in the model is that vehicles performing lane changes are temporarily counted twice in the density total. This new 'effective density', is then used to transform the $q_e(k)$ fundamental diagram, leading to a reversed lambda shape. However, because the current version of the theory employs a small artificial constant to introduce the lane changes, we question its practical applicability when it comes to calibration and validation.

To conclude this overview of the first-order models, we highlight two other successful attempts at increasing the capabilities of the classic LWR model. A first important extension was made by Logghe, who derived a *multi-class formulation*[8] that allows for the correct modelling of heterogeneous traffic streams (e.g., preserving the FIFO property for interacting classes). Classes are distinguished by their maximum speed, vehicle length, and capacity (all intended for a triangular $q_e(k)$ fundamental diagram). A central ingredient to his theory, is the interactions between different user classes that reside on a road: in this respect, each class acts selfishly, with slower vehicles taking on the role of moving bottlenecks. Besides being able to construct analytical and graphical solutions, Logghe also provided a stable numerical scheme, as well as a complete network version with road inhomogeneities, and two-way merges and diverges [177]. A second extension was made by Jin; it actually deals with a whole plethora of extensions, in particular for inhomogeneous links (e.g., lane drops), merge and diverge zones, and mixed-type vehicles (i.e., having different $q_e(k)$ fundamental diagram characteristics). All these models are then combined in a description for a *multi-commodity kinematic wave model for network traffic*, whereby the commodities signify vehicles taking different paths [143].

A finally important aspect that is mainly related to lane changes, is the *anisotropy property* of a traffic stream. This property basically states that drivers are not influenced by the presence upstream vehicles. In a sense, most models describing the acceleration behaviour of a vehicle, only take into account the state of the vehicle directly in front. For most macroscopic traffic flow models, this anisotropy constitutes a necessary ingredient. However, in his original paper, Richards very subtly points out that the fact of whether or not drivers only react to the conditions ahead, remains an open question [251]. In contrast to this, Newell states that a driver is only influenced by downstream conditions, leading to a natural cause-and-effect relation, making the problem mathematically well-posed [216]. Recently, Zhang stated that the anisotropy property is likely to be violated in multi-lane traffic flows. His explanation is closely tied to the concavity character of a $q_e(k)$ fundamental diagram (non-concave regions can lead to characteristics that travel faster than the space-mean speed of the traffic stream). He also provides an intuitive reasoning based on Daganzo's rabbits and slugs, whereby *tailgating* vehicles induce slower downstream vehicles to 'make way'. Note that for single-lane traffic flows, the anisotropy property is expected to hold because of the FIFO property (vehicles can not pass each other), although there are exceptions in the case of some higher-order macroscopic traffic flow models [305].

2.1.6 Higher-order models

The development of higher-order macroscopic models came as a response to the apparent shortcomings of the

[8] At approximately the same time, Chanut and Buisson constructed a first-order model that incorporates vehicles with different lengths and free-flow speeds [49]. Their model can be considered as a trimmed-down version of Logghe's multi-class formulation.

first-order LWR model. Harold Payne was among the first in 1971 to develop such a higher-order model [231]. In those days, ramp metering[9] control strategies were basically an all-empirical occasion. Payne recognised the necessity to include dynamic models in the control of on-ramps; the celebrated LWR model however, was found to perform unsatisfactorily with respect to the modelling of real-life traffic flows. One of these shortcomings, was the model's inability to generate stop-and-go waves. Zhang later traced this to be a consequence of the model's persistent reliance on a single equilibrium curve (i.e., the fundamental diagrams) [306]. In the LWR model, drivers are assumed to adapt their vehicle speed *instantaneously* according to the fundamental diagram when crossing a shock wave, a phenomenon termed the *no-memory effect* (i.e., they encounter infinite accelerations and decelerations [300]). One option that leads to a solution of the previously mentioned problems, is to introduce different fundamental diagrams for vehicles driving under different traffic conditions; this avenue was explored by Newell [213] and Zhang [301]. Another, more popular type of solution was proposed by Payne (as well as by Whitham some years later [291]): they suggested to add an equation to the LWR conservation law (6) and its fundamental diagram[10]. This new *dynamic speed equation* was derived from the classical car-following theories of Gazis et al. [97] (see also section 2.3.1 for more details). An important aspect is this derivation, is the fact that the car-following model includes a *reaction time*, resulting in a *momentum equation* that relates the space-mean speed of a vehicle stream to its density. As a result, vehicles no longer instantaneously change their speed when crossing a shock wave. Payne's second-order macroscopic traffic flow model is now described by the following pair of PDEs, i.e., a conservation law and a momentum equation:

$$k_t + (k\,\overline{v}_s)_x = 0, \tag{14}$$

$$d\overline{v}_s = \overline{v}_{s_t} + \underbrace{\overline{v}_s \overline{v}_{s_x}}_{\text{convection}} = \underbrace{\frac{\overline{v}_{s_e}(k) - \overline{v}_s}{\tau}}_{\text{relaxation}} - \underbrace{\frac{c^2(k)}{k} k_x}_{\text{anticipation}}, \tag{15}$$

with $\overline{v}_{s_t}$ and $\overline{v}_{s_x}$ denoting the partial derivatives of the space-mean speed with respect to time and space, respectively, $\overline{v}_{s_e}$ the traditional fundamental diagram, and τ the *reaction time*. The function $c(k)$ corresponds to the model-dependent *sound speed of traffic* (i.e., the typical speed of a backward propagating kinematic shock wave); examples of $c(k)$ are [307]:

$$-\sqrt{-\frac{1}{2\tau}\frac{d\overline{v}_{s_e}(k)}{dk}} \qquad \text{(Payne)} \tag{16}$$

$$-\sqrt{\frac{v}{\tau}} \qquad \text{(Whitham)} \tag{17}$$

$$k\,\frac{d\overline{v}_{s_e}(k)}{dk} \qquad \text{(Zhang)} \tag{18}$$

with v being a parameter in equation (17).

In equation (15), the left hand side corresponds to the derivative of the speed, i.e., the acceleration of vehicles. As can be seen from the formulation, Payne identified three different aspects for the momentum equation: a *convection* term describing how the space-mean speed changes due to the arrival and departure of vehicles at the time-space location (t, x), a *relaxation* term describing how vehicles adapt their speed to the conditions dictated by the fundamental diagram, but with respect to a certain reaction time (as opposed to the instantaneous adaption in the LWR model), and finally an *anticipation* term describing how vehicles react to downstream traffic conditions.

In continuation of the above derivation, many other higher-order models have been based on the Payne-Whitham (PW) second-order traffic flow model. An example is the work of Phillips, who changed the reaction time τ in the relaxation term of equation (15) from a constant to a value that is dependent on the current density [236]. Another example is due to Kühne, who artificially introduced a viscosity term into equation (15), in order to smooth the shock waves [157]. The physical role that viscosity plays in a vehicular traffic stream is however not entirely understood: according to Zhang, the viscosity reflects the resistance of drivers against sharp changes in speeds [306]. For a rather complete overview of extensions to the PW model, we refer the reader to the work of Helbing [121].

2.1.7 Critiques on higher-order models

Higher-order models have been successfully applied in various computer simulations of traffic flows, e.g., the original FREFLO implementation by Payne [232], the work of Kwon and Machalopoulos who developed KRONOS which is an FDE solver for a motorway corridor [158], the METANET model of Messmer and Papageorgiou [193], ... Despite their success, it was Daganzo who in 1995 published their final requiem, which stood out as an obituary for all higher-order models [67]. From a theoretical perspective, there were some serious *physical flaws* that littered these second-order models. Most notably was the fact that there exist two families of characteristics (called *Mach lines*) in the Payne-Whitham type models. On the one hand, there are characteristics that imply a diffusion-like behaviour, which under certain circumstances can lead to negative speeds at the end of a queue, i.e., vehicles travelling backwards. On the other hand, there are characteristics that have the property of

[9] Ramp metering is an ATMS whereby a traffic light is placed at an on-ramp, such that traffic enters the highway from the on-ramp by drops. We refer the reader to the work of Bellemans [22] and Hegyi [114] for an overview and some recent advancements.

[10] Note that Lighthill and Whitham originally proposed to extend the conservation law in their model with relaxation and diffusion terms, but the idea did not receive much thought at the time [169].

travelling faster than the propagation of traffic flow. This latter gas-like behaviour means that vehicles can get influenced by upstream conditions (because information is sent along the characteristics), which is a clear violation of the anisotropy property for single-lane traffic as explained in the previous section. From a physical point of view, the relaxation term in equation (15) may even introduce a 'suction process' because slower vehicles can get sucked along by leading faster ones [113].

Several years after these critiques, Papageorgiou responded directly to the comments stated in Daganzo's article [227]. In his response, Papageorgiou put a lot of emphasis at the incapabilities of first-order traffic flow models for use in a traffic control strategy (e.g., ramp metering). He very briefly reacts to the anisotropy violation, by mentioning that in multi-lane traffic flows, the space-mean speeds of the different lanes are not all the same, leading to characteristics that are *allowed* to travel faster than the space-mean speed of all lanes combined. With respect to negative speeds (and hence, negative flows), he proposes to simply include an a posteriori check that allows to set the negative flows equal to zero. One year later, in 1999, Heidemann reconsidered these higher-order models, but this time from the perspective of *mathematical flaws*. His main argument was the fact that the models led to an internal inconsistency, because they ignored some aspects related to the conservation law [115]. However, after careful scrutiny, Zhang later refuted Heidemann's claims: the inconsistencies that plague the models are a result of the insistence on the universality of a conservation law and the imposing of arbitrary solutions. As a consequence, the Payne-Whitham type of models are mathematically consistent theories, although they may suffer from the aforementioned physical quirks [307].

Note that the dynamic speed equation (15), can also be cast in another form that is more closely related to a gas-kinetic analogy. With this in mind, we can rewrite the momentum equation as follows [134]:

$$\begin{aligned} d\overline{v}_s &= \overline{v}_{s_t} + \underbrace{\overline{v}_s \overline{v}_{s_x}}_{\text{transport}} \\ &= \underbrace{\frac{\overline{v}_{s_e}(k) - \overline{v}_s}{\tau}}_{\text{relaxation}} - \underbrace{\frac{P_x}{k}}_{\text{pressure}} + \underbrace{\frac{\nu}{k}\,\overline{v}_{s_{xx}}}_{\text{viscosity}}, \end{aligned} \tag{19}$$

with now P denoting the *traffic pressure* and ν the *kinematic traffic viscosity* (as introduced by Kühne [157]). The convection term has been relabelled a *transport term*, describing the propagation of the speed profile with the speed of the vehicles. The *pressure term* reflects the change in space-mean speed due to arriving vehicles having different speeds, and the *viscosity term* reflects changes due to the 'friction' between different successive vehicles. The classic Payne model is obtained if we set $P = kc^2(k)$ and $\nu = 0$.

In contrast to Papageorgiou's response which did not provide a definite answer, Aw and Rascle carefully examined the reason why the PW model exhibited the strange phenomena indicated in Daganzo's requiem [11]. The root cause of this behaviour can be traced back to the spatial derivative P_x of the pressure term in equation (19). Their solution suggests to abandon the transport and relaxation terms, and replace the spatial derivative of the pressure P (which is a function of the density k) with a *convective (Lagrangian) derivative*, i.e., $D/Dt = \partial_t + (\overline{v}_s \cdot \nabla) = \partial_t + \overline{v}_s \partial_x$, with $(\overline{v}_s \cdot \nabla)$ called the *advective derivative term* [240]:

$$(\overline{v}_s + P(k))_t + \overline{v}_s\,(\overline{v}_s + P(k))_x = 0. \tag{20}$$

This new formulation allows to remedy all Daganzo's stated problems [11]. Because of the somewhat limited character of their derivation of equation (20), Rascle add a relaxation term to the equation's right-hand side, and developed a numerically stable discretisation scheme, as well as showing convergence to the classic LWR model when the relaxation tends towards zero [246].

To end our overview of higher-order models, we illustrate two other types. The first model is actually a *third-order model* created by Helbing. It is based on the two PDEs of the Payne-Whitham type models, but is extended with a third equation that describes the change in the *variance of the speed*, denoted by Θ [118]. Helbing derived his equations using a gas-kinetic analogy, resulting in the following Navier-Stokes-like equation (it is typically encountered in the pressure term for P):

$$\begin{aligned} \Theta_t + \overline{v}_s \Theta_x &= \underbrace{\frac{2(\Theta_e(k) - \Theta)}{\tau}}_{\text{relaxation}} + \\ &\quad \underbrace{\frac{2}{k}\,\overline{v}_{s_x}(\nu\,\overline{v}_{s_x} - P)}_{\text{pressure}} + \\ &\quad \underbrace{\frac{\nu}{k}\,\overline{v}_{s_{xx}} + \frac{\kappa}{k}\,\Theta_{xx}}_{\text{viscosity}}, \end{aligned} \tag{21}$$

with now the equilibrium relation $\overline{v}_{s_e}(k, \Theta)$ of equation (19) also depending on the speed variance Θ. In addition to the viscosity ν, the dynamic speed variance equation (21) also contains an equilibrium relation $\Theta_e(k, \overline{v}_s)$ for the variance of the speed, and κ which is a *kinetic coefficient* that is related to the reaction time τ, the density k, and the speed variance Θ. For $\nu = \kappa = 0$, Helbing's model reduces to an inviscid Euler type model as explained in section 2.1.1 [149]. Whereas in the LWR model there is only one family of characteristics, and in the PW model there are two families, the Helbing model generates three different families of characteristics; this implies that small perturbations in the traffic flow propagate

both with the traffic flow itself, as well as in upstream and downstream direction relative to this flow [134].

The second model we illustrate, is the *non-equilibrium model* of Zhang. Because of the relaxation terms in the Payne-Whitham equations, drivers initially tend to 'overshoot' the equilibrium speed as dictated by the $\overline{v}_{s_e}(k)$ fundamental diagram. It takes a certain amount of time for them to adapt to their speed to the new traffic conditions (i.e., a change in density is accompanied by a *smooth* change in space-mean speed), after which they converge on the diagram. This latter aspect gives rise to the empirically observed scatter in the (k,q) phase space, leading Zhang to the terminology of 'non-equilibrium models' because of the deviation from the one-dimensional equilibrium fundamental diagram [300].

In his model, Zhang considers equilibrium traffic to be a state in which $d\overline{v}_s/dt = \partial k/\partial x = 0$. In similar spirit of Payne's theory, Zhang constructs his model using an equilibrium relation between density and space-mean speed (i.e., the fundamental diagram), a reaction time that allows relaxation, and an anticipation term that adjusts the space-mean speed to downstream traffic conditions. This results in a macroscopic model that contains equation (14) as the conservation law, as well as the following momentum equation:

$$\begin{aligned} d\overline{v}_s &= \overline{v}_{s_t} + \overline{v}_s \overline{v}_{s_x} \\ &= \frac{\overline{v}_{s_e}(k) - \overline{v}_s}{\tau} - k\left(\frac{d\overline{v}_{s_e}(k)}{dk}\right)^2 k_x, \quad (22) \end{aligned}$$

with the last anticipation term showing the dependence on the spatial change of the density. Zhang also complements the theory with a finite difference scheme that allows to solve the equations in a numerically stable fashion, based on an extension of the Godunov scheme that satisfies the entropy condition referred to in section 2.1.3 [303].

Just as with the improved PW model of Aw and Rascle, this model alleviates Daganzo's stated problem of wrong-way travel, even though there are also two families of characteristics, travelling slower, respectively faster, than the space-mean speed of traffic. An important fact here is that for the slower characteristics, the associated shock waves and fans correspond perfectly to those of the first-order LWR model. However, the shock waves and fans associated with the faster family of characteristics can still violate the anisotropy property of traffic (although they decay exponentially), but in the end, Zhang questions its universal validity, stating that traffic might occasionally violate this principle due to the heterogeneity of a traffic stream [302, 305]. The violation of anisotropy, i.e., drivers get influenced by upstream traffic, is sometimes referred to as gas-like behaviour, because in contrast to fluid-dynamics, gas particles are not anisotropic. In an attempt to remove this faulty behaviour, Zhang developed yet another non-equilibrium model that removed the gas-like behaviour, thereby respecting the anisotropy property. Moreover, both families of characteristics in his model satisfy the condition of travelling at a lower speed than the space-mean speed of the traffic stream, but still keeping the one-to-one correspondence between the slower characteristics and those of the first-order LWR model. At present, it is however unclear if this new model can generate stop-and-go waves, although there are indications that it can because of the non-equilibrium transitions that can occur [304].

Despite the significant progress that has been made on the front of higher-order macroscopic traffic flow models, the Berkeley school firmly holds its faith in first-order models and their extensions. Its main reason is because of the numerical solution schemes that are well developed and understood. This is not the case for higher-order models, as these contain other characteristics that complicate the finite difference schemes (because information is now carried both up- and downstream, and because their numerical schemes initially were flawed [303, 160]). Related to this critique, is the fact that in contrast to the first-order model, no analytical solutions exist for the higher-order models.

Another reason for sticking with first-order models, is because the school believes that first-order characteristics are sufficient for the description of traffic flows [45]. Using a triangular $q_e(k)$ fundamental diagram that captures the most important traffic flow characteristics (i.e., the free-flow speed $\overline{v}_{\text{ff}}$, the capacity q_{cap}, the jam density k_j, and the backward kinematic wave speed w), results in a further elegance of the models.

2.2 Mesoscopic traffic flow models

The previous section dealt with macroscopic models that described traffic streams at an aggregated level, derived from a fluid-dynamic analogy. This section describes how traffic can be modelled at this aggregate level, but with special consideration for microscopic characteristics (e.g., driver behaviour). Because of the ambiguity that surrounds mesoscopic models, we first elucidate what is meant by the term mesoscopic (i.e., it is something between a macroscopic and a microscopic approach). In the sections thereafter, we zoom in on a derivation of mesoscopic models based on a gas-kinetic analogy. For an outstanding overview of gas-kinetic models, we refer the reader to the work of Tampère [262].

2.2.1 The different meanings of 'mesoscopic'

Considering the amount of literature that has been generated during the last few decades, it seems to us that there exists no unanimous consensus as to what exactly constitutes mesoscopic traffic flow models. In general, there

are three popular approaches when it comes to mesoscopic models [134]:

- **Cluster models**

  When considering vehicles driving on a road, a popular method is to group nearby vehicles together with respect to one of their traffic flow characteristics, e.g., their space-mean speed. Instead of having to perform detailed updates of all vehicles' speeds and positions, the cluster approach allows to treat these vehicles as a set of groups (called *clusters*, *cells*, *packets*, or *macroparticles*); these groups are then propagated downstream without the need for explicit lane-changing manoeuvres (leading to the coalescing and splitting of colliding and separating groups).

  Examples of this kind of models, are the *CONtinuous TRaffic Assignment Model* (CONTRAM) of Leonard et al. [163], the work of Ben-Akiva et al, called *Dynamic network assignment for the Management of Information to Travellers* (DynaMIT), which is based on a cell transmission model with a cell of a link containing a set of vehicles with identical speeds [24], the *Mesoscopic Traffic Simulator* (MesoTS) of Yang, which allows fast predictions of future traffic states [297], ...

- **Headway distribution models**

  This rather unknown and somewhat outdated class of models, places the emphasis on the probability distributions of time headways of successive vehicles (this aggregation makes them mesoscopic). Two popular examples are Buckley's semi-Poisson model [36], and Branston's generalised queueing model [34]. As clarified in the summary of Hoogendoorn and Bovy, the original versions of these headway distribution models assume homogeneous traffic flows and they are inadequate at describing the proper dynamics of traffic flows [134].

- **Gas-kinetic models**

  The third and most important characterisation of mesoscopic models comes from a gas-kinetic analogy. Because macroscopic models aim towards obeying the fundamental diagram (either instantaneously as in the first-order LWR model or through a relaxation process as in higher-order models), the focus there lies on the generation and dissipation of shock and rarefaction waves. As a consequence, more complex and non-linear dynamics can not be reproduced. To remedy this, gas-kinetic models implicitly bridge the gap between microscopic driver behaviour and the aggregated macroscopic modelling approach.

In the next sections, we will first give an overview of the original gas-kinetic model as derived by Prigogine and Herman, after which we discuss some of the recent successful modifications that allow for heterogeneity in the traffic stream (i.e., multi-class modelling), as well as the inclusion of more specific driver behavioural characteristics.

2.2.2 Mesoscopic models considered from a gas-kinetic perspective

As opposed to the macroscopic traffic flows models that are derived from a conservation equation based on the Navier-Stokes equations, mesoscopic models can be derived from a gas-kinetic analogy. From individual driving behaviour (termed a microscopic approach), a macroscopic model is derived. The earliest model can be traced back to the work of the late Nobel laureate Ilya Prigogine, in cooperation with Frank Andrews and Robert Herman [241, 242]. A central component in their theory, is the concept of a phase-space density (PSD):

$$\widetilde{k}(t, x, \overline{v}_s) = k(t, x)\ P(t, x, \overline{v}_s), \tag{23}$$

in which $P(t, x, \overline{v}_s)$ denotes the *distribution* of the vehicles with space-mean speed $\overline{v}_s$ at location x and time t; the concept of this distribution originated in Boltzmann's theory of gas dynamics. For the above density function, a kinetic conservation equation can be derived, looking as follows [121]:

$$\frac{d\widetilde{k}}{dt} = \widetilde{k}_t + \overline{v}_s\ \widetilde{k}_x = \left(\widetilde{k}_t\right)_{\text{acc}} + \left(\widetilde{k}_t\right)_{\text{int}}, \tag{24}$$

with now the two terms on the right hand side denoting the *accelerations* of and *interactions* between the vehicles; they are also called *gains* and *losses*, *relaxation* and *slowing down*, or *continuous* and *discrete* terms, respectively [149, 262]. Equation (24) is called the *Prigogine-Herman kinetic model* and it actually describes three processes:

1. Similar to the macroscopic conservation equation, the term $\overline{v}_s\ \widetilde{k}_x$ describes a convective behaviour: arriving and departing vehicles cause a change in the distribution $\widetilde{k}$ of vehicle speeds.

2. The first term on the equation's right hand side, $\left(\widetilde{k}_t\right)_{\text{acc}}$, describes the acceleration behaviour of vehicles, which is assumed to be a *density-dependent relaxation* process of the speed distribution P of equation (23) towards some pre-specified target speed distribution P_0 (typically based on an equilibrium speed).

3. The second term on the equation's right hand side, $\left(\widetilde{k}_t\right)_{\text{int}}$, describes the interactions between vehicles, as fast vehicles either must slow down or overtake slower ones (hence implying inherently multi-lane traffic). The decision on when to either slow down or to overtake (which is assumed to be a *discrete event*), is governed by the probabilities $(1 - \pi)$ and

π, respectively. The interaction term is called a *collision equation*, in analogy with the physics of the Boltzmann equation (where the collision term describes the scattering of the gas molecules). Because there occur joint distributions in this latter equation (i.e., the probability of a faster vehicle encountering a slower one), a common assumption called *vehicular chaos* is used, which states that vehicles' speeds are uncorrelated, hence allowing to split the joint distribution.

More than a decade later, Paveri-Fontana criticised the assumption of vehicular chaos in the interaction term [230]. He subsequently proposed an improved gas-kinetic model, in which he extended the phase-space density of equation (23) with a dependence on the desired speed v_{des}, i.e., $\widetilde{k}(t, x, \overline{v}_s, v_{\text{des}})$; in Prigogine's original derivation, this desired speed was incorrectly considered to be a property of the road, instead of being a driver-related property [121].

An interesting property of the gas-kinetic modelling approach instigated by the seminal work of Prigogine, is that for densities beyond a certain critical density, Nelson and Sopasakis found that the model solutions split into two distinct families. The current hypothesis surrounding this phenomenon states that this corresponds to the widely observed data scatter in the empirically obtained (k,q) fundamental diagrams [211].

2.2.3 Improvements to the mesoscopic modelling approach

Significant contributions to the gas-kinetic mesoscopic model have been sporadic; after the work of Paveri-Fontana, Nelson was among the first to tackle the computational complexity associated with the four-dimensional phase-space density $\widetilde{k}(t, x, \overline{v}_s, v_{\text{des}})$ [210]. In his derivation, he reformulated the relaxation and interaction terms both as discrete events, based on a bimodal distribution of the vehicles' speeds (i.e., corresponding to stopped and moving vehicles). In contrast to the classic model which uses a relaxation process in the acceleration term, Nelson furthermore based his derivation on a microscopic behavioural model [121, 134, 262].

Building on the work of Nelson (which is, as he describes, just a first initial step towards constructing a suitable kinetic model), Wegener and Klar derived a kinetic model in similar spirit, based on a microscopic description of individual driver behaviour with respect to accelerations and lane changes. Attractive to their work, is the fact that they also pay attention to the numerical solutions of their model, with respect to the description of homogeneous traffic flows [290].

Noting that the correct reproducing of traffic flow behaviour at moderate to higher densities still troubled the existing mesoscopic models, Helbing et al. explored an interesting avenue. Not only did they capture the effect that vehicles require a certain finite space (leading to an Enskog- instead of a Boltzmann-equation), they also generalised the interaction term of equation (24). This last method allowed them to dismiss the traditional assumption of vehicular chaos, i.e., they were now able to treat correlations between vehicles' speeds (which have a substantial influence at higher densities). The trick to obtain this behaviour, was to assume that drivers react to the downstream traffic conditions. This leads to the inclusion of non-local interaction (braking) term, and hence their model is referred to as the *non-local gas-kinetic traffic flow model* [119, 117]. Interestingly, this non-locality can generate effects that are similar to the ones induced by viscosity/diffusion terms in macroscopic traffic flows models, causing smooth behaviour at density jumps [121]. The power of their model is also demonstrated as it is able to reproduce all traffic regimes encountered in Kerner's three-phase traffic theory [148].

Central to some of the recently proposed models, is the step process that transforms one model class into another. Starting from microscopic driver behavioural principles (e.g., accelerating, braking, ...), a mesoscopic model is deduced. This mesoscopic model can then be translated into an equivalent macroscopic one by applying the *method of moments*. This allows to obtain PDEs that describe the dynamic evolution of the density k, space-mean speed $\overline{v}_s$, and its variance Θ (an exception to this methodology is the previously mentioned model of Wegener and Klar that obtains dynamic solutions directly [290]). As an example, Helbing et al. also devised a numerical scheme for their previously discussed model. It was implemented in a simulation package called MASTER [120].

Important progress was made by the work of Hoogendoorn et al., who extended the gas-kinetic traffic flow models with *multiple user classes*, in the sense that different classes of drivers have different desired speeds. In order to achieve this, they replaced the traditional phase-space density with a *multi-class phase-space density* (MUC-PSD). The kinetic conservation equation thus describes the tempo-spatial evolution of this MUC-PSD (i.e., the interactions between different user classes), after which an equivalent system of macroscopic model equations is derived. The generalisation power of their model is exemplified as the previously mentioned model of Helbing et al., which is just a special case, having only one class [132, 133]. The developed multiclass gas-kinetic model is currently being integrated in a macroscopic simulation model for complete road networks, called HELENA, which allows prediction of future traffic states, and hence to assess the effectiveness of policy measures [135].

Recently, Waldeer derived a kinetic model that is based on the description of a driver's acceleration behaviour

(as opposed to his *observed* speed behaviour). This novel approach attempts to alleviate the unrealistic jumps in speeds that are typically encountered in kinetic models. To this end, Waldeer extends the phase-space density even further, including a vehicle's acceleration in addition to its position, speed, and desired speed (leading to an even more complex system). Because now the acceleration is updated discretely, the speed will change continuously as a result [287]. Furthermore, Waldeer provided a numerical scheme for solving his model, by employing a Monte Carlo technique that is frequently used in non-equilibrium gas-kinetic theory [286].

To end this overview of gas-kinetic models, we mention the important work of Tampère, who significantly extended the previous modelling approaches [262, 264]. In his work, he used the *generalised phase-space density* (as derived by Hoogendoorn [132]), which incorporates a dependency on the *traffic state* S (e.g., encompassing vehicles' speeds and their desired speeds). As it is an increasingly recognised fact that a complete traffic flow model should contain elements which describe the human behaviour (see for example the comments made by Daganzo [71, 72]), Tampère proposes to include a driver's *activation level*. His *human-kinetic model* (HKM) is, just like that of Helbing et al. and Hoogendoorn, able to reproduce all known traffic regimes. Because of the dependency of the PSD on a behavioural parameter (i.e., the activation level that describes a driver's awareness of the governing traffic conditions), the model is well-suited to evaluate the applicability of *advanced driver assistance systems* (ADAS). As another illustrating example, the phenomenon of a capacity funnel can be realistically explained and reproduced [263]. However, despite the progress related to incorporating human behaviour into mathematical models for traffic flows, Tampère argues that most of the work can currently not be validated because there is no appropriate data yet available.

2.3 Microscopic traffic flow models

Having discussed both mesoscopic and macroscopic traffic flows models, we now arrive at the other end of the spectrum where the microscopic models reside. Whereas the former describe traffic operations on an aggregate scale, the latter kind is based on the explicit consideration of the *interactions between individual vehicles* within a traffic stream. The models typically employ characteristics such as vehicle lengths, speeds, accelerations, and time and space headways, vehicle and engine capabilities, as well as some rudimentary human characteristics that describe the driving behaviour.

The material in this section is organised as follows: we first introduce the classic car-following (and lane-changing) models as well as some of their modern successors, after which we discuss the optimal velocity model, then introduce the more human behaviourally psycho-physiological spacing models, which are subsequently followed by a brief description of traffic cellular automata models. After some words on models based on queueing theory, the section concludes with a concise overview of some of the (commercially) available microscopic traffic flow simulators, as well as some of the issues that are related to the calibration and validation of microscopic traffic flow models.

More detailed information with respect to microscopic models (more specifically, car-following models), can be found in the book of May [190], the overview of Rothery [92], the work of Ahmed [2], and the overview of Brackstone and McDonald [32].

2.3.1 Classic car-following and lane-changing models

Probably the most widely known class of microscopic traffic flow models is the so-called family of *car-following* or *follow-the-leader* models. One of the oldest 'models' in this case, is the one due to Reuschel [250], Pipes [238], and Forbes et al. [90]. It is probably best known as the *"two-second rule"* taught in driving schools everywhere[11] . An earlier example of this line of reasoning is the work of Herrey and Herrey, who specified a *safe driving distance* that also included the distance needed to come to a full stop [129].

It still remains astonishing that the seemingly daunting and complex task that encompasses driving a vehicle, can be executed with such relative ease and little exercise, as is testified by the many millions of kilometres that are driven each year. In spite of this remark, the first mathematical car-following models that have been developed, were based on a description of the interaction between two neighbouring vehicles in a traffic stream, i.e., a follower and its leader. In this section, we historically sketch the development of car-following theories, as they evolved from conclusions about early experiments into more sophisticated models.

The above mentioned model was originally formulated as the following *ordinary differential equation* (ODE) for single-lane traffic:

$$\frac{dv_i(t)}{dt} = \frac{v_{i+1}(t) - v_i(t)}{T}, \tag{25}$$

with $v_i(t)$ and $v_{i+1}(t)$ the speeds of the following, respectively leading, vehicle at time t, and T a relaxation parameter. For the above case, the underlying assumption/justification is that vehicle i (the follower) tries to achieve the speed $v_{i+1}(t)$ of vehicle $i+1$ (its leader), whilst taking a certain relaxation time T into account.

As equation (25) describes a stable system, Chandler et al. were among the first to include an explicit *reaction time* τ into the model (e.g., τ =1.5 s), leading to destabilisation of vehicle platoons [48]. This reaction encompasses both a *perception-reaction time* (PRT), i.e., the driver

[11] Note that, in his article, Pipes actually stated his safe-distance rule as keeping at least a space gap equal to a vehicle length for every 15 km/h of speed you are travelling at [238, 134].

sees an event occurring (for example the brake lights of the leading vehicle), as well as a *movement time* (MT), i.e., the driver needs to take action by applying pressure to the vehicle's brake pedal [92]. Introducing this behaviour, resulted in what is called a *stimulus-response model*, whereby the right-hand side of equation (25) describes the stimulus and the left-hand side the response (the response is frequently identified as the acceleration, i.e., the actions a driver takes by pushing the acceleration or brake pedal). The relaxation parameter is then reciprocally reformulated as the sensitivity to the stimulus, i.e., $\lambda = T^{-1}$, resulting in the following expression:

$$\begin{aligned} \text{response} &= \text{sensitivity} \times \text{stimulus} \\ \frac{dv_i(t+\tau)}{dt} &= \lambda\,(v_{i+1}(t) - v_i(t)). \end{aligned} \tag{26}$$

Additional to this theoretical work, there were also some early controlled car-following experiments, e.g., the ones done by Kometani and Sasaki, who add a non-zero acceleration term to the right-hand side of the stimulus-response relation, in order to describe collision-free driving based on a safety distance [152, 153].

Equation (26) is called a *delayed differential equation* (DDE), which, in this case, is known to behave in an unstable manner, even resulting in collisions under certain initial conditions. Gazis et al. remedied this situation by making the stimulus λ dependent on the distance, i.e., the space gap g_{s_i} between both vehicles [97]:

$$\frac{dv_i(t+\tau)}{dt} = \lambda\,\frac{v_{i+1}(t) - v_i(t)}{x_{i+1}(t) - x_i(t)}. \tag{27}$$

Further advancements to this car-following model were made by Edie, who introduced the current speed of the following vehicle [84]. Gazis et al, forming the club of people working at General Motors' research laboratories, generalised the above set of models into what is called the *General Motors non-linear model* or the *Gazis-Herman-Rothery (GHR) model* [95]:

$$\frac{dv_i(t+\tau)}{dt} = \lambda\, v_i^m(t)\frac{v_{i+1}(t) - v_i(t)}{(x_{i+1}(t) - x_i(t))^l}, \tag{28}$$

with now λ, l, and m model parameters (in the early days, the model was also called the *L&M model* [98]). For a good overview of the different combinations of parameters attributed to the resulting models, we refer the reader to the book of May [190], and the work of Ahmed [2].

A recent extension to the classic car-following theory, is the work of Treiber and Helbing, who developed the *intelligent driver model* (IDM). Its governing equation is the following [268, 267, 269]:

$$\frac{dv_i}{dt} = a_{\max}\left[\underbrace{1 - \left(\frac{v_i}{v_{\text{des}}}\right)^{\delta}}_{\text{acceleration}} - \underbrace{\left(\frac{g_s^*(v_i, \Delta v_i)}{g_{s_i}}\right)^2}_{\text{deceleration}}\right], \tag{29}$$

with $a_{\max}$ the maximum acceleration, v_{des} the vehicles' desired speed, and Δv_i the speed difference with the leading vehicle (we have dropped the dependencies on time t for the sake of visual clarity). The first terms within the brackets denote the tendency of a vehicle to accelerate on a free road, whereas the last term is used to allow braking in order to avoid a collision (the effective desired space gap $g_s^*(v_i, \Delta v_i)$ is based on the vehicle's speed, its relative speed with respect to its leader, a comfortable maximum deceleration, a desired time headway, and a jam space gap). The finer qualities of the IDM are that it elegantly generalises most existing car-following models, and that it has an explicit link with the non-local gas-kinetic mesoscopic model discussed in section 2.2.3 [267]. It is furthermore quite capable of generating all known traffic regimes. Based on the IDM, Treiber et al. also constructed the *human driver model* (HDM), which includes a finite reaction time, estimation errors, temporal and spatial anticipation, and adaptation to the global traffic situation [270].

Similar to the work of Kometani and Sasaki, Gipps proposed a car-following model based on a safe braking distance, leading to collision-free dynamics [100]. The model is interesting because no differential equations are involved (i.e., the speeds are computed directly from one discrete time step to another), and because it can capture underestimation and overreactions of drivers, which can lead to traffic flow instabilities. In similar spirit of Gipps' work, Krauß developed a model that is based on assumptions about general properties of traffic flows, as well as typical acceleration and deceleration capabilities of vehicles. Fundamental to his approach, is that all vehicles strive for collision-free driving, resulting in a model that has the ability to generalise most known car-following models [155, 156].

Another example of a recently proposed car-following model, is the 'simple' model of Newell, who formulates his theory in terms of vehicle trajectories whereby the trajectory of a following vehicle is essentially the same as that of its leader[12]. Remarkable properties are that the model has no driver reaction time, and that it corresponds to the first-order macroscopic LWR traffic flow model with a triangular $q_e(k)$ fundamental diagram (see section 2.1.3) [221]. The model furthermore also agrees quite well with empirical observations made at a signallised intersection, which support the model and consequently also the first-order macroscopic LWR model [3].

As a final example, we briefly illustrate Zhang's car-following theory which is based on a multi-phase vehicu-

[12] A similar model was proposed earlier by Helly [125, 99, 221].

lar traffic flow. This means that the model is able to reproduce both the capacity drop and hysteresis phenomena, because his theory is based on the asymmetry between acceleration and deceleration characteristics of vehicles [182]. The model also holds a generalisation strength, as it is possible to derive all other classic car-following models [308].

With respect to the stability of the car-following models, there exist two criteria, i.e., *local* and *asymptotic stability* (also called *string stability*). The former describes how initial disturbances in the behaviour of a leading vehicle affect a following vehicle, whereas the latter is used to denote the stability of a *platoon of following vehicles*. By such a stable platoon it is then meant that initial finite disturbances exponentially die out along the platoon. Early experiments by Herman et al. already considered these criteria for both real-life as for the developed mathematical car-following models [127].

As an example, we graphically illustrate in Fig. 8 the asymptotic stability of a platoon of some 10 identical vehicles. We have used the simple car-following model of Gazis et al. of equation (27) to describe how a following vehicle changes his acceleration, based on the speed difference and space gap with its direct leader in the platoon (the sensitivity λ was set to 5000 m/s^2 with a reaction time $\tau = 1$ s). The left part of the figure shows all the vehicles' positions, whereas the middle and right parts show the speeds and accelerations of the 2$^{\text{nd}}$, the 5$^{\text{th}}$, and the 10$^{\text{th}}$ vehicle respectively. We can see that all vehicles are initially at rest (homogeneously spaced), after which the leading vehicle applies an acceleration of 1 m/s^2, decelerates with -1 m/s^2, and then comes to a full stop. As can be seen, the first 4 following vehicles mimic the leader's behaviour rather well, but from the 5$^{\text{th}}$ following vehicle on, an instability starts to form (note that all following vehicles suffer from oscillations in their acceleration behaviour). This instability grows and leads to very large accelerations for the last vehicle, which even momentarily reaches a negative speed of some -150 km/h; this is clearly unrealistic (the vehicles shouldn't be driving backwards on the road), indicating that the specified car-following model is unsuitable to capture the realistic behaviour of drivers under these circumstances.

In continuation of this small excerpt on stability, we refer the reader to the work of Zhang and Jarrett who analytically and numerically derive the general stability conditions (in function of the reaction time and the sensitivity to the stimulus) for the previously mentioned classic car-following models [309], the work of Holland who derives general stability conditions and validates them with empirical data containing non-identical drivers (i.e., aggressive and timid ones); central to Holland's work is the source for instability with respect to a breakdown of a traffic flow. He relates this event to a so-called anticipation time that describes the duration for a wave containing an instability to travel to the current driver [131]. Finally, we mention that stability analysis is of paramount importance for e.g., automated vehicle technologies ('smart cars') such as *intelligent* or *adaptive cruise control* (ICC/ACC), as in for example the platooning experiments in the PATH project where a platoon of vehicles autonomously drives close to each other at high speeds.

To conclude this section, we shed some light on the typical mechanisms behind lane-changing models. With respect to microscopic models for multi-lane traffic, it is a frequent approximation to only take lateral movements *between* neighbouring lanes into account (as opposed to the *within-lane* lateral dynamics of a vehicle). In such cases, a vehicle changes a lane based on an incentive: these lane changes can then be classified as being *discretionary* (e.g., to overtake a slower vehicle), or *mandatory* (e.g., to take an off-ramp). When a vehicle (i.e., driver) has decided to perform a lane change, a check is made on whether or not it is physically possible to merge in to the adjacent lane (note this lane changing process also describes vehicles turning at street intersections). This latter process is called the *gap acceptance* behaviour: if there is no such possibility (as it is frequently the case in dense traffic), a driver may initiate at *forced merging*, in which case the following vehicle in the target lane might have to *yield*. This interaction between forced merging and yielding can be frequently observed at on-ramps where heavy duty vehicles enter the motorway. Although it seems intuitive that there is an asymmetry between the frontal and backward space gaps in the target lane (i.e., the former is usually smaller than the latter due to the human behaviour associated with forced merging and yielding), there is in our opinion nevertheless not enough empirical data available to calibrate the microscopic models that describe lane-changing (see for example the work of Ahmed [2]). One way to obtain a correct behaviour is to use a kind of a black box approach, in which for example the downstream capacity of a motorway section is used as a measure for calibrating the interactions (i.e., lane changes) between vehicles in a traffic stream. Note that as technology advances, new detailed data sets are constructed. An example is the work of Hoogendoorn et al. who use a *remote sensing* technique to capture vehicle trajectories based on aerial filming of driving behaviour under congested conditions [136].

2.3.2 Optimal velocity models

Closely related to the previously discussed classic car-following models, are the so-called *optimal velocity models* (OVM) of Newell and Bando et al. Whereas the previous car-following models mostly describe the behaviour of a vehicle that is following a leader, the OVMs modify the acceleration mechanism, such that a vehicle's desired speed is selected on the basis of its space headway, instead of only considering the speed of the leading vehicle [121]. Newell was the first to suggest such an approach, using an equilibrium relation for the desired speed as a function of its space headway (e.g., the $\overline{v}_s(\overline{h}_s)$ fundamental diagram [212].

Bando et al. later improved this model, resulting in the

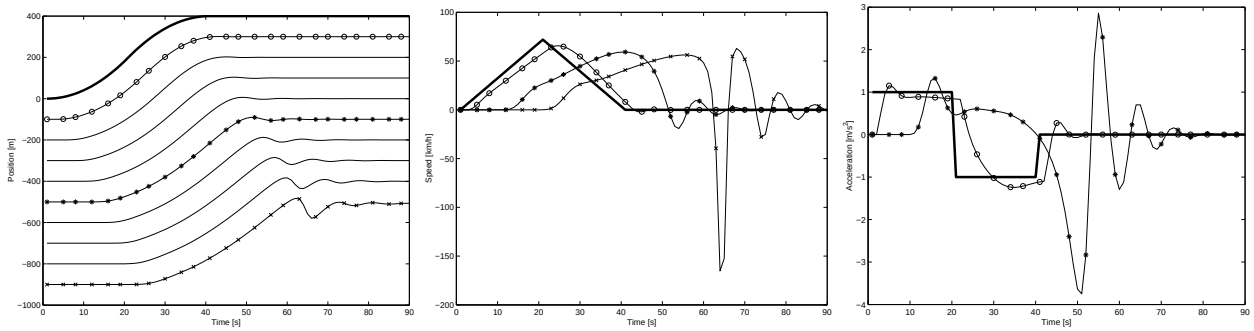


Figure 8: An example based on Gazis et al.'s car-following model of equation (27), indicating an asymptotically unstable platoon of 10 vehicles. *Left:* the time-space trajectories of all ten vehicles (the leading vehicle is shown with a thick solid line). We can see an instability occurring at the 6$^{\text{th}}$ ($\star$) vehicle, growing severely such that the last vehicle even has to drive backwards at a high speed of -150 km/h. *Middle:* the speeds of the first, the 2$^{\text{nd}}$ ($\circ$), the 6$^{\text{th}}$ ($\star$), and the last vehicle ($\times$). Note the oscillations and negative values for the speeds of the vehicles at the end of the platoon. *Right:* the accelerations of the first, the 2$^{\text{nd}}$ ($\circ$), and the 6$^{\text{th}}$ ($\star$) vehicle (example based on [141]).

following equation that describes a vehicle's acceleration behaviour [16]:

$$\frac{dv_i(t)}{dt} = \alpha \left(V(h_{s_i}(t)) - v_i(t) \right), \tag{30}$$

in which $V()$ is called the *optimal velocity function* (OVF). The difference between this desired speed, associated with the driver's current space headway, and the vehicle's current speed, is corrected with an acceleration $\alpha V()$, with now α a coefficient expressing the sensitivity of a driver. Specification of the optimal velocity function (typically a sigmoid function such as $\tanh$) is done such that it is zero for $h_{s_i} \to 0$, and bounded to $v_{\max}$ for $h_{s_i} \to +\infty$; this latter condition means that the model is able to describe the acceleration of vehicles without the explicit need for a leader as in the previous car-following models.

Interestingly, the OVM requires, in contrast to the classic car-following models, no need for a reaction time in order to obtain spontaneous clustering of vehicles [156]. Unfortunately, the model is not always free of collisions, and can result in unrealistically large accelerations [206].

2.3.3 Psycho-physiological spacing models

Instead of using continuous changes in space gaps and relative speeds, it was already recognised in the early sixties that drivers respond to certain *perception thresholds* [32]. For example, a leading vehicle that is looming in front of a follower, will be perceived as having approximately the same small size for a large duration, but once the space gap has shrunk to a certain size, the size of the looming vehicle will suddenly seem a lot bigger (i.e., like crossing a threshold), inducing the following vehicle to either slow down or overtake.

The underlying thresholds with respect to speeds, speed differences, and space gaps, were cast into a model by the work of Wiedemann et al. [292]. In this respect, the models are called *psycho-physiological spacing models*, and although they seem quite successful in explaining the traffic dynamics from a behavioural point of view (even lane-change dynamics can be included based on suitable perception thresholds), calibration of the models has nevertheless been a difficult issue [32].

2.3.4 Traffic cellular automata models

In the field of traffic flow modelling, microscopic traffic simulation has always been regarded as a time consuming, complex process involving detailed models that describe the behaviour of individual vehicles. Approximately a decade ago, however, new microscopic models were being developed, based on the *cellular automata* programming paradigm from *statistical physics*. The main advantage was an *efficient and fast performance* when used in computer simulations, due to their rather low accuracy on a microscopic scale. These so-called *traffic cellular automata* (TCA) are dynamical systems that are discrete in nature, in the sense that time advances with discrete steps and space is coarse-grained (e.g., the road is discretised into cells of 7.5 metres wide, each cell being empty or containing a vehicle). This coarse-graininess is fundamentally different from the usual microscopic models, which adopt a semi-continuous space, formed by the usage of IEEE floating-point numbers. TCA models are very flexible and powerful, in that they are also able to capture all previously mentioned basic phenomena that occur in traffic flows [18, 56]. In a larger setting, these models describe *self-driven, many-particle systems, operating far from equilibrium*. And in contrast to strictly gaseous analogies, the particles in these systems are intelligent and able to learn from past experience, thereby opening the door to the incorporation of behavioural and psychological aspects [55, 294, 121].

Not only in the field of vehicular traffic flow modelling, but also in other fields such as *pedestrian behaviour*, *escape* and *panic dynamics*, ...the cellular au-

tomata approach proved to be quite useful. It is now feasible to simulate large systems containing many 'intelligent particles', such that is it possible to observe their interactions, collective behaviour, self-organisation, ... [140, 274, 123, 121, 199, 200, 54]

2.3.5 Models based on queueing theory

In this final section dealing with types of microscopic traffic flow models, we briefly summarise some of the models that are based on the paradigm of queueing theory. Early applications of queueing theory to the field of transportation engineering are mostly related to descriptions of the behaviour signallised and unsignallised intersections, overtaking on two-lane roads with opposing traffic, ... [60]. Another more theoretically oriented application can be traced back to the work of Newell, who gives a nice summary of the mathematical details related to the *practical* application of the methodology. Newell was one of the few people who directly questioned the usefulness of cleverly devising a lot of methods and solutions, whereby corresponding problems remained absent [214]. In his later work, Newell reintroduced the concept of arrival and departure functions (i.e., the cumulative curves as briefly described in section 2.1.2), giving an analytical but still highly intuitive method for solving traffic flow problems, and drawing parallels with the well-known and studied first-order macroscopic LWR model [215, 216, 217].

During the mid-nineties, Heidemann developed several queueing-based traffic flow models, of which the most powerful version deals with non-stationary conditions and is able to model the capacity drop and hysteresis phenomena, as well as providing an explanation for the wide scatter observed in empirical fundamental diagrams [116, 182].

Central to the approach in this field, is the partitioning of a road into equal pieces of width $1/k_{\text{jam}}$. Each of these pieces is then considered as a *service station* operating with a *service rate* $\mu = k_{\text{jam}} \cdot \overline{v}_{\text{ff}}$. Equivalently, vehicles arrive at each service station with an *arrival rate* $\lambda = k \cdot \overline{v}_{\text{ff}}$, with the assumption that k is the prevailing density and that traffic can flow unimpeded in the free-flow traffic regime. When vehicles enter the motorway, they can get stuck inside the queues, thereby reducing the space-mean speed in the system. Different *queueing policies* can be specified in the form of service and arrival distributions. In queueing theory, the *Kendall notation* is adopted, whereby a system is described as $A/S/m$ with A the arrival distribution, S the service distribution, and m the number of servers (i.e., service stations). Typical forms are the $M/M/1$ queues that have an exponentially distributed arrival time, exponentially distributed service time, and one server (with an infinite buffer).

Recently, Van Woensel extended the existing queueing models for traffic flows, leading to e.g., analytical derivations of fundamental diagrams based on $G/G/m$ queues that have general distributions for the arrival and service rates with multiple servers [293]. The methodology also includes queues with finite buffers, and has been applied to the estimation of emissions, although we question the validity of this latter approach (which is essentially based on a one-dimensional fundamental diagram) as we believe dynamic models are necessary, e.g., to capture transients in traffic flows [275].

Queue-based models were also used to describe large-scale traffic systems, e.g., complete countries, as was mentioned in section 1.2.5 [47]. In that section, we already mentioned that queues with finite buffer capacities are to be preferred in order to correctly model queue spill back. However, with respect to a proper description of traffic flow phenomena, some of the problems can not be so easily solved, e.g., the speed of a backward propagating kinematic shock wave. Take for example vehicles queued behind each other at a traffic light: once the light turns green, the first-order macroscopic LWR model correctly shows the dispersal of this queue. In a queue-based model however, once a vehicle exits the front of the queue, all vehicles simultaneously and instantly move up one place, thus the kinematic wave propagates backwards at an infinite speed !

To conclude this short summary on queueing models, we mention the work of Júlvez and Boel, who present a similar approach, based on the use of *Petri nets*[13]. Their work allows them to construct complete urban networks, based on the joining together of short sections, with continuous Petri nets for the propagation of traffic flows, and discrete Petri nets for the description of the traffic lights [19, 146].

2.3.6 Microscopic traffic flow simulators

In continuation of the previous sections that gave an overview of the different types of existing microscopic traffic flow models, this section introduces some of the computer implementations that have been built around these models. In most cases, the computer simulators incorporate the car-following and lane-changing processes as submodels, as opposed to strategic and operational modules that work at a higher-level layer (i.e., route choice, ...).

Whereas most microscopic traffic simulators allow to build a road network, specify travel demands (e.g., by means of OD tables), there was quite some effort spent over the last decade, in order to achieve a qualitative visualisation (e.g., complete virtual environments with trees, buildings, pedestrians, bicycles, ... An example of such a virtual environment is shown in Fig. 9, which is based on VISSIM's visualisation module. Note that in our opinion, the usefulness of these virtual scenes should not be underestimated, as in some cases a project's approval

[13] Petri nets (invented in the sixties by Carl Adam Petri) are a formalism for describing discrete systems [235]; they consist of directed graphs of 'transitions' and 'places', with arcs forming the connections between them. Places can contain 'tokens', which can be 'consumed' when a transition 'fires'.

might hinge on a good visual representation of the results. It is one thing for policy makers to judge the effects of replacing a signallised intersection with a roundabout, based on a report of the observed downstream flows of each intersection arm, but it gives a whole other feeling when they are able to *see* how the traffic streams will interact ! Even in the early sixties, it was recognised that a visual representation of the underlying traffic flow process, was an undeniable fact for promoting its acceptance among traffic engineers [99]. With respect to this latest comment, Lieberman even states that *"There is a need to view vehicle animation displays, to gain an understanding of how the system is behaving, in order to explain why the resulting statistics were produced"* [92].



Figure 9: A screenshot of the VISSIM microscopic traffic flow simulator, showing a detailed virtual environment containing trees, buildings, pedestrians, ... (image reproduced after [243]).

Quite a large amount of microscopic traffic flow models have been developed, in most cases starting from a research tool, and — by the law of profit — naturally evolving into full-blown commercial packages, including e.g., dynamic traffic assignment and other transportation planning features. Note the sad observation that this commercialisation inherently tends to obscure the underlying models. In such cases, privacy concerns, company policies, and project contracts and agreements prohibit a total disclosure of the mathematical details involved. Some of these computer models are listed here. For starters, the *Generic Environment for TRaffic Analysis and Modeling* (GETRAM) couples the multi-modal traffic assignment model EMME/2 to the *Advanced Interactive Microscopic Simulator for Urban and Non-Urban Networks* (AIMSUN2) model [17]. Next, the *Parallel microscopic traffic simulator* (Paramics), initially developed at the Edinburgh Parallel Computing Centre, but afterwards bought by Quadstone [43, 170]. Subsequently, Yang developed a *MIcroscopic Traffic flow SIMulator* (MITSIM) [297], and Maerivoet constructed a *MIcroscopic TRAffic flow SIMulator* (Mitrasim *2000*) which was mostly based on and influenced by MITSIM's and Paramics' dynamic behaviour [179]. Two further examples are the Open Source Software (OSS) package called *Simulation of Urban MObility* (SUMO), developed at the Deutsches Zentrum für Luft- und Raumfahrt [154], and the VISSIM programme developed by the German PTV group [243]. In addition, there is the *INteractive DYnamic traffic assignment* (INDY) model [185], and the INTEGRATION software package developed by Van Aerde et al. This latter simulator deserves a special mention: it is microscopic in nature, but the speeds of the vehicles that are propagated through the network, are based on a macroscopic $\overline{v}_{s_e}(\overline{h}_s)$ fundamental diagram for each link [1]. Finally, we mention the *TRansportation ANalysis and SIMulation System* (TRANSIMS) project [205], ...

An extensive overview of all existing microscopic traffic flow simulators until 1998 is provided by the *Simulation Modelling Applied to Road Transport European Scheme Tests*, or better known as the SMARTEST report [4].

When using one of these microscopic simulators, it is important to understand the assumptions and limitations inherent to the implemented models, in order to judge the results objectively. Indeed, as with any model, the question on whether some observed behaviour arises due to the implemented model, or as a result of the imposed boundary conditions, should always be asked, understood, and answered.

2.3.7 Calibration and validation issues

Due to the sometimes large amount of parameters typically involved in microscopic traffic flow models, their computational complexity is often a significant disadvantage when compared to meso- or macroscopic models (although there are some exceptions, e.g., the traffic cellular automata models of section 2.3.4). From the point of view of model calibration and validation, this poses an interesting conundrum, as in many cases not all parameters are equally influential on the results (thus requiring some sensitivity analyses). In this sense, microscopic models contain a real danger of purporting to convey a sort of fake accuracy. Different parameter combinations can lead to the same phenomenological effects, leaving us pondering as to what exactly is causing the observed behaviour [262]. As there is no clear road map on how to calibrate microscopic traffic flow models, we here give a small sample of some of the numerous attempts that have been made.

There is the work of Jayakrishan and Sahraoui who distinguish between calibration in the conceptual (i.e., at the level of the underlying mathematical model) and operational (i.e., within the global context of the study) phases; they apply their operational methodology to both PARAMICS (micro) and DYNASMART (macro), using the *California Freeway Performance Measurement System* (PeMS) database from the PATH project to feed and couple both models [142].

Based on a publicly available data set of a one-lay road

corridor of six kilometres long (the data contained detailed cumulative curves), Brockfeld et al. systematically tested the predicted travel times of some ten well-known microscopic traffic flow models. As a result of a non-linear optimisation process to calibrate the models, they found that the intelligent driver model and the cell-transmission model perform the best (i.e., below an error rate of 17%), due to the fact that these models require the least amount of parameters (there were even some models such as the Gipps-based ones that had *hidden* parameters). Their final conclusion is noteworthy, as they state that *"creating a new model is often done, however calibrating this model to reality is a formidable task, which explains why there currently are more models than results about them"* [35].

Related to the previous study, Hourdakis et al. present an automated systematic calibration methodology based on an optimisation process, applied to the AIMSUN2 simulator. The data used for the calibration procedure stem from a twenty kilometres long motorway in Minneapolis, Minnesota. The process first involves a calibration of the global model parameters (i.e., to get the macroscopic flows and speeds correct), after which the local parameters are dealt with (i.e., ramp metering setups, et cetera). In their results, Hourdakis et al. state an obtained average correlation coefficient of 0.961 for manual calibration (the results for the automated calibration are similar), which is quite high (they mostly explain this due to the data's high level of detail, as well as the quality of the simulator software) [138].

Recently, Chu et al. extended the systematic, multi-stage calibration approach for the PARAMICS simulator. Based on data of a highly congested six kilometres long corridor network in the city of Irvine, Orange County, California, they first calibrate the driving behaviour models, then the route choice model, after which estimation and fine-tuning of the OD tables is done. Despite the good reproduction of travel times, their calibration methodology was done manually, and an automated optimisation procedure remains future work [57].

Other examples of calibration of microscopic traffic flow models, include the work of Dowling et al., who give an extensive account on the application of commercially available simulation tools to typically encountered traffic engineering problems [82], and the work of Mahanti which is primarily based on the correct representation of OD tables [184].

To end this section, we state some important principles that are — in our opinion — related to a correct calibration methodology. First and foremost, we believe that all traffic flow models (whether they are macro-, meso- or microscopic in nature), should be able to accurately reproduce and predict the encountered delays, queue lengths, and other macroscopic *first-order characteristics* (i.e., the kinematic wave speed, a correct and realistic road capacity, ...). One way to test this is the use of cumulative curves, as they provide an elegant way to automatically perform a good calibration. It is for example possible to consider the difference between observed and simulated curves, and then use a Kolmogorov-Smirnov goodness-of-fit statistical test to decide on whether the difference is statistically significant, or if it is just a Brownian motion with a zero mean. Only when these first-order effects can be correctly reproduced, the next step can be to consider *second-order effects* such as waves of stop-and-go traffic, oscillations, ...

Furthermore, it is important to take into account the *spatial nature* of the study area, i.e., a detailed description of the road infrastructure, with bottleneck locations as well as up- and downstream boundary conditions. With respect to the model that is created within the computer, it is paramount to know how the model behaves on both the link as well as the node level. Because the models are most of the time working with fairly homogeneous road links (e.g., constant elevations, no road curvature, ...), it might be necessary to allow for small deviations from (or fixes to) reality (e.g. inserting extra intermediate nodes in the network in order to artificially obtain bottlenecks).

2.4 Submicroscopic traffic flow models

As the level of modelling detail is increased, we enter the realm of submicroscopic models. Traditional microscopic models describe vehicles as single operating units, putting emphasis on the interactions between different (successive) vehicles. In addition to this, submicroscopic models push the boundaries even further, giving detailed descriptions of a vehicle's inner workings. This typically entails modelling of the *physical characteristics* such as engine performance, detailed gearbox operations, acceleration, braking, and steering manoeuvres, ... Complementary to the functioning of a vehicle's physical components, submicroscopic models can also describe a *human driver's decision taking process* in much more detail than is usually done. Some examples of submicroscopic models are:

- van Arem's *Microscopic model for Simulation of Intelligent Cruise Control* MIXIC: it contains a driver model (for deciding on and executing of lane changes, car-following behaviour, and the application of intelligent (or adaptive) cruise control – ICC/ACC) and a vehicle model (dealing with the engine, the

transmission, road friction, aerodynamic, rolling, and slope resistance) [271].

- In similar spirit, Minderhoud has developed the *Simulation model of Motorways with Next generation vehicles* (Simone); this model focusses on intelligent driver support systems, such as ICC/ACC, platoon driving, centralised control of vehicles, et cetera. In contrast to most other (sub)microscopic models, Simone explicitly allows for rear-end collisions to occur under certain parameter combinations. As there is a close coupling between driver behaviour related parameters and those of the simulation, these collision dynamics enable the modeller to find realistic values (or ranges) for these parameters [195].
- Ludmann's *Program for the dEvelopment of Longitudinal micrOscopic traffic Processes in a System-relevant environment* (PELOPS), is akin to the previous two models. It is however more technologically oriented with respect to the car-following behaviour of vehicles, aiming at merging both a driver's perceptions and decisions, the car's handling, and the surrounding traffic conditions. At the core of the model, there are four modules that respectively describe vehicle routing in a road network, human decision taking (i.e., car-following, tactical decisions with respect to lane-changing, ...), vehicle handling (i.e., a driver's physical acts of steering, accelerating and braking, ...), and finally a module that describes physical vehicle characteristics (traction on elevations, engine capabilities, exhaust gas modelling, ...) [178, 85].

To conclude this section, we like to mention an often scientifically neglected area of research, namely the popular field of simulation in the *computer gaming industry*. Over the last couple of decades, numerous arcade-style *racing simulations* have been developed, allowing a *player* to be completely immersed in a three-dimensional virtual world in which racing at high speeds is paramount. Examples of these kinds of programmes are the highly addictive world of *Formula 1* racing, street racing in city environments, off-road rally races, ... The underlying submicroscopic models in these games, have over the course of several years been evolved to incorporate all sorts of physical effects. Friction characteristics (e.g., pavement versus asphalt), road elevation, wet conditions, air drag and wind resistance (including effects such as slip streaming and downforce), car weight depending on fuel consumption, tyre wear, ... have had influences on what we commonly refer to as *car handling*, i.e., realistic behaviour with respect to car acceleration, braking, and steering. Thanks to the increasing computational power of desktop computers, graphics cards, as well as dedicated gaming consoles (e.g., Microsoft's *Xbox*, Sony's *PlayStation*, Nintendo's *GameCube*, ...), the path to a whole plethora of extra realistic effects has been paved: skidding, under- and oversteering, sun glare, overly realistic collision dynamics (in our opinion, this is where the arcade sensation plays a major role), ...

2.5 The debate between microscopic and macroscopic models

Deciding which class of models, i.e., microscopic, or macroscopic (and we also include the mesoscopic models), is the correct one to formulate traffic flow problems, has been a debate among traffic engineers ever since the late fifties. Although the debate was not as intense as say, the one between first- and higher-order macroscopic traffic flow models (see section 2.1.7 for more details), it nevertheless sparkled some interesting issues. As is nearly always the case, the true answer to the above question depends on the kind of problem one is interested in solving [98].

In the beginning years of traffic flow engineering, a bridge was formed between the microscopic General Motors car-following model of equation (28), and the Greenberg macroscopic model [104, 95]. This proved to be quite a significant breakthrough, as it was now possible to obtain all known steady-state macroscopic fundamental diagrams, by integrating the car-following equation with suitably chosen parameter values [98]. A recent example of this kind of linking, was done by Treiber and Helbing, who provided a *micro-macro link* between their non-local gas-kinetic mesoscopic model (see section 2.2.3) and the intelligent driver model (see section 2.3.1) [119, 117].

Besides this explicit translating of microscopic into macroscopic (mesoscopic) models and vice versa, it is also possible to develop *hybrid models* that couple macroscopically modelled road links to microscopically modelled ones. Examples include the work of Magne et al., who develop a hybrid simulator that couples a METANET-like second-order macroscopic traffic flow model with the *SImulation TRAfic* (SITRA-B+) microscopic traffic flow model. Special attention is given to the interfaces between macroscopically and microscopically modelled road segments; each macroscopic time iteration in the simulator, is accompanied by a number of microscopic iterations [183]. In similar spirit, the work of Bourrel and Henn links macroscopic representations of traffic flows to microscopic ones, using interfaces that describe the transitions between them. As an application of their methodology, they describe the translation between the first-order macroscopic LWR model and a vehicle representation of this model (based on trajectories) [29]. Another avenue was pursued by the Wilco [41] and Wilco et al. [42], who developed an integration framework between the MITSIMLab microscopic model and the Mezzo mesoscopic model. By building upon a mesoscopic approach, the strength of their work lies in the fact that no aggregation and disaggregation of flows needs to be performed.

3 Conclusions

The material elaborated upon in this paper, spanned a broad range going from transportation planning models that operate on a high level, to traffic flow models that explicitly describe the physical propagation of traffic flows.

As explained in the introduction, we feel there is a frequent confusion among traffic engineers and policy makers when it comes to transportation planning models and the role that traffic flow models play therein. To this day, many transportation planning bureaus continue to use static tools for evaluating policy decisions, whereas the need for dynamic models is getting more and more pronounced [180].

Even after more than sixty years of traffic flow modelling, the debate on what is the correct modelling approach remains highly active. On the transportation planning side, many agencies still primarily focus on the traditional four-step model (4SM), because it is the best intuitively understood approach. In contrast to this, activity-based modelling (ABM) is gaining momentum, although it remains a rather obscure discipline to many people. At the basis of this scrutiny towards the ABM, lies the absence of a generally accepted framework such as the one of the 4SM. It is tempting to translate the ABM approach to the 4SM, by which e.g., the ABM's synthetic population generation (including activity generation, household choices and scheduling) corresponds to the 4SM's production and attraction, distribution, and modal split (or to discrete choice theory in a broader setting), thereby generating (time dependent) OD tables. Similarly, the ABM's agent simulation can be seen as an implementation of the 4SM's traffic assignment. However, it remains difficult to gain insight into this kind of direct translation and the resulting travel behaviour, although the ABM's scientific field is continuously in a state of flux thanks to the increasing computational power.

On the traffic flow modelling side, the debate on whether or not to use macro-/meso- or microscopic models still continues to spawn many intriguing discussions. Despite the respective criticisms, it is widely agreed upon that modelling driver behaviour entails complex human-human, human-vehicle, and vehicle-vehicle interactions. These call for interdisciplinary research, drawing from fields such as mathematics, physics, and engineering, as well as sociology and psychology (see e.g., the overview of Helbing and Nagel [122]).

A Glossary of terms

A.1 Acronyms and abbreviations

4SM	four step model
AADT	annual average daily traffic
ABM	activity-based modelling
ACC	adaptive cruise control
ACF	average cost function
ADAS	advanced driver assistance systems
AIMSUN2	Advanced Interactive Microscopic Simulator for Urban and Non-Urban Networks
AMICI	Advanced Multi-agent Information and Control for Integrated multi-class traffic networks
AON	all-or-nothing
ASDA	Automatische StauDynamikAnalyse
ASEP	asymmetric simple exclusion process
ATIS	advanced traveller information systems
ATMS	advanced traffic management systems
BCA	Burgers cellular automaton
BJH	Benjamin, Johnso, and Hui
BJH-TCA	Benjamin-Johnson-Hui traffic cellular automaton
BL-TCA	brake-light traffic cellular automaton
BML	Biham, Middleton, and Levine
BML-TCA	Biham-Middleton-Levine traffic cellular automaton
BMW	Beckmann, McGuire, and Winsten
BPR	Bureau of Public Roads
CA	cellular automaton
CA-184	Wolfram's cellular automaton rule 184
CAD	computer aided design
CBD	central business district
CFD	computational fluid dynamics
CFL	Courant-Friedrichs-Lewy
ChSch-TCA	Chowdhury-Schadschneider traffic cellular automaton
CLO	camera Linkeroever
CML	coupled map lattice
CONTRAM	CONtinuous TRaffic Assignment Model
COMF	car-oriented mean-field theory
CPM	computational process models
CTM	cell transmission model
DDE	delayed differential equation
DFI-TCA	deterministic Fukui-Ishibashi traffic cellular automaton
DGP	dissolving general pattern
DLC	discretionary lane change
DLD	double inductive loop detector
DNL	dynamic network loading
DRIP	dynamic route information panel
DTA	dynamic traffic assignment
DTC	dynamic traffic control
DTM	dynamic traffic management
DUE	deterministic user equilibrium
DynaMIT	Dynamic network assignment for the Management of Information to Travellers
DYNASMART	DYnamic Network Assignment-Simulation Model for Advanced Roadway Telematics
ECA	elementary cellular automaton
EP	expanded congested pattern
ER-TCA	Emmerich-Rank traffic cellular automaton
FCD	floating car data

FDE	finite difference equation
FIFO	first-in, first-out
FOTO	Forecasting of Traffic Objects
GETRAM	Generic Environment for TRaffic Analysis and Modeling
GHR	Gazis-Herman-Rothery
GIS	geographical information systems
GNSS	Global Navigation Satellite System (e.g., Europe's Galileo)
GoE	Garden of Eden state
GP	general pattern
GPRS	General Packet Radio Service
GPS	Global Positioning System (e.g., USA's NAVSTAR)
GRP	generalised Riemann problem
GSM	Groupe Spéciale Mobile
GSMC	Global System for Mobile Communications
HAPP	household activity pattern problem
HCM	Highway Capacity Manual
HCT	homogeneously congested traffic
HDM	human driver model
HKM	human-kinetic model
HRB	Highway Research Board
HS-TCA	Helbing-Schreckenberg traffic cellular automaton
ICC	intelligent cruise control
IDM	intelligent driver model
INDY	INteractive DYnamic traffic assignment
ITS	intelligent transportation systems
IVP	initial value problem
JDK	Java™ Development Kit
KKT	Karush-Kuhn-Tucker
KKW-TCA	Kerner-Klenov-Wolf traffic cellular automaton
KWM	kinematic wave model
LGA	lattice gas automaton
LOD	level of detail
LOS	level of service
LSP	localised synchronised-flow pattern
LTM	link transmission model
LWR	Lighthill, Whitham, and Richards
MADT	monthly average daily traffic
MC-STCA	multi-cell stochastic traffic cellular automaton
MesoTS	Mesoscopic Traffic Simulator
MFT	mean-field theory
MITRASIM	MIcroscopic TRAffic flow SIMulator
MITSIM	MIcroscopic Traffic flow SIMulator
MIXIC	Microscopic model for Simulation of Intelligent Cruise Control
MLC	mandatory lane change moving localised cluster
MOE	measure of effectiveness
MPA	matrix-product ansatz
MPCF	marginal private cost function
MSA	method of successive averages
MSCF	marginal social cost function
MSP	moving synchronised-flow pattern
MT	movement time
MUC-PSD	multi-class phase-space density
NaSch	Nagel and Schreckenberg
NAVSTAR	Navigation Satellite Timing and Ranging
NCCA	number conserving cellular automaton
NSE	Navier-Stokes equations
OCT	oscillatory congested traffic
OD	origin-destination
ODE	ordinary differential equation
OSS	Open Source Software
OVF	optimal velocity function
OVM	optimal velocity model
Paramics	Parallel microscopic traffic simulator
PATH	California Partners for Advanced Transit and Highways Program on Advanced Technology for the Highway
PCE	passenger car equivalent
PCU	passenger car unit
PDE	partial differential equation
PELOPS	Program for the dEvelopment of Longitudinal micrOscopic traffic Processes in a Systemrelevant environment
PeMS	California Freeway Performance Measurement System
PHF	peak hour factor
PLC	pinned localised cluster
pMFT	paradisiacal mean-field theory
PRT	perception-reaction time
PSD	phase-space density
PW	Payne-Whitham
QoS	quality of service
SFI-TCA	stochastic Fukui-Ishibashi traffic cellular automaton
Simone	Simulation model of Motorways with Next generation vehicles
SLD	single inductive loop detector
SMARTEST	Simulation Modelling Applied to Road Transport European Scheme Tests
SMS	space-mean speed
SOC	self-organised criticality
SOMF	site-oriented mean-field theory
SP	synchronised-flow pattern
SSEP	symmetric simple exclusion process
STA	static traffic assignment
STCA	stochastic traffic cellular automaton
STCA-CC	stochastic traffic cellular automaton with cruise control
SUE	stochastic user equilibrium
SUMO	Simulation of Urban MObility
T^2-TCA	Takayasu-Takayasu traffic cellular automaton
TASEP	totally asymmetric simple exclusion process
TCA	traffic cellular automaton
TDF	travel demand function
TMC	Traffic Message Channel
TMS	time-mean speed

TOCA	time-oriented traffic cellular automaton
TRANSIMS	TRansportation ANalysis and SIMulation System
TRB	Transportation Research Board
TSG	triggered stop-and-go traffic
UDM	ultra-discretisation method
UMTS	Universal Mobile Telecommunications System
VDR-TCA	velocity-dependent randomisation traffic cellular automaton
VDT	total vehicle distance travelled
VHT	total vehicle hours travelled
VMS	variable message sign
VMT	total vehicle miles travelled
VOT	value of time
WSP	widening synchronised-flow pattern
WYA	whole year analysis

A.2 List of symbols

$a_{\max}$	the maximum acceleration in the IDM
$c(k)$	the sound speed of traffic
$C(q)$	the economical cost associated with the travel demand q
$\Delta f(x)$	the forward difference operator applied to $f(x)$
Δk	the difference in density up- and downstream of a shock wave
Δq	the difference in flow up- and downstream of a shock wave
ΔX	the width of a cell in a numerical discretisation scheme
ΔT	the size of a time step in a numerical discretisation scheme
D_j	a destination zone j
ϵ	a small diffusion constant for the viscosity ν
$g_s^*(v_i, \Delta v_i)$	the effective desired space gap in the IDM
κ	a kinetic coefficient related to τ, k, and Θ
k_t	the partial derivative of $k(t,x)$ with respect to time
k_x	the partial derivative of $k(t,x)$ with respect to space
$\widetilde{k}(t,x,\overline{v}_s)$	the phase-space density at (t,x) associated with SMS $\overline{v}_s$
$\widetilde{k}_t$	the partial derivative of $\widetilde{k}(t,x)$ with respect to time
$\widetilde{k}_x$	the partial derivative of $\widetilde{k}(t,x)$ with respect to space
λ	the sensitivity to the stimulus in a car-following model
	the arrival rate at a server in queueing theory
μ	the service rate of a server in queueing theory
$\nabla f(x)$	the backward difference operator applied to $f(x)$
	the gradient vector of $f(x)$
ν	the kinematic traffic viscosity coefficient
O_i	an origin zone i
π	the probability of overtaking (as opposed to slowing down)
P	the traffic pressure
P_x	the partial derivative of the traffic pressure with respect to space
$P(t,x,\overline{v}_s)$	the distribution of the vehicles with SMS $\overline{v}_s$ at (t,x)
q_{pc}	the practical capacity
q_{so}	travel demand associated with a system optimum
q_{ue}	travel demand associated with a user equilibrium
S	a traffic state in the human-kinetic model
τ	a driver's reaction time
Θ	the variance of the speed
$\Theta_e(k,\overline{v}_s)$	an equilibrium relation between the speed variance, the density, and the SMS
T	a travel time
	a relaxation parameter (in Pipes' car-following model)
T_{ff}	a travel time under free-flow conditions
u	the velocity (in the context of a Navier-Stokes fluid)
v_{des}	the desired speed of drivers
$\overline{v}_{s_t}$	the partial derivative of the space-mean speed with respect to time
$\overline{v}_{s_x}$	the partial derivative of the space-mean speed with respect to space
$\overline{v}_{s_e}(k,\Theta)$	an equilibrium relation between the SMS, the density, and the speed variance
$V()$	the optimal velocity function
w_{shock}	the speed of a shock wave

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