Echo cancellation for Discrete Multitone Frame-Asynchronous ADSL Transceivers

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***Abstract*—In the past, several papers have reported on echo canceler (EC) structures developed for discrete multitone transmission (DMT) based asymmetric digital subscriber lines (ADSL) transmission. The most commonly known scheme, developed by Ho, Cioffi and Bingham, is based on canceling the received echo noise efficiently in time as well as in the frequency domain. The time domain processing part, which is commonly called Cyclic Echo Synthesis (CES), can be minimized by choosing the optimal temporal alignment between the echo transmitter and the EC in the modem. This paper improves previously published work by minimizing the CES-complexity without choosing a specific temporal alignment. Moreover, the obtained structure is highly suitable for making the update process of the echo canceler coefficients independent of the far end signal, leading to improved convergence behavior.**

I. Introduction

In ADSL, modulation is performed by means of discrete multi tone modulation (DMT) in which the available bandwidth is divided into numerous subchannels or tones using a fast Fourier transform (FFT). Typically, the subchannels are assigned to one or both of the two transmission directions: upstream and downstream. Signals in opposite directions have to be separated using *duplexing schemes*. Since duplexing is performed over one twisted-pair, the ADSL standard allows two distinct duplexing methods: frequency division duplexing (FDD) and echo canceling [1].

As transmission in both directions takes place over the same loop, the transmitter and receiver at one end are coupled to the line by a *hybrid*. A perfectly balanced hybrid prevents leakage of transmitted signals into the receiver. However, due to large variations in the subscriber loops, a fixed hybrid can not be exactly balanced for all loops and hence leakage occurs. This leakage is called *echo*.

The existing ADSL standard allows bidirectional communication over one twisted-pair using *echo cancellation* to separate up- and downstream channels. If properly designed, echo cancellation can improve the reach and/or noise margin of an ADSL system by allowing both up- and downstream signals to share the low frequency portion of the available frequency band.

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Several echo cancellation structures for DMT transceivers have been studied in literature [2][3][4][5]. All the proposed structures exploit a common principle: the echo channel is estimated through an adaptive updating process and an emulated version of the echo is subtracted from the received signal. In [2], the *emulation* is performed in time domain, while the *updating* process is mainly executed in the frequency domain. Ho *et al.* modified this scheme in [3] by exploiting the 'circular' aspects of the DMT line code. The resulting structure is a structure which performs echo canceling partially in time and in frequency domain. The time domain processing element comprises a cyclic echo synthesizer (CES) to produce an echo signal that appears to have come from a periodic transmit signal. The resulting echo signal can then be cancelled with low complexity in the frequency domain.

In [6], the author investigated *the CES complexity as a function of the frame misalignment* between the transmitted echo reference symbols and the received echo symbols and calculated the optimal alignment that results in the minimal CES complexity. In this way, an ADSL modem can set its alignment between its transmit and receive part to obtain a reduction in the CES complexity with a factor two, compared to the zero misalignment assumed in [3]. Since only one modem of the transmission system can choose its alignment, an extra FFT-IFFT pair is added at the *receiver* of the other modem to allow for the transmitter/EC misalignment to be set independently of the misalignment between transmitter and receiver [6]. The receive FFT in central office (CO) is typically much smaller than in remote terminal (RT). Hence, in the resulting transmission setup the modem at RT selects its optimal alignment while the extra FFT-processing at CO certifies also there a minimal CES complexity.

In [7][8], the authors independently presented a practical way to remove the far end signal in the update process of the EC, usually referred to as *double talk cancellation*. The advantage of double talk cancellation is mainly improved convergence. The cheapest possible double talk cancellation is obtained when the FFT at the receive side of the ADSL modem is shared between the EC and the far end demodulator [7][8]. This is not the case at CO in [3][6] where an extra FFT-IFFT pair is used for the EC, as mentioned earlier. This makes the EC at CO not practical for cheap double talk cancellation.

In this paper, we present a new technique to *minimize the CES*

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complexity without imposing a specific temporal alignment. The proposed scheme requires only one extra FFT instead of the FFT/IFFT pair of [6], this time at the *transmit* side of the modem. Since the receive FFT of the modem remains shared for demodulating the far end signal and performing echo cancellation (both at CO and RT), double talk cancellation is straightforward to add, leading to increased convergence speed at total minimal complexity of the overall EC.

The paper is organized as follows. In section II, notation is introduced and the new EC structure is presented for symmetric rate and multi rate setups. Complexity numbers are given in section III. Section IV contains simulation results and we conclude with section V.

II. Frame-Asynchronous Echo Cancellation

A. Symmetric Setup

Initially, assume a symmetric ADSL setup with FFT/IFFT size and cyclic prefix equal to N and ν respectively. The received echo is modeled as a linear convolution of a finite impulse response (FIR) echo channel, $\mathbf{h}_E$, with cyclically prefixed echo reference symbols[1] $\mathbf{u}^k$, where k is the symbol index. We assume that a time domain equalizer (TEQ), which is commonly used in DSL systems to reduce inter-symbol-interference and inter-carrier-interference of the far end signal ([5] and references therein), is inserted in front of the echo canceler. The echo channel $\mathbf{h}_E$ is assumed to be of length $M \leq N$ and is zero padded to length N. It contains the influence of the analog and digital front end filters, the hybrid circuitry and the TEQ.

When the echo reference symbols are misaligned with respect to the received symbols by a delay Δ, the echo channel output of size N – as 'seen' by the receive FFT – can be written as a function of samples u_j^k originating from three consecutive echo reference symbols $\mathbf{u}^{k-1}$, $\mathbf{u}^k$ and $\mathbf{u}^{k+1}$ [6], i.e.

$$\mathbf{y}^k = \mathbf{U}^{k-1,k,k+1}\mathbf{h}_E + \mathbf{n}^k \tag{1}$$

where $\mathbf{n}^k$ contains the received far end signal plus additive Gaussian noise and $\mathbf{U}^{k-1,k,k+1}$ is an $N \times N$ Toeplitz matrix with first column and first row equal to $[u^k_{\Delta+1} \dots u^k_N u^{k+1}_{N-\nu+1} \dots u^{k+1}_N u^{k+1}_1 \dots u^{k+1}_{\Delta-\nu}]^{\mathrm{T}}$ and $[u^k_{\Delta+1} \dots u^k_1 u^k_N \dots u^k_{N-\nu+1} u^{k-1}_N \dots u^{k-1}_{\Delta+\nu+2}]$ respectively, with $\{\cdot\}^{\mathrm{T}}$ denoting transpose. For restrictions on Δ for this '3-symbol'-model to be valid, see [6]. In [6] a circulant matrix $\mathbf{C}^k$ is defined as an $N \times N$ matrix with first column equal to $[u^k_{\Delta+1} \dots u^k_N u^k_1 \dots u^k_\Delta]^{\mathrm{T}}$ such that the echo cancelled output after the demodulating FFT can be written as

$$E^k = \mathcal{F}_N(\mathbf{y}^k - (\mathbf{U}^{k-1,k,k+1} - \mathbf{C}^k + \mathbf{C}^k)\mathbf{w}^k_E) \tag{2}$$

$$= \mathcal{F}_N \underbrace{(\mathbf{y}^k - \chi^{k-1,k,k+1}\mathbf{w}^k_E)}_{\text{CES}} \underbrace{-\mathrm{diag}(U^k)\mathrm{diag}(\mathbf{p})W^k_E}_{\text{Freq. Domain EC}} \tag{3}$$

where $\mathcal{F}_N$ represents the N-points DFT-matrix shared between the EC and the far end demodulator, $\mathbf{w}^k_E$ are the EC coefficients, $W^k_E = \mathcal{F}_N\mathbf{w}^k_E$, $U^k = \mathcal{F}_N\mathbf{u}^k$ are the transmitted echo frequency domain symbols at instance k and vector $\mathbf{p} = [1 \dots e^{j2\pi\frac{i-1}{N}\Delta} \dots e^{j2\pi\frac{N-1}{N}\Delta}]^{\mathrm{T}}$ contains the phase shifts for all tones i due to the alignment difference. The matrix $\chi^{k-1,k,k+1}$ is defined as

$$\chi^{k-1,k,k+1} = \mathbf{U}^{k-1,k,k+1} - \mathbf{C}^k \tag{4}$$

and is typically composed of two triangles of non zero elements. The first term in (3) represents the CES in the time domain and the second term cancels the remaining echo efficiently in the frequency domain.

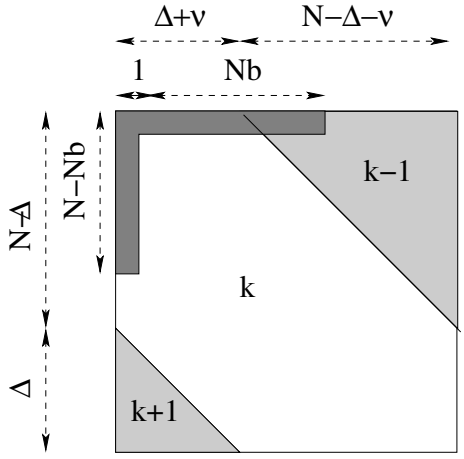


Fig. 1. Illustration of the $N \times N$ Toeplitz matrix $\mathbf{U}^{k-1,k,k+1}$ and the choice of N_b.

In [4] it is pointed out that the extra multiplications due to the phase shift $\mathbf{p}$ can be avoided by redefining the frequency domain filter coefficients: $\tilde{W}^k_E = \mathrm{diag}(\mathbf{p})W^k_E$. Because multiplication with a phase shift in the frequency domain is equivalent with a time domain shift, the time domain filter coefficients $\tilde{\mathbf{w}}^k_E = \mathcal{I}_N\tilde{W}^k_E$, will be shifted over Δ elements, where $\mathcal{I}_N$ denotes the N-points IDFT matrix. One needs to undo the time domain shift to obtain $\mathbf{w}^k_E$ as needed for CES.

Since the overall EC complexity is mainly determined by the CES, Δ can be optimized to minimize the number of multiply accumulates in $\chi^{k-1,k,k+1}\mathbf{w}_E$. If one modem regulates its alignment, the frame misalignment of the modem at the other side of the loop is automatically established as well. To minimize the CES complexity of the second modem also, an extra FFT/IFFT-pair must be added[2] [6]. Typically, this extra processing is appointed to CO since it requires the lowest amount of computation because of the smaller receive FFT.

Here, we propose a different split of $\mathbf{U}^{k-1,k,k+1}$ in $\tilde{\chi}^{k-1,k,k+1}$ and $\tilde{\mathbf{C}}$, where $\tilde{\mathbf{C}}$ is an $N \times N$ circulant matrix constructed with $N_b + 1$ samples from the first row and $N - N_b$ samples from the first column of $\mathbf{U}^{k-1,k,k+1}$ (see **Fig. 1**), i.e.

[1] $\mathbf{u}^k$ is composed of N consecutive time domain samples without cyclic prefix, i.e. $\mathbf{u}^k = [u^k_1 \dots u^k_N]^{\mathrm{T}}$.

[2] The IFFT is required for canceling the echo in time domain, independently of the alignment between echo and far end signal, while the FFT transforms the echo cancelled signal back to the frequency domain to obtain E^k for the EC update.

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the first column of $\tilde{\mathbf{C}}$ equals

$$\begin{aligned}\tilde{\mathbf{C}}_1 &= [u^k_{\Delta+1}\dots u^k_{N-N_b+\Delta}u^{k-1}_{N-N_b+\nu+\Delta+1}\dots u^{k-1}_N \\ &\quad u^k_{N-\nu+1}\dots u^k_N u^k_1\dots u^k_\Delta]^{\mathrm{T}}. \end{aligned} \quad (5)$$

Hence, $\tilde{\mathbf{C}}$ is constructed with N elements of the upper left corner of $\mathbf{U}^{k-1,k,k+1}$. Compared to [3] and [6], $\tilde{\mathbf{C}}$ includes elements of two different symbol instances.

The echo cancelled output can now be written as

$$E^k = \mathcal{F}_N(\mathbf{y}^k - \tilde{\chi}^{k-1,k,k+1}\mathbf{w}^k_E) - \mathrm{diag}(\tilde{U}^k)W^k_E \quad (6)$$

with $\tilde{\chi}^{k-1,k,k+1} = \mathbf{U}^{k-1,k,k+1} - \tilde{\mathbf{C}}$ and $\tilde{U}^k = \mathcal{F}_N\tilde{\mathbf{C}}_1$. Hence, an extra FFT is required at the *transmit* side of the echo reference symbols. Again, $\tilde{\chi}^{k-1,k,k+1}$ is composed of two triangles of non zero elements. Observe that N_b can be chosen such that $\tilde{\mathbf{C}}$ is constructed with samples from only one symbol. In that case, our scheme reduces to (3) making an extra FFT superfluous.

Advantages of this scheme are:

- N_b can be determined to minimize the number of operations in $\tilde{\chi}^{k-1,k,k+1}\mathbf{w}^k_E$ without choosing a specific alignment. It can be easily shown that the optimal N_b equals $\frac{M}{2}$. Hence, minimal CES complexity is always possible by choosing N_b in this way.
- The FFT at receive side is shared between the EC and the far end demodulator leading to straightforward application of double talk cancellation for the EC update process [7][8]. Now, the far end signal can easily be removed by using the slicer error instead of the FFT output in the update equations of the EC coefficients. In [7][8] it is pointed out that the advantage of double talk cancellation is faster convergence of the EC coefficients.
- The proposed scheme can be easily combined with (3): if one modem minimizes its CES complexity by choosing a specific alignment and by using a value for N_b such that $\tilde{\mathbf{C}}$ is only a function of one symbol, then no extra FFT is needed at that side. The other side can apply (6), adding only one extra FFT at transmit side. Since the RT-transmitter is typically operating at the lowest rate ($N = 64$), it is clear that adding the extra FFT at RT results in the lowest overall complexity.

Similar to [3][2] an LMS like procedure can be developed to update the EC coefficients in the frequency domain.

$$W^{k+1}_E \leftarrow W^k_E + \mu\mathrm{diag}(\tilde{U}^k)^* E^k, \quad (7)$$

with $\{\cdot\}^*$ denoting complex conjugation and μ the step size. The vector $\mathbf{w}^{k+1}_E$ is obtained by taking the IFFT of W^{k+1}_E.

Under the assumption that all tones are transmitted with equal normalized power one can prove that (7) converges in the mean if the step size μ satisfies the following condition (proof is omitted)

$$0 < \mu < 2. \quad (8)$$

In reality these assumptions are not valid. To guarantee convergence a reduced power is allocated to the unused tones, similar as in [2].

B. Multi Rate Structures

Extension of the proposed EC to multi rate structures, i.e. different transmit and receive rates, can easily be made and will only be treated graphically. For mathematical details on multi rate structures for echo canceling, we refer to [3]. **Fig. 2** and **Fig. 3** show signal flow graphs (SFG) of the proposed scheme, which are an extension of the corresponding SFGs in [3].

Fig. 2 shows the proposed EC for a typical RT setup. Due to the asymmetric bit rates in ADSL, the upstream rate is lower than the downstream rate. This is clearly reflected in **Fig. 2**, where the transmit IFFT is κ times smaller than the receive FFT at RT. As a result, the proposed EC exploits the multi rate structure with only one additional N-points FFT at the transmit side. The figure also shows the time and frequency domain equalizers (TEQ and FEQ) required for a correct detection of the far end signal. Typical values for RT in ADSL are $\kappa = 8$, $N = 64$ and $\nu = 4$.

Compared to [3][6], **Fig. 2** also includes double talk cancellation [7][8]. Here, the far end signal is removed by updating the EC with the slicer error instead of the output of the receive FFT (assuming correct decisions). Since the proposed EC shares the receive FFT for echo canceling and far end demodulation, the double talk cancellation requires almost no extra complexity.

At CO the transmit IFFT is κ times larger than the receive FFT, resulting in an extra FFT of κN for the EC, see **Fig. 3**. Note that the transmit IFFT size at RT is not necessarily equal to the receive FFT size at CO and vice versa. Some vendors may choose to oversample the transmit IFFT – i.e. using a larger transmit IFFT than strictly necessary – in order to realize cheaper digital and/or analog front ends. In this case, also the EC would become more complex. However, the echo reference signal can be decimated before sending it to the EC block, requiring a smaller FFT in the echo canceling path. Typical values for CO in ADSL are $\kappa = 8$, $N = 64$ and $\nu = 4$. Compared to [8], even the double talk cancellation at CO remains fairly simple, since in [8] an extra FFT and IFFT is needed at CO to remove the far end signal.

III. Complexity

Table I summarizes the complexity of three different schemes for CO and RT. Scheme I represents ordinary CES as described in [3], scheme II includes CES with optimal alignment [6] and scheme III is the proposed EC, with the assumption that CO has chosen its alignment and N_b to avoid the extra FFT. The computational complexity of the algorithms is approximated by the number of real multiplications per iteration. A distinction is made between the *echo emulation* and the *adaptation process*. For the adaptation process, the complexity of the double talk canceler is included as well.

For the double talk canceler it is assumed that no extra FFT processing is required at RT since the receiver FFT is shared between far end demodulation and EC. In schemes I and II CO requires two extra IFFTs and one extra FFT. The two IFFTs are needed to transform two consecutive blocks of slicer errors to the time domain and the extra FFT calculates the correctly aligned

	Scheme I	Scheme II	Scheme III
Emul., RT	$\frac{(M-\kappa\nu-1)^2}{2\kappa}+2\kappa N$	$\frac{(M-\kappa\nu)^2}{4\kappa}+2\kappa N$	$\frac{M^2}{4\kappa}+2\kappa N$ $+N(\log\frac{N}{2}+2)$
Adapt., RT	$5\kappa N+\frac{\kappa N(\log\frac{\kappa N}{2}+2)}{m}$	$5\kappa N+\frac{\kappa N(\log\frac{\kappa N}{2}+2)}{m}$	$5\kappa N+\frac{\kappa N(\log\frac{\kappa N}{2}+2)}{m}$
Emul., CO	$\frac{(M-1)^2}{2\kappa}+2\kappa N$ $+N(\log\frac{N}{2}+2)$	$\frac{M^2}{4\kappa}+2\kappa N$ $+N(\log\frac{N}{2}+2)$	$\frac{(M-\kappa\nu)^2}{4\kappa}+2\kappa N$
Adapt., CO	$2\kappa N+\frac{\kappa N(\log\frac{\kappa N}{2}+2)}{m}$ $+3N+3N(\log\frac{N}{2}+2)$	$2\kappa N+\frac{\kappa N(\log\frac{\kappa N}{2}+2)}{m}$ $+3N+3N(\log\frac{N}{2}+2)$	$2\kappa N+\frac{\kappa N(\log\frac{\kappa N}{2}+2)}{m}$ $+3N$

TABLE I

COMPLEXITY FIGURES FOR THE ECHO EMULATION AND EC ADAPTATION AT CO AND RT.

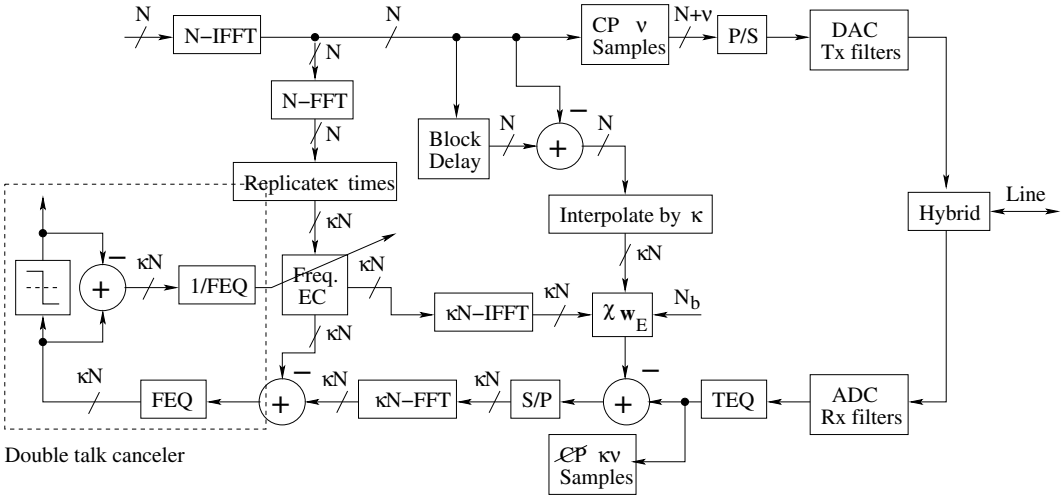


Fig. 2. Echo cancellation at RT, with time domain equalizer (TEQ) included in the echo path. Typical values for RT are $\kappa = 8$, $N = 64$ and $\nu = 4$.

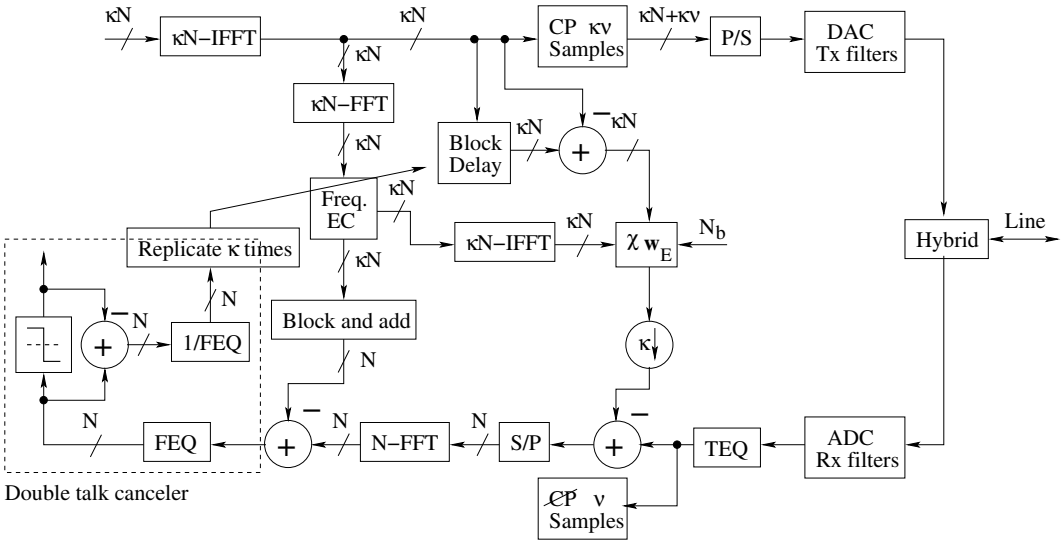


Fig. 3. Echo cancellation at CO, with time domain equalizer (TEQ) included in the echo path. Typical values for CO are $\kappa = 8$, $N = 64$ and $\nu = 4$. If the alignment and N_b are chosen such that $\tilde{\mathbf{C}}_1$ contains only elements of symbol k, the extra κN-points FFT can be omitted.

E^k for the EC adaptation [8]. The complexity of the real to complex (I)FFTs is based on [9].

In the table, M is the number of time domain EC taps, N and ν are the symbol length and cyclic prefix of the smallest side, κ is the up- or downsampling factor due to the multi rate structure of the modem and m indicates that the frequency domain

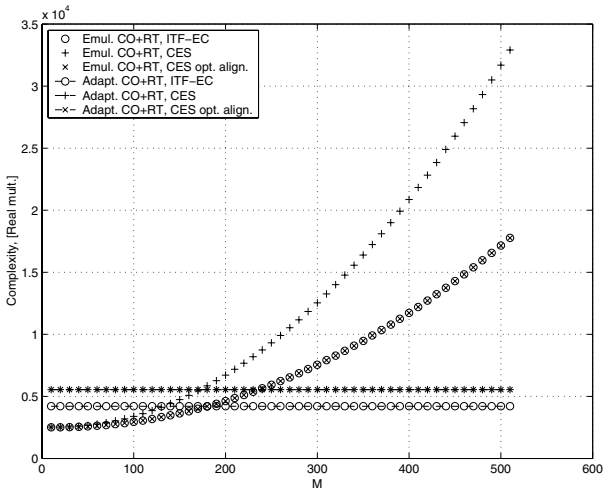


Fig. 4. Complexity numbers in real multiplications per iteration for echo emulation and adaptation.

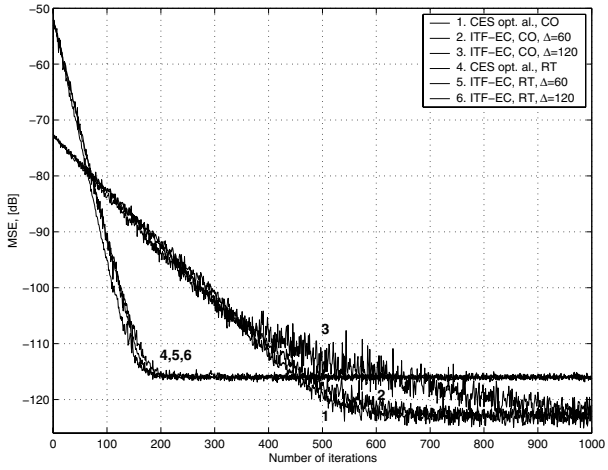


Fig. 5. Learning curves for CES with optimal alignment and CES with optimal N_b at CO and RT.

EC taps are infrequently transformed to the time domain by an IFFT [3]. The complexity of the time domain echo emulation is approximated. For complexity figures of the first and the second scheme we refer to [3] and [4].

The overall adaptation and emulation complexity of CO plus RT is depicted in **Fig. 4** as a function of M, with $\nu = 4$, $N = 64$, $\kappa = 8$ and $m = 24$. Scheme I is represented by plus-marks, scheme II by cross-marks and scheme III by circles. Observe that the proposed EC has the same complexity for echo emulation as CES with optimal alignment [6], while the overall update complexity is lower.

IV. Simulation Results

The results above are verified by simulations for an ADSL 26awg line of 3000m and are depicted in **Fig. 5**. For upstream tones 7-31 and for downstream tones 33-255 were used. Since there is no overlap between up- and downstream tones the EC is only needed to reduce echo due to DFT leakage. In these simulations, each tone transmits a 4-QAM signal constellation. The downstream and upstream signal transmit with -40 dBm/Hz and -38 dBm/Hz respectively. To ensure convergence the echo reference signal at RT and CO contains 20 dB lower power on the unused tones. The external additive noise is white Gaussian noise at -140 dBm/Hz. At RT, the transmit block length is 64 and the receive block length is 512, while at CO the opposite is valid. The true echo channel contains 512 samples at 2.2MHz, while the number of used EC taps is $M = 220$. Hence, the optimal N_b is equal to $\frac{M}{2} = 110$. The EC coefficients are updated using a normalized LMS procedure [10] and the EC taps were initialized with all zeros.

In **Fig. 5** the CES with optimal N_b (scheme III) is compared with the EC of [6] (scheme II) for RT and CO. The learning curves for two different misalignments, i.e. $\Delta = 60$ and $\Delta = 120$, are depicted for scheme III. The curves are calculated by summing the mean square error of the EC update procedure over all the tones after the receive FFT. It can be observed that after convergence the white noise floor is reached. From the simulations, it was noticed that the convergence is weakly dependent on the alignment. The convergence of the CES method with optimal N_b is almost as fast as the EC of [6].

V. Conclusions

In this paper we proposed a modification to the existing mixed time and frequency domain echo canceling for DMT based ADSL modems. In contrast to currently known EC structures, the obtained EC always achieves minimal CES complexity without choosing a specific alignment. Under certain assumptions, convergence in the mean can be assured. Since the receive FFT of the ADSL modem can be shared between the EC and the far end demodulator, double talk cancellation can easily be added, resulting in improved convergence. The resulting structure requires only one additional FFT at the transmit side of the RT modem and hence offers a cheap alternative to existing echo cancelers.

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